Monetary Policy and Wealth Effects: The Role of Risk and Heterogeneity*

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March 25, 2025

Abstract

We study the role of asset revaluation in the monetary transmission mechanism. We build an analytical heterogeneous-agents model with two main ingredients: i) rare disasters; ii) heterogeneous beliefs. The model captures time-varying risk premia and precautionary savings in a setting that nests the textbook New Keynesian model. The model generates large movements in asset prices after a monetary shock but these movements can be neutral on real variables. Real effects depend on the redistribution among agents with heterogeneous precautionary motives. In a quantitative exploration, we find that this channel can account for a large fraction of the transmission to aggregate consumption.

JEL Codes: E21, E44, E52, G12
Keywords: Monetary Policy, Wealth Effects, Asset Prices, Aggregate Risk, Heterogeneity beliefs

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Conflict-of-interest disclosure statement

Nicolas Caramp
I have nothing to disclose.

Dejanir Henrique Silva
I have nothing to disclose.

1 Introduction

A long tradition in monetary economics emphasizes the role of the revaluation of real and financial assets in shaping the economy's response to changes in monetary policy. Its importance can be traced back to both classical and Keynesian economists.[1] Keynes himself described the effects of interest rate changes as follows:

> Perhaps the most important influence, operating through changes in the rate of interest, on the readiness to spend out of a given income, *depends on the effect of these changes on the appreciation or depreciation in the price of securities and other assets*.
>
> - John M. Keynes, The General Theory of Employment, Interest, and Money (emphasis added).

These revaluation effects caused by monetary policy have been documented by an extensive empirical literature. Bernanke and Kuttner (2005) study the effects of monetary shocks on stock prices. Gertler and Karadi (2015) and Hanson and Stein (2015) consider the effects on bonds. A robust finding of this literature is that changes in asset prices are explained mainly by fluctuations in future excess returns, related to changes in the risk premia, rather than changes in the risk-free rate.[2]

The extent to which changes in asset prices play a relevant role in the transmission of monetary policy to the real economy, however, has been controversial. One view highlights the importance of *wealth effects*. Cieslak and Vissing-Jorgensen (2020) show that policymakers track the behavior of stock markets because of their impact on households' consumption, while Chodorow-Reich, Nenov and Simsek (2021) study the importance of this channel empirically. An alternative view defends that changes in asset valuations have no real implications. Cochrane (2020) and Krugman (2021) argue that movements in discount rates lead to changes in "paper wealth," without an impact on consumption.

In this paper, we study how monetary policy affects the real economy through changes

[1]The revaluation of government liabilities was central to Pigou (1943) and Patinkin (1965), while Metzler (1951) considered stocks and money. Tobin (1969) focused on the revaluation of real assets.

[2]For a recent review of this work, see Bauer and Swanson (2023).

in asset prices in a New Keynesian setting. We provide a new framework that generates rich asset-pricing dynamics and heterogeneous portfolios while preserving the simplicity of the textbook model. In particular, we propose a new solution technique that delivers time-varying risk premium and precautionary savings motive without having to resort to higher-order approximations.[3] We derive necessary conditions for changes in risk premia to affect the real economy. Under special conditions, we obtain a *risk-premium neutrality result*, where changes in risk premia caused by monetary shocks affect asset prices, but they have no effect on aggregate consumption and inflation. We identify the redistribution generated by heterogeneous portfolios revaluations among agents with different precautionary motives as the main channel through which risk premia affect the real economy. Moreover, and despite being stylized, the model captures quantitatively central aspects of the monetary transmission mechanism, including the term premium, the equity premium, and corporate spreads, as well as the differential responses of borrowers and savers to monetary shocks observed in the data. We then use the model to assess the quantitative importance of the risk-premium channel and find that changes in risk premia can account for a large fraction of the response of aggregate consumption and inflation to changes in monetary policy in our model.

We consider an economy populated by workers and savers with two main ingredients: i) rare disasters, and ii) heterogeneous beliefs. Rare disasters enable us to capture both a precautionary savings motive and realistic risk premia.[4] Savers invest in stocks, long-term government bonds, and short-term debt, and have heterogeneous beliefs, as in Caballero and Simsek (2020).[5] This has two consequences. First, they hold heteroge-

[3]As shown in e.g. Schmitt-Grohé and Uribe (2004), a standard perturbation around the non-stochastic steady state can only generate time-varying risk premia with at least a third-order approximation.

[4]Rare disasters have been widely used to explain a range of asset-pricing "puzzles"; see Tsai and Wachter (2015) for a review.

[5]For recent evidence from bond returns consistent with belief heterogeneity, see Bauer and Chernov (2023). A large literature on asset pricing studies models with heterogeneous beliefs, see e.g. Detemple and Murthy (1994), Basak (2005), and Atmaz and Basak (2018).

neous portfolios in equilibrium. Second, they have heterogeneous marginal propensities to consume (MPCs) out of changes in wealth due to different precautionary motives. This generates *time-variation* in risk premia in response to monetary shocks.

Our first contribution is methodological and consists of an aggregation result. Given investor heterogeneity, we must characterize not only the dynamics of aggregate consumption and inflation, but also the behavior of portfolios, asset prices, and individual consumption. This increases the dimensionality of the problem and typically makes deriving analytical results infeasible. We show that our economy satisfies an *as if* result: the economy with heterogeneous savers behaves as an economy with a representative saver, but the probability of disaster, as implied by market prices, is time-varying and responds to monetary policy. This *market-implied disaster probability* is a key determinant of asset prices, and it is the main channel through which investor heterogeneity affects the real economy.

Our second contribution identifies conditions under which time-varying risk premia plays a role in the monetary transmission mechanism. Consistent with the evidence, a contractionary monetary shock leads to an increase in risk premia and a reduction in the price of risky assets. One could then conclude that this reduction in households' wealth leads to a reduction in consumption. However, as the discount rate increases, the amount of wealth required to finance the same amount of consumption also decreases. The net effect of changes in risk premium is ambiguous and depends on whether households are net buyers or net sellers of risky assets. As recently articulated by Cochrane (2020) and Krugman (2021), a household who consumes the dividends from their financial assets can still afford the same level of consumption after a change in discount rates.

Formally, we show that the aggregate wealth effect corresponds to the sum of all households' wealth *net* of the change in the cost of the original consumption bundle. Interestingly, the aggregate wealth effect does not depend on the equity premium. Movements in equity prices redistribute wealth among investors but do not generate gains or losses for the household sector as a whole. In a closed economy, the government is the only counterpart to the household sector, so the aggregate wealth effect depends on the revaluation of government bonds and the amount of trading in these bonds.

Risk also affects the households' precautionary motive, given the redistribution among savers after a monetary shock. Because optimists hold a larger fraction of their wealth in risky assets, an increase in the interest rate disproportionately reduces their wealth. Holding the *aggregate* wealth effect constant, this redistribution of wealth is then reflected in the market-implied probability of disaster, which increases after the monetary shock. This is the "as-if" result in action: redistribution between optimists and pessimists is akin to an increase in the objective probability of disaster risk in a representative-agent model.

We perform next a quantitative exploration of the importance of risk and heterogeneity for the transmission of monetary shocks to the real economy. While the model lacks some important dimensions present in state-of-the-art quantitative HANK models (e.g., rich MPC heterogeneity), this exercise is useful to get a first evaluation of the economic relevance of these channels. We find that the time-varying precautionary motive accounts for roughly 60% of the response of aggregate consumption on impact, while the response coming from the aggregate wealth effect accounts for roughly 30% of the overall consumption response. The intertemporal-substitution effect accounts for less than 10% of the response of aggregate consumption on impact. Heterogeneous beliefs are crucial for this result. The response of consumption in the economy with heterogeneous beliefs is more than three times larger than in the economy with homogeneous beliefs. Finally, we introduce long-term defaultable household debt and find that it amplifies the response of aggregate consumption. Hence, risk and heterogeneity play a large role in how monetary policy affects the real economy.

Literature review. Wealth effects have a long tradition in monetary economics. Pigou (1943) relied on a wealth effect to argue that full employment could be reached even in a liquidity trap. Kalecki (1944) argued that these effects apply only to government liabilities, as inside assets cancel out in the aggregate, while Tobin highlighted the role of private assets and high-MPC borrowers. Recently, wealth effects have regained relevance. Kaplan, Moll and Violante (2018) build a quantitative HANK model and find only a minor role for the standard intertemporal-substitution channel, leading the way to a more important role for wealth effects. Much of the literature has focused on the role of heterogeneous marginal propensities to consume (MPCs) in settings with idiosyncratic income risk. Instead, we focus on aggregate risk and heterogeneous portfolios.

Our work is closely related to two strands of literature. First, it is related to work on the interaction between monetary policy and changes in asset prices, including models with sticky prices, such as Caballero and Simsek (2020), and models with financial frictions, such as Brunnermeier and Sannikov (2016) and Drechsler, Savov and Schnabl (2018).[6] In a recent contribution, Kekre and Lenel (2022) consider the role of the marginal propensity to take risk in determining the risk premium and shaping the response of the economy to monetary policy. Kekre, Lenel and Mainardi (2023) consider the role of market segmentation in the determination of the term premium. We contribute to this literature by presenting an analytical framework that features aggregate risk and generates a sizable time-varying risk premium while preserving the tractability of standard New Keynesian models. Also related is Campbell, Pflueger and Viceira (2020) and Pflueger and Rinaldi (2022), which use a habit model to study the role of monetary policy in determining bond and equity premia. Their models generate an exact log-linear Euler equation that is independent of risk, which implies that aggregate consumption and inflation are also independent of risk, consistent with our risk-premium neutrality result. In contrast, aggregate risk, through the precautionary motives they generate, are a crucial channel of transmission in our model.

[6] Our work is also related to the literature on unconventional monetary policy and asset prices, see e.g. Silva (2020), Caballero and Simsek (2021), and Corhay, Kind, Kung and Morales (2023).

The paper is also closely related to the analytical HANK literature, such as Werning (2015) and Debortoli and Galí (2017). While this literature focuses primarily on how the cyclicality of income interacts with differences in MPCs, we focus instead on how heterogeneous asset positions interact with differences in MPCs. As e.g. Eggertsson and Krugman (2012), we consider the role of household debt, but they abstract from risk and focus instead on the implications of deleveraging. Iacoviello (2005) considers a monetary economy with private debt but focuses instead on housing as collateral. Our work is also related to Auclert (2019), which studies the redistribution channel of monetary policy arising from portfolio heterogeneity. Our paper emphasizes the redistribution channel in the context of a general equilibrium setting with aggregate risk.

Finally, a literature studies rare disasters and business cycles. Gabaix (2011) and Gourio (2012) consider a real business cycle model with rare disasters, while Andreasen (2012) and Isoré and Szczerbowicz (2017) allow for sticky prices. They focus on changes in disaster probability while we study monetary shocks in a heterogeneous-agent model.

2 D-HANK: A Rare Disasters Analytical HANK Model

In this section, we consider an analytical HANK model with two main ingredients: i) the possibility of rare disasters, and ii) heterogeneous beliefs.

2.1 The Model

Environment. Time is continuous and denoted by $t \in \mathbb{R}_+$. The economy is populated by households, firms, and a government. There is a continuum of households that can be of three types: *workers*, *optimistic savers*, and *pessimistic savers* (denoted by w, o and p, respectively), who differ in their discount rates and beliefs about the probability of the economy being hit by an aggregate shock. We let $\mu_j \geq 0$ denote the mass of households of type $j \in \{w, o, p\}$, where $\mu_b + \mu_o + \mu_p = 1$. Households can borrow or lend at a riskless rate subject to a borrowing constraint, and they can save on long-term nominal government bonds and corporate equity. In this section, we assume that the borrowing limit is zero. We study the case of a positive borrowing limit and defaultable long-term household debt in Section 5. Workers are the only ones who supply labor, and they are relatively impatient, so their borrowing constraint is binding in equilibrium.

Firms can produce final or intermediate goods. Final-goods producers operate competitively and combine intermediate goods using a CES aggregator with elasticity $\epsilon > 1$. Intermediate-goods producers use labor as their only input and face quadratic (Rotemberg, 1982) pricing adjustment costs. Intermediate-goods producers are subject to an aggregate productivity shock: with Poisson intensity $\overline{\lambda} \geq 0$, their productivity is permanently reduced. This shock captures the possibility of rare disasters: low-probability, large drops in productivity and output, as in the work of Barro (2006, 2009). Periods that predate the realization of the shock are in the *no-disaster state*, and periods that follow the shock are in the *disaster state*. The disaster state is absorbing, and there are no further shocks after the disaster is realized.[7]

The government sets fiscal policy, comprising of transfers to workers and savers, and monetary policy, specified by an interest rate rule subject to monetary shocks.

Savers' problem. Savers face a portfolio problem where they choose how much to invest in short-term bonds, long-term bonds, and corporate equity.

A long-term bond issued in period t trades at a nominal market price $Q_{L,t}$ in the no-disaster state and promises to pay coupons $e^{-\psi_L(s-t)}$ at all dates $s \geq t$. Because of the structure of the coupon payments, the prices of the bonds issued at previous dates are proportional to new issues, i.e. a bond issued in $t - z$ trades at $Q_{L,t}e^{-\psi_L z}$ in period t. The

[7] Assuming an absorbing disaster state simplifies the presentation, but it is not essential for our results. Allowing for partial recovery, as in e.g. Barro, Nakamura, Steinsson and Ursúa (2013), introduces dynamics in the disaster state, but it does not change the implications for the no-disaster state, which is our focus.

rate of decay ψ_L is inversely related to the bond's duration, where a consol corresponds to $\psi_L = 0$ and the limit $\psi_L \to \infty$ corresponds to the case of short-term bonds. We denote by $Q^*_{L,t}$ the price of the bond in the disaster state, where the star superscript is used throughout the paper to denote variables in the disaster state. Then, the nominal return on the long-term bond is given by

$$dR_{L,t} = \left[\frac{1}{Q_{L,t}} + \frac{\dot{Q}_{L,t}}{Q_{L,t}} - \psi_L\right] dt + \frac{Q^*_{L,t} - Q_{L,t}}{Q_{L,t}} d\mathcal{N}_t,$$

where $\mathcal{N}_t$ is a Poisson process with arrival rate $\overline{\lambda}$ (under the objective measure).

The price of a claim on real aggregate corporate profits is denoted by $Q_{E,t}$ and the real return on equities evolves according to

$$dR_{E,t} = \left[\frac{\Pi_t}{Q_{E,t}} + \frac{\dot{Q}_{E,t}}{Q_{E,t}}\right] dt + \frac{Q^*_{E,t} - Q_{E,t}}{Q_{E,t}} d\mathcal{N}_t,$$

where Π_t denotes real profits and $Q^*_{E,t}$ is the equity price in the disaster state.

Savers have heterogeneous beliefs regarding the probability of a disaster. Subjective beliefs about the arrival rate of the aggregate productivity shock are given by λ_j, for $j \in \{o, p\}$, where $\lambda_o \leq \lambda_p$. We follow Chen, Joslin and Tran (2012) and assume that savers are dogmatic in their beliefs about disaster risk, so we abstract from any learning process.

Savers' subjective discount rate is a function of their consumption share, $\rho_{j,t} = \rho_j\left(\frac{C_{j,t}}{C_{s,t}}\right)$, where $C_{s,t} = \frac{\mu_o}{\mu_o+\mu_p}C_{o,t} + \frac{\mu_p}{\mu_o+\mu_p}C_{p,t}$ denotes savers' aggregate consumption. Following Schmitt-Grohé and Uribe (2003), we assume that $\rho_j(\cdot)$ depends on the average consumption of type-j savers, so it is taken as given by any individual saver. This formulation, a form of Uzawa (1968) preferences, implies that there is a unique stationary wealth distribution, but it is otherwise not central to our results.

Let $B_{j,t} = B^S_{j,t} + B^L_{j,t} + B^E_{j,t}$ denote the net worth of a type-j saver, the sum of short-term bonds $B^S_{j,t}$, long-term bonds $B^L_{j,t}$, and equity holdings $B^E_{j,t}$. A type-j saver chooses consumption $C_{j,t}$, long-term bonds $B^L_{j,t}$, and equity holdings $B^E_{j,t}$, given an initial net worth $B_{j,t} > 0$, to solve the following problem:

$$V_{j,t}(B_{j,t}) = \max_{[C_{j,z}, B^L_{j,z}, B^E_{j,z}]_{z \geq t}} \mathbb{E}_{j,t}\left[\int_t^{t^*} e^{-\int_t^z \rho_{j,u} du} \frac{C_{j,z}^{1-\sigma}}{1-\sigma} dz + e^{-\int_t^{t^*} \rho_{j,u} du} V^*_{j,t^*}(B^*_{j,t^*})\right],$$

subject to the flow budget constraint

$$dB_{j,t} = \left[(i_t - \pi_t)B_{j,t} + r_{L,t}B^L_{j,t} + r_{E,t}B^E_{j,t} + T_{j,t} - C_{j,t}\right]dt + \left[B^*_{j,t} - B_{j,t}\right]d\mathcal{N}_t,$$

and borrowing constraint $B_{j,t} \geq 0$, given $B_{j,0} > 0$, where $B^*_{j,t} = B_{j,t} + B^L_{j,t}\frac{Q^*_{L,t} - Q_{L,t}}{Q_{L,t}} + B^E_{j,t}\frac{Q^*_{E,t} - Q_{E,t}}{Q_{E,t}}$ denotes savers' net worth after the disaster is realized, i_t is the nominal interest rate, π_t is the inflation rate, $r_{L,t} \equiv \frac{1}{Q_{L,t}} + \frac{\dot{Q}_{L,t}}{Q_{L,t}} - \psi_L - i_t$ is the excess return on long-term bonds conditional on no disasters, $r_{E,t} \equiv \frac{\Pi_t}{Q_{E,t}} + \frac{\dot{Q}_{E,t}}{Q_{E,t}} - (i_t - \pi_t)$ is the excess return on equities conditional on no disasters, and $T_{j,t}$ denotes government transfers. The random arrival time t^* represents the period in which the aggregate shock hits the economy. V^*_{j,t^*} denotes the value function in the disaster state. The savers' problem in the disaster state corresponds to a deterministic version of the problem above. The non-negativity constraint on $B_{j,t}$ captures the assumption that households cannot borrow on net.

The savers' Euler equation for short-term bonds is given by

$$\frac{\dot{C}_{j,t}}{C_{j,t}} = \sigma^{-1}(i_t - \pi_t - \rho_{j,t}) + \frac{\lambda_j}{\sigma}\left[\left(\frac{C_{j,t}}{C^*_{j,t}}\right)^{\sigma} - 1\right], \tag{1}$$

where $C^*_{j,t}$ is the consumption of a type-j saver in the disaster state. The first term captures the usual intertemporal-substitution force present in RANK models. The second term captures the *precautionary savings motive* generated by the disaster risk, and it is analogous to the precautionary motive that emerges in HANK models with idiosyncratic risk.

The Euler equation for long-term bonds is given by

$$r_{L,t} = \underbrace{\lambda_j \left(\frac{C_{j,t}}{C^*_{j,t}}\right)^{\sigma}}_{\substack{\text{price of} \\ \text{disaster risk}}} \underbrace{\frac{Q_{L,t} - Q^*_{L,t}}{Q_{L,t}}}_{\substack{\text{quantity of} \\ \text{risk}}}. \tag{2}$$

This expression captures a risk premium on long-term bonds, which pins down long-term interest rates in equilibrium. The premium on long-term bonds is given by the product of the *price of disaster risk*, the compensation for a unit exposure to the risk factor, and the *quantity of risk*, the loss the asset suffers conditional on switching to the disaster state. Similarly, the Euler equation for equities is given by

$$r_{E,t} = \lambda_j \left(\frac{C_{j,t}}{C^*_{j,t}}\right)^{\sigma} \frac{Q_{E,t} - Q^*_{E,t}}{Q_{E,t}}. \tag{3}$$

The expression above pins down the (conditional) equity premium. Note that differences in the quantity of risk drive the differences in expected returns between stocks and bonds.

Workers' problem. Workers supply labor and have GHH preferences (Greenwood, Hercowitz and Huffman, 1988) over consumption and labor. Their problem is given by

$$V_{w,t}(B_{w,t}) = \max_{[C_{w,z}, N_{w,z}]_{z \geq t}} \mathbb{E}_{w,t} \left[\int_t^{t^*} \frac{e^{-\rho_w (z-t)}}{1-\sigma} \left(C_{w,z} - \frac{N_{w,z}^{1+\phi}}{1+\phi} \right)^{1-\sigma} dz + e^{-\rho_w (t^* - t)} V^*_{w,t^*}(B_{w,t^*}) \right],$$

subject to $\dot{B}_{w,t} = (i_t - \pi_t) B_{w,t} + \frac{W_t}{P_t} N_{w,t} + T_{w,t} - C_{w,t}$, and the borrowing constraint $B_{w,t} \geq 0$, where W_t is the nominal wage, P_t is the price level, and $T_{w,t}$ denote fiscal transfers.

We focus on the case where $B_{w,0} = 0$ and ρ_b is sufficiently large, so workers are constrained at all periods.[8] As workers are constrained, their beliefs about the disaster prob-

[8]In Appendix D.1, we introduce "wealthy hand-to-mouth" households into the model. We show that

ability play no role in the determination of equilibrium. The labor supply is determined by $\frac{W_t}{P_t} = N_{w,t}^{\phi}$. GHH preferences imply that there is no income effect on labor supply, roughly in line with the evidence (see e.g. Auclert, Bardóczy and Rognlie, 2021).[9]

Market-implied probabilities and the SDF. From equations (2) and (3), we can see that, even though savers disagree on the probability of a disaster, they agree on the *value* of a unit of consumption in that state.[10] We can then price any cash flow using the beliefs and marginal utility of either optimistic or pessimistic savers. Instead of using the beliefs of a specific saver, it is convenient to define the economy's stochastic discount factor (SDF) using the aggregate consumption of savers, and the corresponding disaster probability implied by asset prices, as shown in Proposition 1. Proofs omitted in the text are provided in the appendix.

Proposition 1 (Market-implied disaster probability)**.** *Define the market-implied disaster probability λ_t as follows:*

$$\lambda_t \equiv \left[\frac{\mu_o C_{o,t}}{\mu_o C_{o,t} + \mu_p C_{p,t}} \lambda_o^{\frac{1}{\sigma}} + \frac{\mu_p C_{p,t}}{\mu_o C_{o,t} + \mu_p C_{p,t}} \lambda_p^{\frac{1}{\sigma}}\right]^{\sigma}, \tag{4}$$

and let $\mathbb{E}_t[\cdot]$ denote the expectation operator associated with the arrival rate λ_t for the disaster shock. Then, $\eta_t = e^{-\int_0^t \rho_{s,z} dz} C_{s,t}^{-\sigma}$ is a valid stochastic discount factor, i.e., η_t correctly prices all tradeable assets given the disaster probability λ_t and an appropriately chosen process for $\rho_{s,t}$.

The market-implied probability λ_t is a CES aggregator of individual probabilities, weighted by the corresponding consumption share. Expression (4) is reminiscent of the

our results hold in this case with two types of constrained agents.

[9]GHH preferences avoid the counterfactual implications caused by income effects on labor supply in sticky-price heterogeneous-agent models emphasized by Broer, Harbo Hansen, Krusell and Öberg (2020).

[10]The value of a consumption unit in the disaster state for saver j is $\lambda_j (C_{j,t}^*/C_{j,t})^{-\sigma}$, the continuous-time version of the standard expression for state prices, which is equalized for all savers from equations (2)-(3).

complete-markets formula with heterogeneous beliefs in Varian (1985).[11] In our setting, consumption shares can potentially move over time, which leads to endogenous time-variation in the perceived probability of a disaster. We can then price assets as-if the economy has a representative saver with (endogenous) time-varying beliefs.

Firms' problem. Intermediate-goods producers are indexed by $i \in [0,1]$ and operate in monopolistically competitive markets. Final good producers are price takers and combine intermediate goods to produce the final good. Their demand for variety i is given by $Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} Y_t$, and the equilibrium price level is given by $P_t = \left(\int_0^1 P_{i,t}^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}$.

Intermediate-goods producers operate the linear technology $Y_{i,t} = A_t N_{i,t}$. Productivity in the no-disaster state is given by $A_t = A$, and productivity in the disaster state is given by $A_t = A^*$, where $0 < A^* < A$. Intermediate-goods producers choose $\pi_{i,t} = \dot{P}_{i,t}/P_{i,t}$, given the initial price $P_{i,0}$, to maximize the expected discounted value of real profits subject to Rotemberg quadratic adjustment costs.[12] These costs are rebated back to shareholders, so they do not represent real resource costs. The optimality condition for the firms' problem delivers the non-linear New Keynesian Phillips curve (NKPC):

$$\dot{\pi}_t = \left(i_t - \pi_t + \lambda_t \frac{\eta_t^*}{\eta_t}\right) \pi_t - \frac{\epsilon}{\varphi A}\left(\frac{W_t}{P_t} - (1-\epsilon^{-1})A\right) Y_t, \tag{5}$$

assuming a symmetric initial condition $P_{i,0} = P_0$, for all $i \in [0,1]$, and $\pi_{i,t}^* = 0$.

Government. The government is subject to a flow budget constraint

$$\dot{D}_{G,t} = (i_t - \pi_t + r_{L,t}) D_{G,t} + \sum_{j \in \{w,o,p\}} \mu_j T_{j,t},$$

[11] For a discussion of similar aggregation results in heterogeneous-agents asset pricing models, see Panageas (2020).

[12] For a version of the model with sticky wages, see Appendix D.2.

and a No-Ponzi condition $\lim_{t\to\infty} \mathbb{E}_0[\eta_t D_{G,t}] \leq 0$, where $D_{G,t}$ denotes the real value of government debt, $D_{G,0} = D_G$ is given, and analogous conditions hold in the disaster state. Transfers to workers are given by the policy rule $T_{w,t} = T_w(Y_t)$. We assume $T_{o,t} = T_{p,t}$, and the government adjusts transfers to savers such that the No-Ponzi condition is satisfied.

In the no-disaster state, monetary policy is determined by the policy rule

$$i_t = r_n + \phi_\pi \pi_t + u_t, \tag{6}$$

where $\phi_\pi > 1$, u_t is a monetary shock, and r_n denotes the real rate when $\pi_t = u_t = 0$ at all periods. We assume that in the disaster state there are no monetary shocks, that is, $i_t^* = r_n^* + \phi_\pi \pi_t^*$. By abstracting from the policy response after a disaster, we isolate the impact of changes in monetary policy during "normal times."

Market clearing. The market-clearing conditions are given by

$$\sum_{j\in\{w,o,p\}} \mu_j C_{j,t} = Y_t, \quad \sum_{j\in\{w,o,p\}} \mu_j B_{j,t}^S = 0, \quad \sum_{j\in\{w,o,p\}} \mu_j B_{j,t}^L = D_{G,t}, \quad \sum_{j\in\{w,o,p\}} \mu_j B_{j,t}^E = Q_{E,t},$$

and $\mu_w N_{w,t} = N_t$, where $Y_t = \left(\int_0^1 Y_{i,t}^{\frac{\epsilon}{\epsilon-1}} di\right)^{\frac{\epsilon-1}{\epsilon}}$ and $N_t = \int_0^1 N_{i,t} di$.

2.2 Equilibrium dynamics

Stationary equilibrium. We define a stationary equilibrium as an equilibrium in which all variables are constant in each aggregate state. The economy will be in a stationary equilibrium in the absence of monetary shocks, that is, $u_t = 0$ for all $t \geq 0$. Since variables are constant in each state, we drop time subscripts and write, for instance, $C_{j,t} = C_j$ and $C_{j,t}^* = C_j^*$. For ease of exposition, we follow Bilbiie (2018) and assume that T_w implements $C_w = Y$ and $C_w^* = Y^*$, and a symmetric allocation in the disaster state: $C_w^* = C_o^* = C_p^*$.

We discuss a more general case in Appendix A.

The natural interest rate, the real rate in the stationary equilibrium, is given by

$$r_n = \rho_s - \lambda \left[\left(\frac{C_s}{C_s^*} \right)^{\sigma} - 1 \right],$$

where ρ_s and λ are the values of $\rho_{s,t}$ and λ_t in the stationary equilibrium, and $0 < C_s^* < C_s$. We assume that the natural rate is positive, $r_n > 0$. The precautionary motive depresses the natural interest rate relative to the one that would prevail in a non-stochastic economy.

In a stationary equilibrium where both types of savers are unconstrained, the following condition must hold $\rho_o + \lambda_o = \rho_p + \lambda_p$. As ρ_j depends on the consumption share, this condition pins down the stationary-equilibrium consumption and wealth distributions. For simplicity, we assume that this equality holds when both types have the same net worth, i.e, $B_o = B_p$, which implies $C_o > C_p$.

From equation (2), we can pin down the term spread, the difference between the yield on the long-term bond and the short-term rate, which is given by $r_L = \lambda \left(\frac{C_s}{C_s^*} \right)^{\sigma} \frac{Q_L - Q_L^*}{Q_L}$, and $Q_L^* < Q_L$. It can be shown that $r_L = i_L - r_n$, where $i_L = Q_L^{-1} - \psi_L$ is the yield on the long-term bond. Thus, our model generates an upward-sloping yield curve, where long-term yields exceed the short rate, consistent with the data.[13] Similarly, the equity premium (conditional on no-disaster) is given by $r_E = \lambda \left(\frac{C_s}{C_s^*} \right)^{\sigma} \frac{Q_E - Q_E^*}{Q_E}$, and $Q_E^* < Q_E$.[14] Therefore, the equity premium is positive in the stationary equilibrium.

Households have heterogeneous portfolios in equilibrium. Workers are against the borrowing constraint and hold no equities or long-term bonds. Optimistic savers are more exposed to disaster risk than pessimistic investors. The exact composition of their portfolio is indeterminate, as we have one redundant asset. For concreteness, we focus

[13]The upward-sloping yield curve is caused by the lack of precautionary savings in the disaster state. We would obtain similar results by introducing expropriation and inflation in a disaster, as in Barro (2006).

[14]The unconditional equity premium equals r_E minus the expected loss on a disaster. Using λ to compute the expected loss, the (unconditional) equity premium would be given by $\lambda \left[(C_s/C_s^*)^{\sigma} - 1 \right] (Q_E - Q_E^*)/Q_E$.

on the case $B_o^E = B_p^E$, so optimists hold more long-term bonds, i.e. $B_o^L > B_p^L$. This leads to a simpler presentation in the analysis that follows.

Log-linear dynamics. We focus on a log-linear approximation of the equilibrium conditions. However, instead of linearizing around the non-stochastic steady state, we linearize the equilibrium conditions around the (stochastic) stationary equilibrium described above. Formally, we perturb the allocation around the economy where $u_t = 0$ and $\lambda > 0$, while the standard approach would perturb around the economy where $u_t = \lambda_t = 0$. This enables us to capture the effects of (time-varying) precautionary savings and risk premia in a linear setting, as shown below.[15]

Let lower-case variables denote log-deviations from the stationary equilibrium, e.g., $y_t \equiv \log Y_t/Y$ and $c_{w,t} \equiv \log C_{w,t}/C_w$. Workers' consumption is given by

$$c_{w,t} = \frac{WN_w}{PY}(w_t - p_t + n_{w,t}) + T_w'(Y)y_t \Rightarrow c_{w,t} = \chi_y y_t, \tag{7}$$

using $w_t - p_t = \phi y_t$ and $n_{w,t} = y_t$, where $\chi_y \equiv \frac{WN_w}{PY}(1+\phi) + T_w'(Y)$. The coefficient χ_y controls the cyclicality of income inequality among workers and savers. We focus on the case $0 < \chi_y < \mu_w^{-1}$, such that the consumption of savers, which is given by $c_{s,t} = \frac{1-\mu_w\chi_y}{1-\mu_w}y_t$ from the market clearing condition for goods, is also increasing in y_t.

Linearizing equation (1) and aggregating across savers, we obtain

$$\dot{c}_{s,t} = \sigma^{-1}(i_t - \pi_t - r_n) + \frac{\lambda}{\sigma}\left(\frac{C_s}{C_s^*}\right)^{\sigma} p_{d,t}, \tag{8}$$

where

$$p_{d,t} \equiv \sigma(c_{s,t} - c_{s,t}^*) + \hat{\lambda}_t \tag{9}$$

[15]This method differs from the procedure considered by Coeurdacier, Rey and Winant (2011) or Fernández-Villaverde and Levintal (2018), as we linearize around a stochastic steady state of an economy with no monetary shocks, instead of the stochastic steady state of the economy with both shocks.

denotes the price of (disaster) risk, $\hat{\lambda}_t \equiv \log \frac{\lambda_t}{\lambda}$, and we used the linearized discount-rate function: $\rho_{j,t} = \rho_j + \sigma\xi(c_{j,t} - c_{s,t})$.[16] The expression for the price of risk has two terms. The first term captures the change in the savers' marginal utility of consumption if the disaster shock is realized. The second term represents the change in the market-implied disaster probability after a monetary shock.

Combining condition (7) for borrowers' consumption, equation (8) for savers' Euler equation, and the market-clearing condition for goods, we obtain the evolution of aggregate output. Proposition 2 characterizes the dynamics of aggregate output and inflation, given the paths of i_t and $p_{d,t}$.

Proposition 2 (Aggregate dynamics)**.** *Given $[i_t, p_{d,t}]_{t\geq 0}$, the dynamics of output and inflation is described by the conditions:*

i. ***Aggregate Euler equation:***

$$\dot{y}_t = \tilde{\sigma}^{-1}(i_t - \pi_t - r_n) + \chi_{p_d} p_{d,t}, \tag{10}$$

where $\tilde{\sigma}^{-1} \equiv \frac{1-\mu_w}{1-\mu_w \chi_y}\sigma^{-1}$ *and* $\chi_{p_d} \equiv \frac{\lambda}{\tilde{\sigma}}\left(\frac{C_s}{C_s^*}\right)^{\sigma}$.

ii. ***New Keynesian Phillips curve:***

$$\dot{\pi}_t = \rho\pi_t - \kappa y_t, \tag{11}$$

where $\rho \equiv \rho_s + \lambda$ *and* $\kappa \equiv \varphi^{-1}(\epsilon - 1)\phi Y$.

Condition (10) represents the *aggregate Euler equation*. This equation has two terms, capturing the effects of heterogeneous MPCs, aggregate risk, and heterogeneous beliefs. The first term is the product of the aggregate elasticity of intertemporal substitution (EIS),

[16] Uzawa preferences correspond to the case $\xi > 0$ and constant discount rates correspond to $\xi = 0$. To simplify the model's aggregation, we assume that the slope coefficient $\sigma\xi$ is the same for both types.

$\tilde{\sigma}^{-1}$, and the real interest rate. The aggregate EIS depends on the cyclicality of inequality among workers and savers, as captured by χ_y. As in the work of Werning (2015) and Bilbiie (2019), heterogeneous MPCs amplify the effect of changes in interest rates if workers' consumption share is procyclical (i.e., $\chi_y > 1$), as it implies that $\tilde{\sigma}^{-1} > \sigma^{-1}$.

The second term, $\chi_{p_d} p_{d,t}$, captures the effect of aggregate risk. To understand the economic forces behind this expression, it is useful to rewrite equation (9) as $p_{d,t} = \tilde{\sigma} y_t + \hat{\lambda}_t$ where we used that $y_t^* = 0$. Then, the aggregate Euler equation can written as

$$\dot{y}_t = \tilde{\sigma}^{-1}(i_t - \pi_t - r_n) + \delta y_t + \chi_{p_d} \hat{\lambda}_t,$$

where $\delta \equiv \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma}$. In the absence of belief heterogeneity, so $\hat{\lambda}_t = 0$, we can write output as $y_t = -\tilde{\sigma}^{-1} \int_t^{\infty} e^{-\delta(s-t)} (i_s - \pi_s - r_n) ds$. Hence, a positive δ dampens the effect of future real interest rates, as in the discounted Euler equation of McKay, Nakamura and Steinsson (2017). In our setting, this is the result of a precautionary motive in response to aggregate disaster risk instead of idiosyncratic income risk. The last term, $\chi_{p_d} \hat{\lambda}_t$, captures the effect of heterogeneous beliefs. An increase in the market-implied disaster probability implies that pessimistic investors have a higher consumption share, as shown in Proposition 1. This increase in pessimism triggers a stronger precautionary motive in the aggregate.

Finally, Proposition 2 derives the NKPC. As in a textbook New Keynesian model, inflation is given by the present discounted value of future output gaps, $\pi_t = \kappa \int_t^{\infty} e^{-\rho(s-t)} y_s ds$.

Fiscal backing. The log-linearized government's flow budget constraint is given by

$$\overline{d}_G \dot{d}_{G,t} = i_L \overline{d}_G d_{G,t} + \overline{d}_G (i_t - \pi_t + r_{L,t} - i_L) - (\chi_\tau y_t + \tau_t), \tag{12}$$

where $\overline{d}_G \equiv \frac{D_G}{Y}$, and $\chi_\tau y_t + \tau_t$ denotes the primary surplus. The coefficient $\chi_\tau \equiv -\mu_w T_w'(Y)$ captures the elasticity of tax revenues to output and $\tau_t \equiv -\sum_{j \in \{o,p\}} \mu_j \frac{T_{j,t} - T_j}{Y}$ represents taxes on savers. As the government adjusts τ_t to ensure the No-Ponzi condition is satisfied, we refer to τ_t as the *fiscal backing* to the monetary shock.

2.3 Monetary policy and risk premia

Asset prices. The response of asset prices to monetary policy depends crucially on the behavior of the price of disaster risk, as shown in equations (2) and (3). Given the (linearized) price of risk in equation (9), we can price any financial asset in this economy. For example, the price of the long-term bond in period zero is given by

$$q_{L,0} = -\underbrace{\int_0^\infty e^{-(\rho+\psi_L)t}(i_t - r_n)dt}_{\text{path of nominal interest rates}} - \underbrace{\int_0^\infty e^{-(\rho+\psi_L)t} r_L p_{d,t} dt}_{\text{term premium}}. \tag{13}$$

The yield on the long-term bond, expressed as deviations from the stationary equilibrium, is given by $-Q_L^{-1} q_{L,0}$, which can be decomposed into two terms: the path of nominal interest rates, as in the expectations hypothesis, and a *term premium*, capturing variations in the compensation for holding long-term bonds. The term premium depends on the price of risk, $p_{d,t}$, and the asset-specific loading r_L. Because the term premium responds to monetary shocks, the expectation hypothesis does not hold in this economy.

The pricing condition for equities is analogous to the one for long-term bonds:

$$q_{E,0} = \underbrace{\frac{Y}{Q_E}\int_0^\infty e^{-\rho t}\hat{\Pi}_t dt}_{\text{dividends}} - \underbrace{\int_0^\infty e^{-\rho t}\left[i_t - \pi_t - r_n + r_E p_{d,t}\right] dt}_{\text{discount rate}}, \tag{14}$$

where $\hat{\Pi}_t = y_t - \frac{WN}{PY}(w_t - p_t + n_t)$. Equity prices respond to changes in monetary policy through two channels: a *dividend channel*, capturing changes in firms' profits, and a *discount rate channel*, capturing changes in real interest rates and risk premia. Risk premia depends on the price of risk, $p_{d,t}$, and the asset-specific loading r_E.

Market-implied disaster probability. Recall that the price of risk depends on y_t and $\hat{\lambda}_t$. We now characterize $\hat{\lambda}_t$. Log-linearizing equation (4), we obtain

$$\frac{1}{\sigma}\lambda^{\frac{1}{\sigma}}\hat{\lambda}_t = \mu_{c,o}\mu_{c,p}\left(\lambda_p^{\frac{1}{\sigma}} - \lambda_o^{\frac{1}{\sigma}}\right)\left[c_{p,t} - c_{o,t}\right], \tag{15}$$

where $\mu_{c,j} \equiv \frac{\mu_j C_j}{\mu_o C_o + \mu_p C_p}$, for $j \in \{o, p\}$. The market-implied disaster probability increases when the monetary shock redistributes wealth towards pessimistic savers. As shown in Appendix A.3, the relative consumption of the two types of savers evolves according to

$$\dot{c}_{p,t} - \dot{c}_{o,t} = -\xi(c_{p,t} - c_{o,t}), \tag{16}$$

and the law of motion of relative net worth $b_{p,t} - b_{o,t}$ is given by

$$\dot{b}_{p,t} - \dot{b}_{o,t} = \rho(b_{p,t} - b_{o,t}) - \chi_{b,c}(c_{p,t} - c_{o,t}) + \chi_{b,c_s}c_{s,t},$$

where the coefficients $\chi_{b,c}$ and χ_{b,c_s} are functions of portfolios and returns in the stationary equilibrium. Given that the evolution of relative net worth depends on $c_{s,t}$, and $c_{s,t}$ depends on y_t, we must simultaneously solve for $[c_{p,t} - c_{o,t}, b_{p,t} - b_{o,t}]_0^\infty$ and $[i_t, y_t, \pi_t]_0^\infty$. In this case, obtaining analytical results would likely be infeasible. We show next that this system satisfies an *approximate block recursivity* property, where we can solve for $c_{p,t} - c_{o,t}$ and $b_{p,t} - b_{o,t}$ independently of (y_t, π_t), provided the effect of $c_{s,t}$ on risk premia is small.

Proposition 3 (Approximate block recursivity)**.** *Suppose $r_k\sigma c_{s,t}$ is small for $k \in \{L, E\}$, i.e. $r_k\sigma c_{s,t} = \mathcal{O}(||i_t - r_n||^2)$. Then, the market-implied disaster probability $\hat{\lambda}_t$ and relative net worth $b_{p,t} - b_{o,t}$ can be solved independently of (y_t, π_t), and they are given by*

$$\hat{\lambda}_t = e^{-\psi_\lambda t}\hat{\lambda}_0, \tag{17}$$

$b_{p,t} - b_{o,t} = e^{-\psi_\lambda t}(b_{p,0} - b_{o,0})$, and $\psi_\lambda = \xi$. If $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$, then $\hat{\lambda}_0$ is given by

$$\hat{\lambda}_0 = \epsilon_\lambda (i_0 - r_n), \tag{18}$$

where $\epsilon_\lambda \geq 0$ and the inequality is strict if and only if $\lambda_p > \lambda_o$.

Proposition 3 shows that we can solve for $\hat{\lambda}_t$ and $b_{p,t} - b_{o,t}$ independently of output and inflation if $r_k \sigma c_{s,t}$ is small. If $r_k \sigma c_{s,t}$ is second-order on the size of the monetary shock, its first-order impact on risk premia is negligible. In this case, we can solve for $\hat{\lambda}_t$ and $b_{p,t} - b_{o,t}$ independently of (y_t, π_t). As the dynamics of (y_t, π_t) depends on $\hat{\lambda}_t$, but $\hat{\lambda}_t$ does not depend on (y_t, π_t), we say the system is (approximately) block recursive. In Appendix A.4, we assess the quantitative importance of the term $r_k \sigma c_{s,t}$. For our calibrated parameters, we find that risk premium effects on stocks and bonds when we include the term $r_k \sigma c_{s,t}$ are nearly identical to the solution when these terms are omitted.

Uzawa preferences ensure that the effects of the monetary shock on the price of risk are transitory. If $\xi = 0$, so subjective discount rates are constant, then $\psi_\lambda = 0$ and a temporary monetary shock has a permanent effect on $\hat{\lambda}_t$. The reason is that a monetary policy surprise leads to permanent changes in relative net worth and relative consumption in this case. With Uzawa preferences, savers' net worth eventually converge to their stationary-equilibrium level, so the effect on $\hat{\lambda}_t$ is transitory.

An important implication of equation (18) is that the price of risk increases after a contractionary monetary shock. A monetary tightening redistributes wealth away from optimistic investors, as they are more exposed to risky assets. The economy becomes on average more pessimistic, which raises the required compensation for holding risky assets. The increase in risk premia in response to contractionary monetary shocks is consistent with the evidence in, e.g., Gertler and Karadi (2015) and Hanson and Stein (2015). Notice that investor heterogeneity is necessary for this result, as $\hat{\lambda}_t = 0$ when $\lambda_o = \lambda_p$.

The four-equation system. Proposition 3 allows us to write the price of risk as follows:

$$p_{d,t} = \tilde{\sigma} y_t + e^{-\psi_\lambda t} \hat{\lambda}_0, \tag{19}$$

where $\hat{\lambda}_0$ is a function of the path of nominal interest rates. Combining the expression above for the price of risk with the interest rate rule (6), the aggregate Euler equation (10), and the NKPC (11), we obtain a four-equation system describing the economy's aggregate dynamics. The system is similar to the textbook three-equation model (see, e.g., Galí, 2015). The interest rate rule and the NKPC are isomorphic to the ones in the simple model. Equation (10) is analogous to the standard Euler equation but features an additional term that depends on the price of risk, $p_{d,t}$. It is this term that connects aggregate risk, asset prices, and macroeconomic variables. Finally, equation (19) characterizes how the price of risk depends on aggregate output and changes in monetary policy.

The approximate block-recursivity is crucial to allow us to write the system in terms of aggregate variables, without having to simultaneously solve for the dynamics of individual balance sheets. The portfolio dynamics is summarized by two coefficients: ϵ_λ, which captures the pass-through of nominal rates to the initial price of risk, and ψ_λ, which controls the persistence of the price of risk. Both coefficients depend on investors' beliefs and their portfolio holdings in the stationary equilibrium.

Comparison with uncertainty shocks. Monetary shocks lead to an endogenous change in the market-implied disaster probability. A related literature studies the effects of exogenous uncertainty shocks in New Keynesian models (see e.g. Basu and Bundick 2017 and Caballero and Simsek 2020). In this literature, monetary policy leans against the uncertainty shock, so interest rates and risk premia move in opposite directions. In contrast, a monetary shock causes an increase in real rates and risk premia in our setting. Movements in risk premia then amplify the impact of monetary policy on asset prices. Similar to these papers, our analysis requires that the monetary policy rule does not track the natural rate in response to changes in the risk premium.

The mechanism through which monetary policy affects $\hat{\lambda}_t$ is reminiscent of the redistribution channel in Kekre and Lenel (2022). Our approximate block recursivity property allows us to incorporate this mechanism into a New Keynesian model in a tractable way, as the aggregate impact of heterogeneous portfolios is summarized by $\hat{\lambda}_t$.

3 Monetary Policy and Wealth Effects

We considered so far how monetary policy affects risk premia and asset prices through their impact on the price of risk, $p_{d,t}$, and the market-implied disaster probability, $\hat{\lambda}_t$. We study next how the revaluation of real and financial assets affects the real economy.

3.1 Risk-premium neutrality

Asset revaluations caused by monetary policy have received significant attention recently. For instance, Cieslak and Vissing-Jorgensen (2020) show that policymakers follow stock market movements due to its potential (consumption) wealth effect. In contrast, Cochrane (2020) and Krugman (2021) argue that wealth gains on "paper" are not relevant for households who simply consume their dividends. The next proposition provides exact conditions under which such neutrality result would hold in our model.

Proposition 4 (Risk-premium neutrality)**.** *Suppose the government uses a consumption tax on savers to neutralize movements in $\hat{\lambda}_t$, that is, τ_t^c satisfies $\dot{\hat{\tau}}_t^c = \lambda\left(\frac{C_s}{C_s^*}\right)\hat{\lambda}_t$, where $\hat{\tau}_t^c \equiv \log(1+\tau_t^c)$, $\tau_t^c = \tau_t^{c,*}$, and the revenue is rebated back to savers such that it is budget neutral for them. Then, $[y_t, \pi_t, i_t]_0^\infty$ is independent of $\hat{\lambda}_t$. Moreover, the fiscal backing τ_t is independent of $\hat{\lambda}_t$ if one of the following conditions hold: i) $\overline{d}_G = 0$; ii) $\overline{d}_G > 0$ and $\psi_L = \infty$; iii) $\overline{d}_G > 0$ and $\psi_L = 0$.*

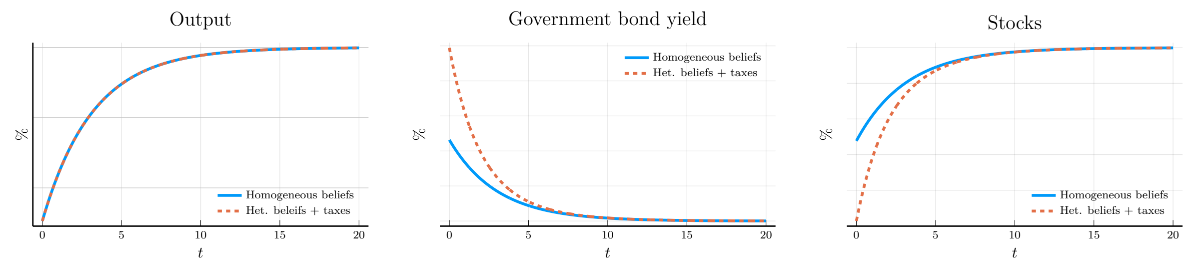

Figure 1: Output and asset prices response

Proof. Savers' Euler equation for the riskless bond is now given by $\dot{c}_{s,t} = \sigma^{-1}(i_t - \pi_t - r_n - \dot{\hat{\tau}}^c_t) + \frac{\lambda}{\sigma}\left(\frac{C_s}{C_s^*}\right)^{\sigma}[\hat{\lambda}_t + \sigma c_{s,t}]$, which is independent of $\hat{\lambda}_t$ if $\dot{\hat{\tau}}^c_t = \lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma}\hat{\lambda}_t$. As $\tau^c_t = \tau^{c,*}_t$, Euler equations for risky assets are not affected. The aggregate Euler equation takes the same form as in Eq. (10), but with $\chi_{p_d} = 0$. Combining it with Eq. (6) and Eq. (11), we obtain $[y_t, \pi_t, i_t]$ independently of $\hat{\lambda}_t$. The fiscal backing τ_t will also be independent of $\hat{\lambda}_t$ if one of the following three conditions hold: i) $\overline{d}_G = 0$, so the fiscal backing simply offsets the present value of transfers to workers; ii) $\overline{d}_G > 0$ and $\psi_L \to \infty = 0$, so $r_L = 0$; iii) $\overline{d}_G > 0$ and $\psi_L = 0$, so the government does not need to issue new debt. □

Under the conditions of Proposition 4, asset revaluations caused by monetary policy have no real effects. Portfolio heterogeneity among savers helps improve the model's asset-pricing implications, but it has no bearing on how monetary shocks ultimately affect the real economy. In particular, output and inflation are independent of $\lambda_p - \lambda_o$. Due to the increase in the risk premium, an economy with heterogeneous beliefs would have a larger drop in asset prices after a monetary contraction than an economy where $\lambda_p = \lambda_o$. Importantly, the real rate would be the same in the two economies, so they only differ in the behavior of risk premia. Even though stocks and bonds suffer a larger drop in value, the response of output and inflation is the same as in the economy without belief heterogeneity. Figure 1 illustrates this result in a numerical example, which shows output and asset prices in two economies, with and without belief heterogeneity. Despite large differences in asset prices, the response of output is the same in both economies.

3.2 Wealth effects

But why do households in the economy that suffered a larger drop in asset prices consume as much as households in the economy with a smaller fall in asset prices? How do households in the former economy, who have lower initial wealth, even afford the same level of consumption as households in the latter economy? To better understand this result, we need to look at the household's intertemporal budget constraint.

Asset revaluation. Household j's intertemporal budget constraint (IBC) is given by:

$$\mathbb{E}_0\left[\int_0^\infty \frac{\eta_t}{\eta_0} C_{j,t} dt\right] = B_{j,0} + \mathbb{E}_0\left[\int_0^\infty \frac{\eta_t}{\eta_0} T_{j,t} dt\right], \tag{20}$$

Consider first the revaluation of the household's financial assets after a monetary shock. Up to first order, it is given by $B_{j,0} - B_j \approx B_j^E q_{E,0} + B_j^L q_{L,0}$. When interest rates increase, the value of households' stocks and bonds decreases, making them poorer. However, an increase in the interest rate also impacts the *cost* of the households' consumption bundle. Denote the value of a claim on consumption (i.e., the left-hand side of the IBC) by $Q_{C_j,t} \equiv \mathbb{E}_t\left[\int_t^\infty \frac{\eta_z}{\eta_t} C_{j,z} dz\right]$. Linearizing the value of the consumption claim, we obtain a pricing condition analogous to the one for stocks and bonds (see equations 13 and 14):

$$q_{C_j,0} = \frac{C_j}{Q_{C_j}} \int_0^\infty e^{-\rho t}(c_{j,t} + \chi_{c_j^*} c_{j,t}^*) dt - \int_0^\infty e^{-\rho t}\left(i_t - \pi_t - r_n + r_{C_j} p_{d,t}\right) dt, \tag{21}$$

where $\chi_{c_j^*} \equiv \frac{\delta}{r_n^*} \frac{C_j^*}{C_j}$ and $r_{C_j} \equiv \lambda \left(\frac{C_s}{C_s^*}\right)^\sigma \frac{Q_{C_j} - Q_{C_j}^*}{Q_{C_j}}$. The first term represents the change in the household's consumption bundle, while the second term represents the change in its cost. We define household j's *wealth effect* as the difference between the revaluation of the household's wealth plus the transfers claim and the change in the cost of the consumption bundle, that is,

$$\underbrace{\Omega_{j,0}}_{\text{wealth effect}} = \underbrace{\frac{1}{C_j}\left[B_j^L q_{L,0} + B_j^E q_{E,0} + Q_{T_j} q_{T_j,0}\right]}_{\text{asset-revaluation effect}} + \underbrace{\frac{Q_{C_j}}{C_j}\int_0^\infty e^{-\rho t}\left(i_t - \pi_t - r_n + r_{C_j} p_{d,t}\right) dt}_{\text{consumption's discount-rate effect}},$$

where $Q_{T_j,t} \equiv \mathbb{E}_t\left[\int_t^\infty \frac{\eta_z}{\eta_t} T_{j,z} dz\right]$ corresponds to the value of the transfers claim. Notice that this implies that we can write the IBC as

$$\int_0^\infty e^{-\rho t}\left(c_{j,t} + \chi_{c_j^*} c_{j,t}^*\right) dt = \Omega_{j,0}. \tag{22}$$

Expression (22) shows that a shock relaxes the household's budget constraint if $\Omega_{j,0} > 0$, as households can then consume more in every period or state. The opposite happens if $\Omega_{j,0} < 0$. As $\Omega_{j,0}$ captures a shift in the household's intertemporal budget constraint, we refer to $\Omega_{j,0}$ as a wealth effect, consistent with its use in microeconomic theory.[17] Finally, we define the aggregate wealth effect as the sum of the individual households' wealth effect, that is, $\Omega_0 \equiv \sum_{j\in\{w,o,p\}} \frac{\mu_j C_j}{Y}\Omega_{j,0}$.

A special case. To illustrate the economics behind the risk-premium neutrality result, consider a household who simply consumes the dividends from her equity investments, so $B_j^L = Q_{T_j} = 0$. To focus on discount-rate changes, assume $y_t = 0$. Then, $\Omega_{j,0}$ is

$$\Omega_{j,0} = -\frac{B_j^E}{C_j}\int_0^\infty e^{-\rho t}\left[i_t - \pi_t - r_n + r_E p_{d,t}\right] dt + \frac{Q_{C_j}}{C_j}\int_0^\infty e^{-\rho t}\left[i_t - \pi_t - r_n + r_{C_j} p_{d,t}\right] dt. \tag{23}$$

The first term corresponds to the drop in equity prices due to an increase in discount rates. The second term corresponds to the drop in the amount of wealth required to finance the initial path of consumption. As consumption equals dividends, the value

[17] We show in Appendix B.2 that $\Omega_{j,0}$ corresponds to (minus) the Hicksian wealth compensation, as defined in Mas-Colell, Whinston and Green (1995).

of the consumption claim equals the value of equity investments, $B_j^E = Q_{C_j}$, and the risk compensation on the consumption claim equals the risk compensation on equities, $r_E = r_{C_j}$. Therefore, the wealth effect is zero in this case: $\Omega_{j,0} = 0$. This result is analogous to the textbook effect of interest rate changes. If the household is neither a net buyer nor a net seller of assets, interest rate changes have no wealth effect.[18]

A similar point emerges in the discussion of capital-gains taxation. Discussing the impact of a drop in interest rates for an investor (Bob) whose consumption equals dividends every period, Cochrane (2020) says

> "When the interest rate goes down, it takes more wealth to finance the same consumption stream. The present value of liabilities – consumption – rises just as much as the present value of assets, so on a net basis Bob is not at all better."

In our terms, the increase in financial wealth does not translate into a positive wealth effect, as the increase in the stock price exactly cancels out the increase in the value of the consumption claim after a drop in interest rates when consumption equals dividends.

Proposition 4 shows that a similar logic holds for the aggregate economy. For instance, in the absence of government debt, the household sector's consumption equals the dividends from their assets, which includes their human wealth. With government debt, changes in discount rates may cause a redistribution between the household sector and the government, which can be offset by movements in the fiscal backing τ_t.

The role of taxes and comparison with log utility. We have seen that the wealth effect measures the extent a shock tightens households' budget constraints. As the revenue from the consumption tax introduced in Proposition 4 is rebated back to savers, the tax does not affect the wealth effect. The consumption tax just offsets the precautionary effect.

[18] In a two-period model, we would have $C_0 + \frac{1}{1+r}C_1 = Y_0 + \frac{1}{1+r}Y_1 \equiv B_0$. If $C_t = Y_t$, a small increase in r would not tighten the household's budget constraint, despite a fall in the value of the assets (B_0).

The wealth effect is independent of preferences, so the discussion above holds even with log utility. In the absence of Uzawa preferences, consumption is proportional to wealth in that case. Nevertheless, the wealth effect is zero when consumption equals dividends. Appendix B.3 shows that, in this case, the consumption response coincides with the compensated (Hicksian) demand, reflecting substitution and precautionary effects.

3.3 Precautionary motives and heterogeneous MPCs

The behavior of the wealth effect illustrates why changes in asset prices are not sufficient to generate a drop in consumption. Next, we show that differences in MPCs across agents with heterogeneous beliefs is at the core of the real effects of movements in risk premia.

Redistribution and iMPCs. The next lemma shows that differences in beliefs translate into differences in intertemporal MPCs (iMPCs).

Lemma 1 (Intertemporal MPCs). *The iMPC at time t for saver $j \in \{o, p\}$ is given by*

$$\mathcal{M}_{j,t} \equiv \frac{1}{C_j}\frac{\partial C_{j,t}}{\partial \Omega_{j,0}} = \frac{(\rho+\xi)}{1+\overline{\chi}\lambda_j^{\frac{1}{\sigma}}}e^{-\xi t}, \qquad \mathcal{M}^*_{j,t} \equiv \frac{1}{C_j}\frac{\partial C^*_{j,t}}{\partial \Omega_{j,0}} = \frac{(\rho+\xi)\overline{\chi}^*\lambda_j^{\frac{1}{\sigma}}}{1+\overline{\chi}\lambda_j^{\frac{1}{\sigma}}}e^{-\xi t},$$

where $\overline{\chi}$ and $\overline{\chi}^*$ are positive constants. Moreover, iMPCs satisfy the following condition:

$$\int_0^\infty e^{-\rho t}\left[\mathcal{M}_{j,t} + \frac{\delta}{r_n^*}\mathcal{M}^*_{j,t}\right]dt = 1. \tag{24}$$

The iMPC at period t corresponds to the response of consumption at this date with respect to a change in wealth at period 0.[19] Given the assumption of Uzawa preferences, iMPCs are decaying over time, consistent with the recent evidence by, e.g., Fagereng,

[19] The standard definition of MPC corresponds to the iMPC at $t = 0$. For a discussion of iMPCs in the context of HANK models, see Auclert, Rognlie and Straub (2018). Auclert (2019) analyzes the redistribution channel of monetary policy in a model without aggregate risk.

Holm and Natvik (2021) and Borusyak, Jaravel and Spiess (2024).[20] An important implication of Lemma 1 is that optimistic investors have a higher iMPC in the no-disaster state than pessimistic investors, and the reverse pattern holds in the disaster state:

$$\mathcal{M}_{o,t} > \mathcal{M}_{p,t}, \qquad \mathcal{M}^*_{o,t} < \mathcal{M}^*_{p,t},$$

when $\lambda_o < \lambda_p$. These differences in iMPCs are tightly connected to movements in $\hat{\lambda}_t$.

Proposition 5. *The market-implied disaster probability is given by*

$$\hat{\lambda}_t = \overline{\chi}_\lambda \left(\mathcal{M}_{o,t} - \mathcal{M}_{p,t}\right)\left(\Omega_{p,0} - \Omega_{o,0}\right), \tag{25}$$

where $\overline{\chi}_\lambda$ is a positive constant given in the appendix.

Proposition 5 shows that $\hat{\lambda}_t$ reflects the interaction between differences in iMPCs, $\mathcal{M}_{o,t} - \mathcal{M}_{p,t}$, and redistribution induced by monetary policy, $\Omega_{p,0} - \Omega_{o,0}$. Hence, differences in iMPCs play an important role in how asset prices respond to monetary shocks.

Precautionary motive and heterogeneity. Differences in iMPCs create a wedge in the precautionary motive of the heterogeneous-beliefs economy relative to an economy with a representative saver. To see this fact, consider the (linearized) Euler equation for saver j:

$$\dot{c}_{j,t} = \underbrace{\sigma^{-1}(i_t - \pi_t - r_n)}_{\text{intertemporal substitution}} + \frac{\lambda_j}{\sigma}\left(\frac{C_j}{C_j^*}\right)^{\sigma} \times \underbrace{\sigma(c_{j,t} - c^*_{j,t})}_{\text{precautionary motive}} - \underbrace{\xi(c_{j,t} - c_{s,t})}_{\text{Uzawa preferences}}.$$

[20] Note that we obtain time-varying iMPCs even in the case of log utility, given the endogeneity of discount rates. We recover the standard log-utility result when discount rates are constant.

Using the fact that $c_{s,t} = \sum_{j\in\{o,p\}} \frac{\mu_j C_j}{\mu_o C_o + \mu_p C_p} c_{j,t}$, we can aggregate across savers:

$$\dot{c}_{s,t} = \sigma^{-1}(i_t - \pi_t - r_n) + \frac{\lambda}{\sigma}\left(\frac{C_s}{C_s^*}\right)^{\sigma} \sum_{j\in\{o,p\}} \frac{\mu_j C_j}{\mu_o C_o + \mu_p C_p} \sigma(c_{j,t} - c_{j,t}^*).$$

The aggregate Euler equation depends on the intertemporal-substitution channel and the average precautionary motive. Interestingly, the average precautionary motive is *not* equal to $\sigma(c_{s,t} - c_{s,t}^*)$, the precautionary motive in an economy with a representative saver:

$$\sum_{j\in\{o,p\}} \frac{\mu_j C_j}{\mu_o C_o + \mu_p C_p} \sigma(c_{j,t} - c_{j,t}^*) = \sigma(c_{s,t} - c_{s,t}^*) + \hat{\lambda}_t,$$

where the equality comes from the optimality condition for risky assets. Notice that we typically have $c_{s,0} < 0$ after a contractionary shock, and we assume that $c_{s,t}^* = 0$, which would weaken the precautionary motive in the absence of heterogeneity. In contrast, $\hat{\lambda}_t > 0$ after a contractionary shock, which tends to strengthen the precautionary motive. The difference is explained by the negative correlation between iMPCs and wealth redistribution after the shock.

Macro-finance separation. A previous literature considered the related concept of *macro-finance separation*. For instance, Tallarini Jr. (2000) and Gourio (2012) studied economies where fluctuations in risk premia do not affect consumption, investment, or output. It is crucial for their results, however, that the EIS is equal to one. This ensures that the real interest rate exactly offsets movements in risk premia, so the price of the risky asset remains unchanged.

In contrast, our result presents conditions under which changes in asset prices, driven by changes in risk premia, have no impact on real equilibrium outcomes. Unlike the macro-finance separation result, our analysis requires that asset prices adjust following the initial shock, allowing us to evaluate the impact of the portfolio revaluations on savers' consumption decisions. To isolate the transmission via asset prices, we introduce consumption taxes designed to counterbalance the time-varying precautionary motive associated to fluctuations in $\hat{\lambda}_t$. Proposition 4 then shows that two economies with different levels of belief disagreement and, therefore, risk premia, will exhibit not only identical paths of output and inflation but also the same path of nominal and real interest rates. As the real rate is the same while the price of the risky assets varies between the two economies, we can conclude that differences in financial wealth *alone* do not affect the real variables of the economy.[21]

3.4 Intertemporal substitution, risk, and wealth effect

We consider next the response of output and inflation to changes in monetary policy in the absence of the consumption tax. Consider the dynamic system in Proposition 2:

$$\begin{bmatrix} \dot{y}_t \\ \dot{\pi}_t \end{bmatrix} = \begin{bmatrix} \delta & -\tilde{\sigma}^{-1} \\ -\kappa & \rho \end{bmatrix} \begin{bmatrix} y_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} \nu_t \\ 0 \end{bmatrix},$$

where we have substituted $p_{d,t}$ with the expression in equation (9), and $\nu_t \equiv \tilde{\sigma}^{-1}(i_t - r_n) + \chi_{p_d}\hat{\lambda}_t$ depends only on the path of nominal interest rates. The eigenvalues of the system are given by

$$\overline{\omega} = \frac{\rho + \delta + \sqrt{(\rho+\delta)^2 + 4(\tilde{\sigma}^{-1}\kappa - \rho\delta)}}{2}, \qquad \underline{\omega} = \frac{\rho + \delta - \sqrt{(\rho+\delta)^2 + 4(\tilde{\sigma}^{-1}\kappa - \rho\delta)}}{2}.$$

The following assumption, which we assume holds for all subsequent analysis, guarantees that the eigenvalues are real-valued and have opposite signs, i.e., $\overline{\omega} > 0$ and $\underline{\omega} < 0$.

Assumption 1. *The following condition holds: $\tilde{\sigma}^{-1}\kappa > \rho\delta$.*

[21] In Online Appendix D.3 we show that our model can reproduce the macro-finance separation result. We also extend our risk-premium neutrality result to an economy with investment.

This assumption implies that local uniqueness of the equilibrium requires a positive coefficient on inflation in the Taylor rule. We show in Appendix B.6 that the equilibrium is locally unique if $\phi_\pi \geq 1 - \frac{\rho\delta}{\tilde{\sigma}^{-1}\kappa} \equiv \overline{\phi}_\pi$. Assumption 1 ensures that $\overline{\phi}_\pi > 0$.

Output. We extend next the analysis in Caramp and Silva (2023), which decomposes the equilibrium path of output into an *intertemporal substitution effect* (ISE) and a *wealth effect*, to our setting with aggregate risk. We focus on the case in which the monetary shock induces an exponentially decaying path for the nominal interest rates; that is, we assume $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$, where ψ_m determines the persistence of interest rates.[22]

Proposition 6 (Aggregate output in D-HANK)**.** *Suppose that $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$ and $\psi_k \neq -\underline{\omega}$, for $k \in \{m, \lambda\}$. The path of aggregate output is then given by*

$$y_t = \underbrace{\tilde{\sigma}^{-1}\hat{y}_{m,t}}_{\textit{ISE}} + \underbrace{\chi_\lambda \hat{y}_{\lambda,t}}_{\substack{\textit{time-varying}\\ \textit{precautionary motive}}} + \underbrace{(\rho - \underline{\omega})e^{\underline{\omega}t}\Omega_0}_{\substack{\textit{GE factor}\times\\ \textit{aggregate wealth effect}}}, \tag{26}$$

where $\chi_\lambda \equiv \chi_{p_d}\epsilon_\lambda$, $\Omega_0 \equiv \sum_{j\in\{w,o,p\}} \frac{\mu_j C_j}{Y}\Omega_{j,0}$, and $\hat{y}_{k,t}$ is given by

$$\hat{y}_{k,t} = \frac{(\rho - \underline{\omega})\, e^{\underline{\omega}t} - (\rho + \psi_k)\, e^{-\psi_k t}}{(\overline{\omega} + \psi_k)\, (\underline{\omega} + \psi_k)}(i_0 - r_n), \tag{27}$$

and satisfies $\int_0^\infty e^{-\rho t}\hat{y}_{k,t}dt = 0$, $\frac{\partial \hat{y}_{k,0}}{\partial i_0} < 0$, for $k \in \{m, \lambda\}$.

Proposition 6 shows that output can be decomposed into three terms: an intertemporal-substitution effect (ISE), a time-varying precautionary motive, and the aggregate wealth effect Ω_0.

The first term captures the standard intertemporal substitution channel present in RANK models. It depends on the aggregate EIS, $\tilde{\sigma}^{-1} = \frac{1-\mu_w}{1-\mu_w\chi_y}\sigma^{-1}$, and $\hat{y}_{m,t}$ given in

[22] The proof of the proposition contains the general case.

(27). Notice that, even though only a fraction $1-\mu_w$ of agents substitute consumption intertemporally, the ISE does not necessarily get weaker as we reduce the mass of savers in the economy. As we reduce $1-\mu_w$, less agents are capable of intertemporal substitution, but the amplification from hand-to-mouth agents gets stronger. The two effects exactly cancel out when $\chi_y = 1$. Importantly, the ISE is equal to zero on average, i.e. $\int_0^\infty e^{-\rho t}\hat{y}_{m,t}dt = 0$. An increase in interest rates shifts demand from the present to the future, but by itself it does not change the present value of aggregate demand.

The second term captures the effect of the time-varying precautionary motive. This term is equal to zero in the absence of belief heterogeneity. In this case, the model behaves as a TANK model with zero liquidity (see e.g. Bilbiie 2019 and Broer et al. 2020). As with the EIS, the precautionary motive shifts demand from the present to the future without changing its present value, that is, $\int_0^\infty e^{-\rho t}\hat{y}_{\lambda,t}dt = 0$. The persistence of the precautionary effects is controlled by ψ_λ, as it depends on the rate at which the balance sheet of optimistic investors recover after a contractionary shock.

The third term in expression (26) plays an important role, as the aggregate wealth effect determines the average response of output to the monetary shock. The GE factor shifts the impact of the wealth effect over time, as we have that $(\rho-\underline{\omega})\int_0^\infty e^{-(\rho-\underline{\omega})t}dt = 1$. Everything else constant, an increase in Ω_0 raises output in all periods by $\rho\Omega_0$, creating a parallel shift in output over time. In general equilibrium, a positive aggregate wealth effect leads to inflation on impact, which reduces the real rate and shifts consumption to the present. The GE factor shows that the effect of Ω_0 on y_0 exceeds the effect on average consumption, $\rho\Omega_0$, by the factor $\frac{\rho-\underline{\omega}}{\rho} > 1$.

This decomposition will be relevant in the interpretation of our quantitative results in Section 4. In particular, it will allow us to assess the importance of standard channels captured in RANK models (e.g., the intertemporal-substitution effect), relative to channels specific to models with risk (e.g., the precautionary motive).

Inflation. The next proposition characterizes the behavior of inflation.

Proposition 7 (Inflation in D-HANK). *Suppose $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$ and $\psi_k \neq -\underline{\omega}$ for $k \in \{m, \lambda\}$. The path of inflation is given by*

$$\pi_t = \tilde{\sigma}^{-1}\hat{\pi}_{m,t} + \chi_\lambda \hat{\pi}_{\lambda,t} + \kappa e^{\underline{\omega} t}\Omega_0, \tag{28}$$

where $\hat{\pi}_{k,t} = \frac{\kappa(e^{\underline{\omega} t} - e^{-\psi_k t})}{(\underline{\omega}+\psi_k)(\overline{\omega}+\psi_k)}(i_0 - r_n)$, $\hat{\pi}_{k,0} = 0$ and $\frac{\partial \hat{\pi}_{k,t}}{\partial i_0} \geq 0$, for $k \in \{m, \lambda\}$.

Inflation can be analogously decomposed into three terms. The first two terms capture the impact of the ISE and time-varying precautionary motive, while the last term captures the impact of the aggregate wealth effect. Because $\hat{\pi}_{k,0} = 0$, the first two terms are initially zero. Initial inflation is then entirely determined by the aggregate wealth effect.

Implementability. In Appendix B.6, we show that, for any given path of $[u_t]_0^\infty$, there is a unique equilibrium path $[i_t, \tau_t]_0^\infty$ provided $\phi_\pi \geq 1 - \frac{\rho\delta}{\tilde{\sigma}^{-1}\kappa}$. Conversely, for any given path of $[i_t, \tau_t]_0^\infty$, there is a unique path of monetary shocks $[u_t]_0^\infty$ that implements this path of policy variables in equilibrium. Hence, there is no loss of generality in expressing the solution in terms of policy variables i_t and τ_t instead of directly in terms of u_t.

Propositions 6 and 7 solve for output and inflation in terms of the nominal rate and the aggregate wealth effect Ω_0. The next expression shows that the aggregate wealth effect can be written as a function of only $[i_t, \tau_t]_0^\infty$:[23]

$$\Omega_0 = \frac{\rho - \underline{\omega}}{(\rho - \underline{\omega})\chi_\tau + \overline{d}_G \kappa}\left[\int_0^\infty e^{-\rho t}\Delta B_t^L (i_t - r_n + r_L \hat{\lambda}_t)dt - \overline{d}_G \int_0^\infty e^{-\rho t}\hat{\pi}_t dt - \int_0^\infty e^{-\rho t}\tau_t dt\right], \tag{29}$$

where $\hat{\pi}_t \equiv \tilde{\sigma}^{-1}\hat{\pi}_{m,t} + \chi_\lambda \hat{\pi}_{\lambda,t}$ is a function of $[i_t]_0^\infty$, $\Delta B_t^L \equiv (1 - e^{-\psi_L t})\overline{d}_G$, and we assumed $\chi_\tau + \frac{\overline{d}_G \kappa}{\rho - \underline{\omega}} > 0$. Equation (29) shows that changes in discount rates affect the aggregate

[23]To obtain Eq. (29), plug y_t and π_t from Propositions 6 and 7 into Eq. (23), aggregate across households, and solve for Ω_0.

wealth, for a given fiscal backing, only through government bonds. In particular, the excess return on equities, as captured by r_E, does not affect Ω_0.

In our quantitative analysis, we consider two approaches to discipline the monetary shocks. First, we estimate the fiscal backing directly from the data and find the monetary shock that implements the empirically estimated fiscal backing. Second, we consider the monetary shock that implements the *minimum state-variable (MSV)* solution (see McCallum 1999). This corresponds to the method used to compute the solution of the textbook NK model. The MSV corresponds to the unique solution where output and inflation are linear functions of contemporaneous values of i_t and $\hat{\lambda}_t$.

4 The Quantitative Importance of Wealth Effects

In this section, we study the quantitative importance of risk and wealth effects in the transmission of monetary shocks. While the model is stylized and lacks some important dimensions present in state-of-the-art quantitative HANK models (e.g., rich MPC heterogeneity), this exercise is useful to assess the economic relevance of these channels.

4.1 Calibration

The parameter values are chosen as follows. The discount rate of savers is chosen to match a natural interest rate of $r_n = 1\%$. We assume a Frisch elasticity of one, $\phi = 1$, and set the elasticity of substitution between intermediate goods to $\epsilon = 6$, common values adopted in the literature. The fraction of workers is set to $\mu_w = 30\%$, consistent with the fraction of (poor and wealthy) hand-to-mouth agents in the U.S. estimated by Kaplan, Violante and Weidner (2014). The parameter $\overline{d}_G$ is chosen to match a ratio of the market value of public debt in the hands of the private sector to GDP of 28% and ψ_L is chosen to match a duration of five years, roughly in line with the historical average between 1962 and 2007 for the United States (Hall and Sargent 2011). The parameter $T'_w(Y)$ is chosen such that $\chi_y = 1$, which requires countercyclical transfers to balance the procyclical wage income.[24] A value of $\chi_y = 1$ is consistent with the evidence in Cloyne, Ferreira and Surico (2020) on the monetary policy impact on the income of borrowers (proxing for hand-to-mouth agents) and savers, where they show that the net income of mortgagors and non-mortgagors reacts similarly to monetary shocks.

To calibrate the disaster risk parameters, we follow closely Barro (2006). We set λ (the steady-state disaster intensity) to match an annual disaster probability of 1.7%. To better map the model to the data, we consider an extension where the magnitude of the drop in productivity, $\zeta_A \equiv 1 - \frac{A^*}{A}$, is stochastic and draw from a given distribution known by all agents. We adopt the empirical distribution estimated by Barro (2006), where ζ_A ranges from 15% to 64%, with an average of 29%. Introducing a random disaster size has only a minor effect on the analytical expressions, with the term $(C_s^*)^{-\sigma}$ being typically replaced by $\mathbb{E}[(C_s^*)^{-\sigma}]$, where the expectation is taken over the disaster size ζ_A.[25]

The risk-aversion coefficient is set to $\sigma = 4$, a value within the range of reasonable values according to Mehra and Prescott (1985), but substantially larger than $\sigma = 1$, a value often adopted in macroeconomic models. Our calibration implies an equity premium in the stationary equilibrium of 7.0%, in line with the observed equity premium (Campbell 2003). Moreover, by setting $\sigma = 4$ we obtain a micro EIS of $\sigma^{-1} = 0.25$, in the ballpark of an EIS of 0.1 as recently estimated by Best, Cloyne, Ilzetzki and Kleven (2020), and in line with the estimates for asset holders by Havránek (2015) of 0.3. The pricing cost parameter φ is chosen to match a slope of the Phillips curve of $\kappa = 0.30$, which is the value for κ in the textbook model with an average price duration of three quarters and $\sigma = 4$.

[24]In our baseline model, a counterfactually large reaction of transfers is required to achieve $\chi_y = 1$. We show in Appendix C.2 that a version of the model with sticky wages delivers values of χ_y close to one when transfers are calibrated to match the retention function in Heathcote, Storesletten and Violante (2017).

[25]With a risk aversion of $\sigma = 4$ and the estimated distribution of disaster sizes, the expected change in marginal utility conditional on a disaster is given by $\mathbb{E}\left[(1-\zeta_A)^{-\sigma}\right] = 7.69$.

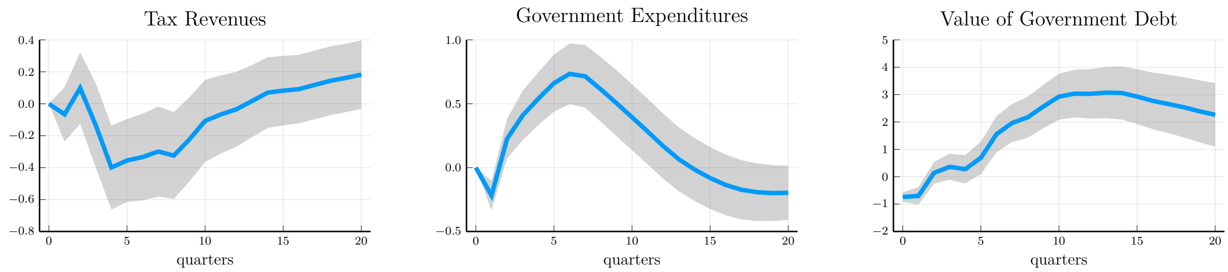

Figure 2: Estimated fiscal response to a monetary policy shock

Note: IRFs computed from a VAR identified by a recursiveness assumption.Variables included: real GDP per capita, CPI inflation, real consumption per capita, real investment per capita, capacity utilization, hours worked per capita, real wages, tax revenues over GDP, government expenditures over GDP, federal funds rate, 5-year constant maturity rate and the real value of government debt over GDP. We estimate a four-lag VAR using quarterly data for the period 1962:1-2007:3. The real value of government debt and the 5-year rate are ordered last, and the fed funds rate is ordered third to last. Gray areas are bootstrapped 68% confidence bands.

For the policy variables, we follow Jiang, Lustig, Van Nieuwerburgh and Xiaolan (2019) and estimate a standard VAR augmented with fiscal variables and compute empirical IRFs applying the recursiveness assumption. We provide the details of the estimation in Appendix E. Figure 2 shows the dynamics of fiscal variables in the estimated VAR in response to a contractionary monetary shock that increases the policy rate by 100 bps on impact. Government revenues fall in response to the contractionary shock, while government expenditures fall on impact and then turn positive, likely driven by the automatic stabilizer mechanisms embedded in the government accounts. The present value of interest payments increases by 36 bps and the initial value of government debt drops by 18 bps.[26] The present value of primary surpluses increases by just 9 bps.

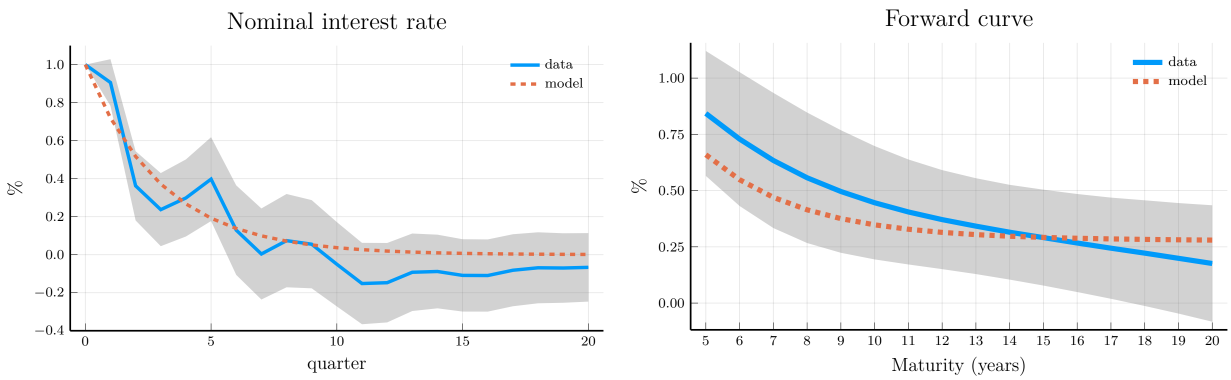

Figure 3: Nominal interest rate and forward curve.

Note: The left panel shows the IRF for the Federal Funds rate in the VAR and the path of nominal interest rates in the model. The right panel shows the response of forward rates to a 100 bps change in the two-year yield, as estimated by Hanson and Stein (2015), and the corresponding forward curve in the model when the monetary shock is scaled such that the two-year yield increases by 100 bps. Grey areas are confidence bands.

4.2 Asset-pricing implications of D-HANK

We focus on a monetary shock that generates a path for the nominal interest rate that can be represented by $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$. We set $\psi_m = 0.33$, which gives a half-life of roughly two quarters, so the path of nominal interest rates closely matches the impulse-response of the Federal Funds rate from the VAR, as shown in the left panel of Figure 3. To obtain $\hat{\lambda}_t$, we need to calibrate ϵ_λ, which determines the elasticity of asset prices to monetary shocks, and ψ_λ, which captures the persistence of changes in risk premia. We calibrate these parameters to match two sets of moments. First, the initial response of the 5-year yield on government bonds to a monetary shock. We find that a 100 bps increase in the nominal interest rate leads to a 32 bps increase in the 5-year yield. Second, the response of the entire *forward curve* around FOMC meetings, as estimated by Hanson and Stein (2015). The solid line in the right panel of Figure 3 shows their estimates of the

[26] The present discounted value of interest payments is calculated as $\sum_{t=0}^{\mathcal{T}} \left(\frac{1+g}{1+i_L}\right)^{\frac{t}{4}} \left[\overline{d}_t^g (\hat{i}_{L,t} - \hat{\pi}_t)\right]$, and similarly for other variables, where $\mathcal{T}$ is the truncation period, $\hat{i}_{L,t}$ is the IRF of the 5-year rate estimated in the data, and $\hat{\pi}_t$ is the IRF of inflation. We set $g = 0.02$ and $i_L = 0.043$. We choose $\mathcal{T} = 60$ quarters, when the main macroeconomic variables, including government debt, are back to their pre-shock values.

response of forwards rates to a 100 bps change in the two-year yield, while the dashed line shows the corresponding response of forward rates in the model.[27] A striking feature of Hanson and Stein's (2015) results was that monetary shocks affected forward rates in the far distant future, a fact at odds with standard models. In contrast, Figure 3 shows that our model is able to closely match their evidence.

The procedure above gives a value of 0.57 to ψ_λ, implying a half-life of roughly 4 months.[28] The value of ϵ_λ is 315, which implies a change of 33 bps in the probability of disaster in response to a 25 bps monetary shock. Given that monetary shocks are typically small in the data, this implies a variability in the market-implied disaster probability induced by monetary shocks that is only a small fraction of the overall volatility in the disaster probability of 114 bps, as estimated by Wachter (2013).[29]

Figure 4 shows the response of the yield on the long-term bond and the contributions of the path of future interest rates and of the term premium. The bulk of the reaction of the yield reflects movements in the term premium, consistent with the findings of e.g. Gertler and Karadi (2015). The model also captures the responses of asset prices that were not directly targeted in the calibration. Consider first the *corporate spread*, the difference between the yield on a corporate bond and the yield on a government bond (without risk of default) with the same promised cash flow. This is consistent with the way the GZ spread is computed in the data by Gilchrist and Zakrajšek (2012). Let $e^{-\psi_F t}$ denote the coupon paid by the bond issued by the representative firm. We assume that the monetary shock is too small to trigger a default, but corporate bonds default if a disaster occurs, where lenders recover the fraction $1 - \zeta_F$ of promised coupons. We calibrate ψ_F and ζ_F to

[27] Appendix C contains the derivation of the partial differential equation (PDE) describing the evolution of forward rates and the procedure we used to numerically solve it.

[28] This value of ψ_λ implies a lower persistence of iMPCs than the estimates of Fagereng et al. (2021), but it is roughly in line with the recent findings of Borusyak et al. (2024).

[29] The value for ϵ_λ is consistent with an annual disaster probability of less than 0.01% for optimistic savers and 7% for pessimistic savers in our preferred interpretation. In Appendix C.3, we show how the mapping between ϵ_λ and the underlying belief heterogeneity changes under different assumptions.

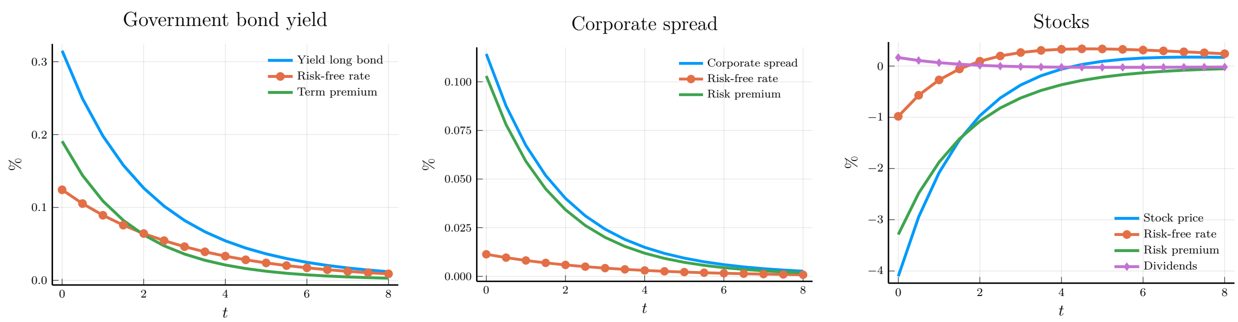

Figure 4: Asset-pricing response to monetary shocks.

match a duration of 6.5 years and a credit spread of 200 bps in the stationary equilibrium, consistent with the estimates reported by Gilchrist and Zakrajšek (2012). Note that the calibration targets the *unconditional* level of the credit spread. We evaluate the model on its ability to generate an empirically plausible *conditional* response to monetary shocks.

Figure 4 shows that the corporate spread responds to monetary shocks by 11 bps. We introduce the excess bond premium (EBP) in our VAR and find an increase in the EBP of 6.5 bps with a standard-error of 3.1 bps, roughly consistent with the model's prediction. Thus, the model produces quantitatively plausible movements in the corporate spread.

Another untargeted moment is the response of equity prices. The model generates a decline in stocks of 4.0% in response to a 100 bps increase in interest rates, which coincides with the point estimate of Bernanke and Kuttner (2005).[30] Consistent with their findings, the response of stocks is explained mostly by movements in the risk premium. Notice the price-dividend ratio falls after a contractionary shock, despite a low EIS. In contrast, Barro (2009) finds that the price-dividend ratio in the endowment disaster model with separable utility *increases* with the probability of disaster when the EIS is less than one. This motivates the adoption of a high EIS in an Epstein-Zin setting.[31] Sticky prices is cru-

[30]We follow standard practice in the asset-pricing literature and report the response of a levered claim on firms' profits, using a debt-to-equity ratio of 0.5, as in Barro (2006).

[31] For a similar reason, a high EIS is adopted in long-run risk models, see e.g. Bansal and Yaron (2004).

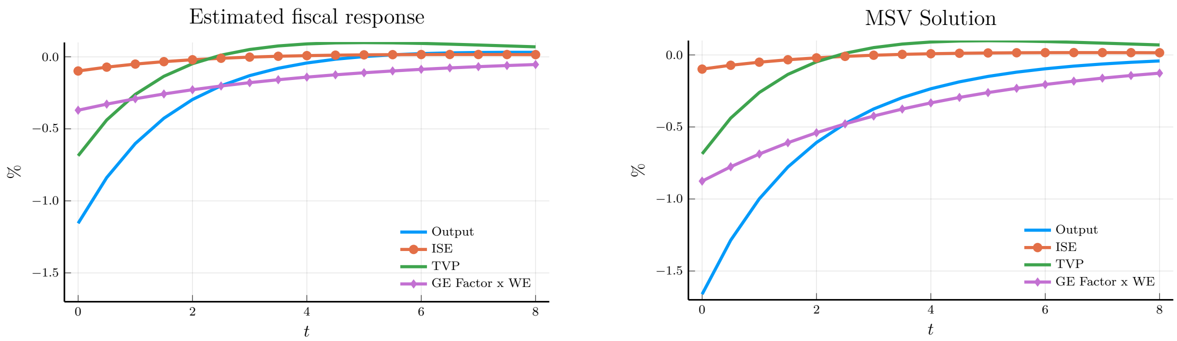

Figure 5: Output in D-HANK.

Note: In both plots, the path of the nominal interest rate is given by $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$, where $i_0 - r_n$ equals 100 bps. The left panel shows the solution with the estimated fiscal backing, while the right panel shows the MSV solution.

cial to avoid counterfactual movements in equity prices in our CRRA setting, as changes in disaster probability would have the opposite effect on stock prices in the flexible-price version of the model. Dividends are roughly acyclical. Due to the assumption of GHH preferences, we avoid the strongly countercyclical profits typical of sticky-prices models.

4.3 Wealth effects in the monetary transmission mechanism

Figure 5 presents the response to a monetary shock of output and its components. The left panel shows the solution when the fiscal backing matches the empirical estimates of Section 4.1, while the right panel shows the conventional MSV solution. In the case with the estimated fiscal backing, output drops on impact by 1.15% in response to an increase of 100 bp in the nominal interest rate, roughly in line with the estimates by Miranda-Agrippino and Ricco (2021). The time-varying precautionary motive (TVP) accounts for 60% of the initial output response, while the aggregate wealth effect (adjusted by the GE factor) accounts for 30%. The ISE accounts for less than 10% of the initial output response, indicating that intertemporal substitution plays only a minor role in our model.

We find stronger real effects with the MSV solution, where output drops by 1.66%

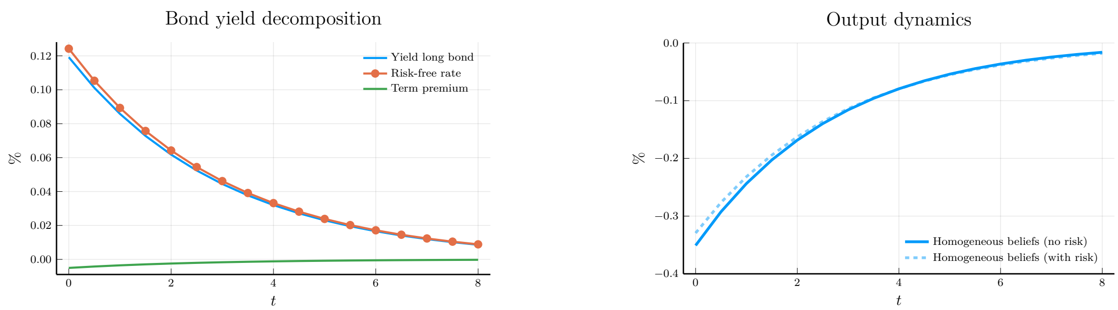

Figure 6: Long-term bond yields and output for economies with and without risk.

on impact. The difference is entirely driven by the aggregate wealth effect, which now explains more than the 50% of the overall effect, with the ISE and TVP being numerically the same as in the case with the estimated fiscal backing. The stronger impact on output, however, requires an increase in the present value of primary surpluses of more than 220 bps, which is more than twenty times bigger than what we estimate in the data.

4.4 The limitations of the homogeneous-beliefs model

The model delivers a substantial response of output, despite a relatively weak intertemporal substitution channel. But is this the result of introducing disaster risk or is it due to heterogeneous beliefs? To answer this question, we consider the behavior of asset prices and output in an economy with homogeneous beliefs (i.e. $\lambda > 0$ but $\epsilon_\lambda = 0$).

Figure 6 (left) shows that the yield on the long bond increases by only 12 bps, less than half of the response estimated by the VAR in Section 4.1. Moreover, the term premium is essentially zero. In this case, stocks would also be mostly driven by interest rates instead of risk premia, inconsistent with the evidence in Bernanke and Kuttner (2005).

Figure 6 (right) shows the response of output for an economy with disaster risk and homogeneous beliefs (solid line) and an economy without disaster risk (dashed line). In both cases, we consider the solution that matches the estimated fiscal backing. In the absence of belief heterogeneity, the impact on output of a monetary shock is substantially weaker, with a drop in output of roughly 0.35%. This is more than three times smaller than the impact on output in the case with belief heterogeneity. Moreover, the solution with disaster risk and homogeneous beliefs is almost identical to the one without disasters.

Introducing disaster risk allows the model to capture important *unconditional* asset-pricing moments, such as the equity premium or an upward-sloping yield curve, but the model is unable to match key *conditional* moments, such as the response of asset prices to monetary policy, which affects how monetary policy impacts the real economy.

5 The Effect of Risk and Maturity of Household Debt

We have focused so far on how monetary policy affects the value of households' assets, such as stocks and bonds. However, movements in risk premia can also affect the real economy through its impact on household debt. In this section, we extend the baseline model to allow workers to borrow a positive amount using long-term risky debt.

5.1 The model with long-term risky household debt

Workers issue long-term debt that promises to pay exponentially decaying coupons given by $e^{-\psi_P t}$ at period $t \geq 0$, where $\psi_P \geq 0$. In response to a large shock, i.e., the occurrence of a disaster, workers default and lenders receive a fraction $1 - \zeta_P$ of the promised coupons, where $0 \leq \zeta_P \leq 1$. Fluctuations in the no-disaster state are small enough such that they do not trigger a default. Thus, workers default only in the disaster state.

Workers can borrow up to $\overline{D}_{P,t} = Q_{P,t}\overline{F}$, which effectively puts a limit on the face value of private debt $\overline{F}$.[32] The (log-linearized) consumption of workers is given by

$$c_{w,t} = \chi_y y_t - \left(\frac{\psi_P}{i_P + \psi_P}(i_{P,t} - i_P) - \pi_t \right) \overline{d}_P, \tag{30}$$

where $\overline{d}_P \equiv \frac{\overline{D}_P}{Y}$ denotes the debt-to-income ratio in the stationary equilibrium, and $i_{P,t} = \frac{1}{Q_{P,t}} - \psi_P$ is the yield on household debt. Equation (30) generalizes the expression for workers' consumption given in Section 2. When debt is short-term, $\psi_P \to \infty$, and riskless, $\zeta_P = 0$, we obtain $i_{P,t} = i_t$. With a consol, $\psi_P = 0$, households simply pay the coupon every period and there is no need to issue new debt. Therefore, they are insulated from movements in nominal rates. For intermediate values of maturity and risk, monetary policy affects workers through changes in the nominal interest rate i_t and the spread $r_{P,t}$.

Proposition 8 (Aggregate output with long-term risky household debt)**.** *Suppose that $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$ and $r_P \sigma c_{s,t} = \mathcal{O}(||i_t - r_n||^2)$. Aggregate output is then given by*

$$y_t = \underbrace{\tilde{\sigma}^{-1}\hat{y}_{m,t}}_{\text{ISE}} + \underbrace{\chi_\lambda \hat{y}_{\lambda,t}}_{\substack{\text{time-varying}\\ \text{precautionary motive}}} + \underbrace{\frac{\mu_w \overline{d}_P \psi_P}{1 - \mu_w \chi_y}\left[\frac{\tilde{\psi}_m \hat{y}_{m,t}}{\rho + \psi_P + \psi_m} + \frac{r_P \epsilon_\lambda \tilde{\psi}_\lambda \hat{y}_{\lambda,t}}{\rho + \psi_P + \psi_\lambda}\right]}_{\text{household-debt effect}} + \underbrace{(\rho - \underline{\omega})e^{\underline{\omega}t}\Omega_0}_{\substack{\text{GE factor}\times\\ \text{aggregate wealth effect}}},$$

where $\tilde{\psi}_k = \psi_k + \rho - r_n$ for $k \in \{m, \lambda\}$.

Proposition 8 extends the decomposition in Proposition 6 to the case of long-term risky household debt. Household debt effectively amplifies the ISE and the time-varying precautionary motive effect. If household debt is safe and short term (i.e, $\zeta_P = 0$ and $\psi_P \to \infty$), then the household-debt effect loads only on $\hat{y}_{m,t}$, amplifying the ISE. When debt is long-term or when households can default, then $r_P > 0$ and the household-debt effect also loads on $\hat{y}_{\lambda,t}$, amplifying the precautionary motive effect.

[32] This formulation guarantees that, after an increase in nominal rates, the value of household debt and the borrowing limit decline by the same amount. This specification of the borrowing constraint, combined with the assumption of impatient borrowers, guarantees that borrowers are constrained at all periods.

5.2 Quantitative implications

We consider next the quantitative effects of introducing household debt. We calibrate $\overline{d}_P$ to match a debt service payment to disposable personal income of 10%. We choose ψ_P to match a duration of 5 years, consistent with the mortgage duration estimated by Greenwald, Leombroni, Lustig and Van Nieuwerburgh (2021) of 5.2 years. We choose ζ_P to match a spread of 2% in a stationary equilibrium relative to the riskless bond with the same promised coupons. Figure 7 shows the role of household debt in the transmission of monetary policy to the economy. The top left panel shows the output decomposition with the estimated fiscal backing. Output on impact drops by 1.6% in response to a 100 bp increase in nominal rates, where the TVP channel accounts for roughly half of the overall response and the aggregate wealth effect (adjusted by the GE factor) accounts for roughly 40%. The top right panel shows the decomposition for the MSV solution. In this case, the drop in output is nearly 50 bp larger than the one with the estimated fiscal backing. However, this requires a present value of primary surplus that is ten times larger than the one we estimated.

The bottom left panel of Figure 7 shows the impact on output for a range of special cases nested by our model. In all cases, we focus on the solution that matches the estimated fiscal backing. The line denoted by RANK corresponds to the solution without disaster risk and zero household debt, which aggregates to the textbook model. The line denoted by HANK corresponds to the solution with positive debt, which given the heterogeneous MPCs between workers and savers captures an important channel of typical HANK models. We also consider two versions of the model with heterogeneous beliefs (D-HANK), with and without household debt. The output response in HANK is 12 bp larger than in RANK. However, the impact on output in HANK is substantially smaller than in either version of D-HANK. Introducing household debt in D-HANK raises the impact on output by 48 bp. Hence, household debt interacts in important ways with

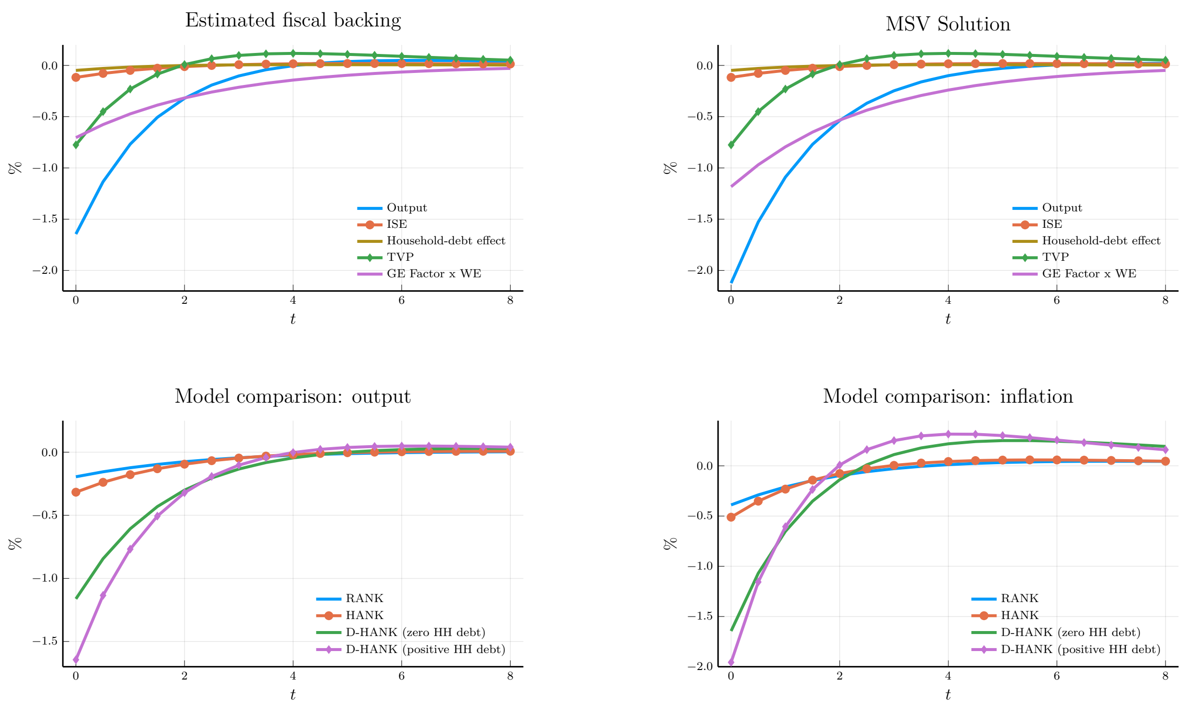

Figure 7: The role of household debt: output decomposition and model comparison

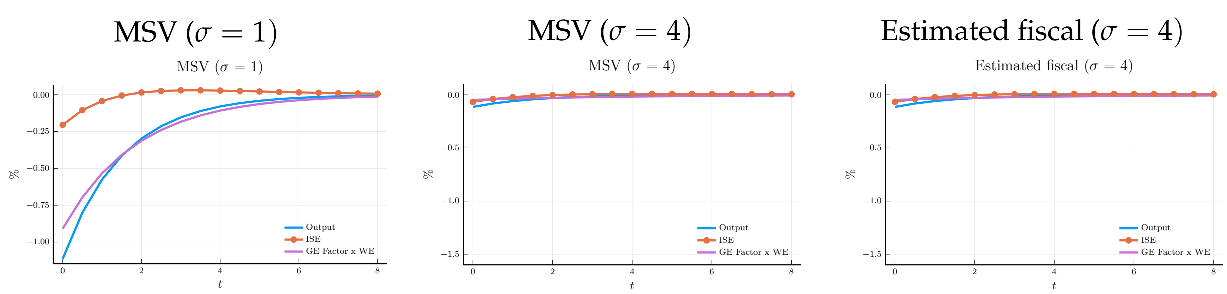

Figure 8: The role of the EIS in RANK's quantitative performance.

disaster risk. The bottom right panel shows the impact on inflation. A similar pattern emerges: we obtain a larger response of inflation under HANK than under RANK, but it is substantially weaker than the inflation response under D-HANK.

5.3 The role of the EIS

We have seen that the real effects of monetary shocks are significantly weaker when we shut down risk and heterogeneity. This appears to be in contrast with standard results from the textbook model, which typically generates large real effects. Figure 8 shows that the calibration of the EIS plays an important role for this result. The left panel shows the MSV solution of the RANK model when we set $\sigma = 1$ and use the persistence of monetary shocks from Galí (2015). Output drops by 1.1% in response to a 100 bp increase in nominal rates, a substantial effect. The aggregate wealth effect, adjusted by the GE factor, accounts for the majority of the output response. The middle panels shows the MSV solution of the RANK model for $\sigma = 4$, as in our baseline calibration. We keep all the other parameters fixed, including the slope of the Phillips curve κ. The response of output is now ten times smaller. The right panel shows the solution that matches the estimated fiscal backing with $\sigma = 4$, which is nearly the same as the MSV solution with $\sigma = 4$.

These results indicate that the quantitative performance of the standard RANK model relies on a counterfactually strong intertemporal-substitution effect, which ends up being amplified in general equilibrium by a large wealth effect. When the model is calibrated to match the observed levels of public debt, this strong wealth effect requires an implied fiscal backing that is too large relative to empirical estimates. This shows that the standard model lacks realistic mechanisms to generate large real effects of monetary policy. Introducing heterogeneous MPCs and household debt improves the model performance, but effects are still not large enough, in particular when debt is long term. We have seen that risk and belief heterogeneity provide a powerful mechanism to generate the strong real effects of monetary shocks observed in the data.

6 Conclusion

In this paper, we provide a novel unified framework to analyze the role of risk and heterogeneity in a tractable New Keynesian model. The methods introduced in this paper can be applied in other settings. For instance, they can be used to introduce time-varying risk premia in a full quantitative HANK model with idiosyncratic risk. One could also introduce a richer capital structure for firms and study the pass-through of monetary policy to households and firms. These methods may enable us to bridge the gap between the existing work on heterogeneous agents and monetary policy and the emerging literature on the role of asset prices in the transmission of monetary shocks.

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Appendix: Proofs

Proof of Proposition 1. To ensure that η_t correctly prices long-term bonds and equities, consistent with equations (2) and (3), the market-implied disaster probability must satisfy the condition $\lambda_t \left(\frac{C_{s,t}}{C^*_{s,t}}\right)^\sigma = \lambda_j \left(\frac{C_{j,t}}{C^*_{j,t}}\right)^\sigma \Rightarrow C^*_{j,t} = \left(\frac{\lambda_j}{\lambda_t}\right)^{\frac{1}{\sigma}} \frac{C^*_{s,t}}{C_{s,t}} C_{j,t}$. Plugging $C^*_{j,t}$ into the definition of savers' average consumption in the disaster state, $C^*_{s,t} \equiv \frac{\mu_o}{\mu_o+\mu_p} C^*_{o,t} + \frac{\mu_p}{\mu_o+\mu_p} C^*_{p,t}$, and rearranging gives equation (4). By setting $\rho_{s,t} \equiv \sum_{j\in\{o,p\}} \frac{\mu_j C_{j,t}}{\mu_o C_{o,t}+\mu_p C_{p,t}} (\rho_{j,t} + \lambda_j) - \lambda_t$, we ensure that η_t correctly prices risk-free bonds, i.e., $\mathbb{E}_t[d\eta_t]/\eta_t = -(i_t - \pi_t)dt$. □

Proof of Proposition 2. Consider the New Keynesian Phillips curve $\dot{\pi}_t = \left(i_t - \pi_t + \lambda_t \frac{\eta^*_t}{\eta_t}\right) \pi_t - \frac{\epsilon}{\varphi A} \left(\frac{W}{P} e^{w_t - p_t} - (1-\epsilon^{-1})A\right) Y e^{y_t}$. Linearizing the above expression, and using $\frac{W}{P} = (1 - \epsilon^{-1})A$, we obtain $\dot{\pi}_t = \left(r_n + \lambda \left(\frac{C_s}{C^*_s}\right)^\sigma\right) \pi_t - \varphi^{-1}(\epsilon - 1) Y (w_t - p_t)$. Using the fact that $w_t - p_t = \phi y_t$, we obtain $\dot{\pi}_t = (\rho_s + \lambda)\pi_t - \kappa y_t$, where $\kappa \equiv \varphi^{-1}(\epsilon-1)\phi Y$ and we used that $r_n + \lambda \left(\frac{C_s}{C^*_s}\right)^\sigma = \rho_s + \lambda$.

Consider next the generalized Euler equation. From the market-clearing condition for goods and workers' consumption, we obtain $c_{s,t} = \frac{1-\mu_w \chi_y}{1-\mu_w} y_t$. Combining this condition with the Phillips Curve and savers' Euler equation, and using the fact that $r_n = \rho - \lambda \left(\frac{C_s}{C^*_s}\right)^\sigma$, we obtain $\dot{y}_t = \tilde{\sigma}^{-1}(i_t - \pi - r_n) + \delta y_t + \chi_\lambda \hat{\lambda}_t$, where the constants $\tilde{\sigma}^{-1}$, δ, and χ_λ are defined in the proposition.

□

Proof of Proposition 3. The linearized Euler equation for saver j is given by $\dot{c}_{j,t} = \sigma^{-1}(i_t - \pi_t - r_n) + \frac{\lambda}{\sigma}\left(\frac{C_s}{C^*_s}\right)^\sigma (\hat{\lambda}_t + \sigma c_{s,t}) - \xi(c_{j,t} - c_{s,t})$. Taking the difference of the Euler equation for the two types, we obtain $\dot{c}_{p,t} - \dot{c}_{o,t} = -\xi(c_{p,t} - c_{o,t})$. Linearizing the savers' flow budget constraint, we obtain $\dot{b}_{p,t} - \dot{b}_{o,t} = \sum_{k\in\{L,E\}} r_k \left[\hat{r}_{k,t}\left(\frac{B^k_p}{B_p} - \frac{B^k_o}{B_o}\right) + \frac{B^k_p}{B_p} b^k_{p,t} - \frac{B^k_o}{B_o} b^k_{o,t}\right] - \left(\frac{C_p}{B_p} c_{p,t} - \frac{C_o}{B_o} c_{o,t}\right) + r_n(b_{p,t} - b_{o,t})$ where $\hat{r}_{k,t} = \hat{\lambda}_t + \sigma c_{s,t} + \frac{Q^*_k}{Q_k - Q^*_k} q_{k,t}$. The relative net worth in the disaster state at $t = t^*$ is given by $\frac{B^*_p}{B_p} b^*_{p,t^*} - \frac{B^*_o}{B_o} b^*_{o,t^*} = b_{p,t^*} - b_{o,t^*} -$

$\sum_{k\in\{L,E\}}\left[\left(\frac{B_p^k}{B_p}-\frac{B_o^k}{B_o}\right)\frac{Q_k^*}{Q_k}q_{k,t^*}+\frac{Q_k-Q_k^*}{Q_k}\left(\frac{B_p^k}{B_p}b_{p,t^*}^k-\frac{B_o^k}{B_o}b_{o,t^*}^k\right)\right]$.

From the revaluation of net worth in the disaster state, shown above, we can solve for the difference in portfolios $\frac{B_p^k}{B_p}b_{p,t^*}^k-\frac{B_o^k}{B_o}b_{o,t^*}^k$. From the optimality condition for risky assets, we obtain $c_{p,t}-c_{o,t}=c_{p,t}^*-c_{o,t}^*$. Savers' consumption in the disaster state is given by $c_{j,t}^*=\frac{r_n^*B_j^*}{C_s^*}b_{j,t}^*$. Combining these expressions, we obtain the relative net worth in the disaster state. We can then solve for the dynamics of relative net worth in the no-disaster state: $\dot{b}_{p,t}-\dot{b}_{o,t}=\rho(b_{p,t}-b_{o,t})-\chi_{b,c}(c_{p,t}-c_{o,t})+\chi_{b,c_s}c_{s,t}$, where $\chi_{b,c_s}\equiv(\sigma-1)\sum_{k\in\{L,E\}}r_k\left(\frac{B_p^k}{B_p}-\frac{B_o^k}{B_o}\right)$, and $\chi_{b,c}\equiv\sigma\mu_{c,o}\mu_{c,p}\left(\frac{\lambda_p^{\frac{1}{\sigma}}-\lambda_o^{\frac{1}{\sigma}}}{\lambda^{\frac{1}{\sigma}}}\right)\sum_{k\in\{L,E\}}r_k\left(\frac{B_o^k}{B_o}-\frac{B_p^k}{B_p}\right)+\mu_{c,p}\frac{C_o}{B_o}+\mu_{c,o}\frac{C_p}{B_p}+\frac{C_s^*(\rho-r_n)}{r_n^*B_s}$, where $\chi_{b,c}>0$. Assuming $\sigma r_k c_s=\mathcal{O}(||i_t-r_n||^2)$, the term involving $c_{s,t}$ can be ignored up to first order. We then obtain a dynamic system in $c_{p,t}-c_{o,t}$ and $b_{p,t}-b_{o,t}$, which has a positive and a negative eigenvalue, so there is a unique bounded solution given by $c_{p,t}-c_{o,t}=\frac{\rho+\xi}{\chi_{b,c}}e^{-\psi_\lambda t}(b_{p,0}-b_{o,0})$ and $b_{p,t}-b_{o,t}=e^{-\psi_\lambda t}(b_{p,0}-b_{o,0})$, where $\psi_\lambda=\xi$. We can then write the market-implied disaster probability as $\hat{\lambda}_t=e^{-\psi_\lambda t}\chi_{\lambda,c}\frac{\rho+\xi}{\chi_{b,c}}(b_{p,0}-b_{o,0})$, where $\chi_{\lambda,c}=\sigma\mu_{c,o}\mu_{c,p}\left(\frac{\lambda_p^{\frac{1}{\sigma}}-\lambda_o^{\frac{1}{\sigma}}}{\lambda^{\frac{1}{\sigma}}}\right)$. The revaluation of the relative net worth is given by $b_{p,0}-b_{o,0}=\left(\frac{B_p^L}{B_p}-\frac{B_o^L}{B_o}\right)q_{L,0}$, using the assumption that $B_o^E=B_p^E$. The price of the long-term bond is given by $q_{L,0}=-\frac{i_0-r_n}{\rho+\psi_L+\psi_m}-\frac{r_L\hat{\lambda}_0}{\rho+\psi_L+\psi_\lambda}$. Combining the expressions for $\hat{\lambda}_t$, relative net worth, and bond prices, we obtain $\hat{\lambda}_t=e^{-\psi_\lambda t}\epsilon_\lambda(i_0-r)$, where ϵ_λ is given by $\epsilon_\lambda\equiv\left[1-\chi_{\lambda,c}\frac{\rho+\xi}{\chi_{b,c}}\left(\frac{B_o^L}{B_o}-\frac{B_p^L}{B_p}\right)\frac{r_L}{\rho+\psi_L+\psi_\lambda}\right]^{-1}\left[\chi_{\lambda,c}\frac{\rho+\xi}{\chi_{b,c}}\left(\frac{B_o^L}{B_o}-\frac{B_p^L}{B_p}\right)\frac{1}{\rho+\psi_L+\psi_m}\right]$.

□

Proof of Lemma **??**. Linearizing the aggregate intertemporal budget constraint, we obtain $Q_C q_{c,0}=D_G q_{L,0}+Q_E q_{E,0}+Q_H q_{H,0}$, where $Q_{H,t}$ is the present discounted value of wages plus transfers. Using the pricing condition for $q_{k,0}$, $k \in \{C, H, E\}$, we obtain

$$\int_0^\infty e^{-\rho t} c_t dt - \frac{Q_C}{Y} \int_0^\infty e^{-\rho t} \left[i_t - \pi_t - r_n + r_C p_{d,t}\right] dt = \int_0^\infty e^{-\rho t} \left[\hat{\Pi}_t + \frac{WN}{PY}(w_t - p_t + n_t) + \hat{T}_t\right] dt$$
$$- \frac{Q_H + Q_E}{Y} \int_0^\infty e^{-\rho t} \left[i_t - \pi_t - r_n\right] dt - \left[\frac{Q_H}{Y} r_H + \frac{Q_E}{Y} r_E\right] \int_0^\infty e^{-\rho t} p_{d,t} dt + \frac{D_G}{Y} q_{L,0}.$$

Using the fact that $Q_C = D_G + Q_E + Q_H$ and $Q_C^* = D_G \frac{Q_L^*}{Q_L} + Q_E^* + Q_H^*$, we obtain $\frac{Q_C}{Y} - \frac{Q_H + Q_E}{Y} = \frac{D_G}{Y} \equiv \overline{d}_G$ and $\frac{Q_C}{Y} r_C - \frac{Q_H r_H + Q_E r_E}{Y} = \overline{d}_G r_L$, given $r_k = \lambda \left(\frac{C_s}{C_s^*}\right)^\sigma \frac{Q_k - Q_k^*}{Q_k}$. Combining these expressions with the equation above, we obtain (**??**) after some rearrangement, and using the fact that $\hat{\Pi}_t + \frac{WN}{PY}(w_t - p_t + n_t) + \hat{T}_t = y_t - (\chi_\tau y_t + \tau_t)$. $\square$

Proof of Lemma 1. Consumption of a type-j saver satisfies $\int_0^\infty e^{-\rho t}(c_{j,t} + \chi_{c_j^*} c_{j,t}^*) = \Omega_{j,0}$, $\dot{c}_{j,t} = \dot{c}_{s,t} - \xi(c_{j,t} - c_{s,t})$, and $\sigma(c_{j,t} - c_{j,t}^*) = \hat{\lambda}_t + \sigma(c_{s,t} - c_{s,t}^*)$. Combining these conditions, we obtain $c_{j,t} = c_{s,t} + \frac{(\rho+\xi)e^{-\xi t}}{1+\overline{\chi}\lambda_j^{\frac{1}{\sigma}}}(\Omega_{j,0} - \Omega_{s,0}) + \frac{\overline{\chi}\lambda_j^{\frac{1}{\sigma}}}{1+\overline{\chi}\lambda_j^{\frac{1}{\sigma}}} \frac{\hat{\lambda}_t}{\sigma}$ and $c_{j,t}^* = \frac{(\rho+\xi)e^{-\xi t}}{1+\overline{\chi}\lambda_j^{\frac{1}{\sigma}}}(\Omega_{j,0} - \Omega_{s,0}) - \frac{1}{1+\overline{\chi}\lambda_j^{\frac{1}{\sigma}}} \frac{\hat{\lambda}_t}{\sigma}$, using $\chi_{c_j^*} = \overline{\chi}\lambda_j^{\frac{1}{\sigma}}$, where $\overline{\chi} \equiv \frac{\delta}{r_n^*}\overline{\chi}^*$ and $\overline{\chi}^* \equiv \lambda^{-\frac{1}{\sigma}} \frac{C_s^*}{C_s}$. From these expressions, we obtain the iMPCs. From $\mathcal{M}_{j,t} + \frac{\delta}{r_n^*}\mathcal{M}_{j,t}^* = (\rho + \xi)e^{-\xi t}$, we obtain (24). $\square$

Proof of Proposition 5. Given $c_{j,t} = c_{s,t} + \frac{(\rho+\xi)e^{-\xi t}}{1+\overline{\chi}\lambda_j^{\frac{1}{\sigma}}}(\Omega_{j,0} - \Omega_{s,0}) + \frac{\overline{\chi}\lambda_j^{\frac{1}{\sigma}}}{1+\overline{\chi}\lambda_j^{\frac{1}{\sigma}}} \frac{\hat{\lambda}_t}{\sigma}$, the difference in consumption at date t is given by $c_{p,t} - c_{o,t} = \mathcal{M}_{p,t}(\Omega_{p,0} - \Omega_{s,0}) - \mathcal{M}_{o,t}(\Omega_{o,0} - \Omega_{s,0}) + \left(\mathcal{M}_{p,0}^* - \mathcal{M}_{o,0}^*\right) \frac{\overline{\chi}}{\overline{\chi}^*} \frac{\hat{\lambda}_t}{\sigma} \frac{1}{\rho+\xi}$, as $\mathcal{M}_{j,0}^* = \frac{(\rho+\xi)\overline{\chi}^* \lambda_j^{\frac{1}{\sigma}}}{1+\overline{\chi}\lambda_j^{\frac{1}{\sigma}}}$. Using $\Omega_{s,0} = \mu_{c,o}\Omega_{o,0} + \mu_{c,p}\Omega_{p,0}$, we can write the expression above as follows: $c_{p,t} - c_{o,t} = \left[\mathcal{M}_{p,t}\mu_{c,o} + \mathcal{M}_{o,t}\mu_{c,p}\right](\Omega_{p,0} - \Omega_{o,0}) + \left(\mathcal{M}_{p,0}^* - \mathcal{M}_{o,0}^*\right) \frac{\overline{\chi}}{\overline{\chi}^*} \frac{\hat{\lambda}_t}{\sigma} \frac{1}{\rho+\xi}$. As $\hat{\lambda}_t = \chi_{\lambda,c}(c_{p,t} - c_{o,t})$, then $c_{p,t} - c_{o,t} = \frac{(\mathcal{M}_{p,t}\mu_{c,o} + \mathcal{M}_{o,t}\mu_{c,p})\left[\Omega_{p,o} - \Omega_{o,0}\right]}{1 - \left(\mathcal{M}_{p,0}^* - \mathcal{M}_{o,0}^*\right) \frac{\overline{\chi}}{\overline{\chi}^*} \frac{\chi_{\lambda,c}}{\sigma(\rho+\xi)}}$. Therefore, $\hat{\lambda}_t$ is given by $\hat{\lambda}_t = \frac{\chi_{\lambda,c} e^{-\xi t}\left(\mathcal{M}_{p,0}\mu_{c,o} + \mathcal{M}_{o,0}\mu_{c,p}\right)}{1 - \left(\mathcal{M}_{p,0}^* - \mathcal{M}_{o,0}^*\right) \frac{\overline{\chi}}{\overline{\chi}^*} \frac{\chi_{\lambda,c}}{\sigma(\rho+\xi)}} \left[\Omega_{p,o} - \Omega_{o,0}\right]$. We show next that $\chi_{\lambda,c}$ is proportional to the differences in iMPCs. From the definition of $\hat{\lambda}_t$, we have $\chi_{\lambda,c} = \frac{\sigma\mu_{c,o}\mu_{c,p}\left(\lambda_p^{\frac{1}{\sigma}} - \lambda_o^{\frac{1}{\sigma}}\right)}{\mu_{c,o}\lambda_o^{\frac{1}{\sigma}} + \mu_{c,p}\lambda_p^{\frac{1}{\sigma}}}$. Then, $\chi_{\lambda,c} = \frac{\overline{\chi}^*}{\overline{\chi}} \frac{\sigma\mu_{c,0}\mu_{c,p}\left(\frac{\mu_o}{\mu_o+\mu_p}(1+\overline{\chi}\lambda_p^{\frac{1}{\sigma}}) + \frac{\mu_p}{\mu_o+\mu_p}(1+\overline{\chi}\lambda_o^{\frac{1}{\sigma}})\right)}{\left[\frac{\mu_o}{\mu_o+\mu_p}\mathcal{M}_{o,0}^* + \frac{\mu_p}{\mu_o+\mu_p}\mathcal{M}_{p,0}^*\right]} \left(\mathcal{M}_{o,0} - \mathcal{M}_{p,0}\right)$, using the expression for $\mu_{c,j}$. This gives $\hat{\lambda}_t = \overline{\chi}_\lambda \left(\mathcal{M}_{o,t} - \mathcal{M}_{p,t}\right)\left[\Omega_{p,0} - \Omega_{o,0}\right]$, for a constant $\overline{\chi}_\lambda$. Notice that $\overline{\chi}_\lambda > 0$ if and only if $1 - \left(\mathcal{M}^*_{p,0} - \mathcal{M}^*_{o,0}\right)\frac{\overline{\chi}}{\overline{\chi}^*}\frac{\chi_{\lambda,c}}{\sigma(\rho+\xi)} > 0$. Using the expressions for $\chi_{\lambda,c}$, $\mathcal{M}^*_{j,0}$, $\mu_{c,j}$, and that $\lambda_o^{-\frac{1}{\sigma}} = \frac{\mu_o+\mu_p}{\mu_o}\lambda^{-\frac{1}{\sigma}} - \frac{\mu_p}{\mu_o}\lambda_p^{-\frac{1}{\sigma}}$, this is equivalent to showing that the function $F\left(\lambda_p,\lambda\right) \equiv 1 - \frac{\mu_p}{\mu_o+\mu_p}\overline{\chi}\left(\frac{\lambda_p^{\frac{1}{\sigma}}}{1+\overline{\chi}\lambda_p^{\frac{1}{\sigma}}} - \frac{1}{\frac{\mu_o+\mu_p}{\mu_o}\lambda^{-\frac{1}{\sigma}} - \frac{\mu_p}{\mu_o}\lambda_p^{-\frac{1}{\sigma}} + \overline{\chi}}\right)\frac{\lambda^{-\frac{1}{\sigma}} - \lambda_p^{-\frac{1}{\sigma}}}{\lambda^{-\frac{1}{\sigma}}}$ is positive for all $\lambda_p \geq \lambda$ and $\lambda > 0$. We have that $F(\lambda,\lambda) = 1$ and $\lim_{\lambda_p\to\infty} F\left(\lambda_p,\lambda\right) = 1 - \frac{\mu_p}{\mu_o+\mu_p}\frac{1}{1+\overline{\chi}\frac{\mu_o}{\mu_o+\mu_p}\lambda^{\frac{1}{\sigma}}} > 0$. Moreover, it is immediate to see that, given λ , $F\left(\lambda_p,\lambda\right)$ is strictly decreasing in λ_p. Hence, $\overline{\chi}_\lambda > 0$ for all $\lambda_p \geq \lambda$ and $\lambda > 0$. □

Proof of Propositions 6 and 7. We can write dynamic system in matrix form as $\dot{Z}_t = AZ_t + B\nu_t$, where $B = [1,0]'$. Applying the eigendecomposition to matrix A, we obtain $A = V\Omega V^{-1}$ where $V = \begin{bmatrix} \frac{\rho-\overline{\omega}}{\kappa} & \frac{\rho-\underline{\omega}}{\kappa} \\ 1 & 1 \end{bmatrix}$, $V^{-1} = \frac{\kappa}{\overline{\omega}-\underline{\omega}}\begin{bmatrix} -1 & \frac{\rho-\underline{\omega}}{\kappa} \\ 1 & -\frac{\rho-\overline{\omega}}{\kappa} \end{bmatrix}$, and $\Omega = \begin{bmatrix} \overline{\omega} & 0 \\ 0 & \underline{\omega} \end{bmatrix}$. Decoupling the system, we obtain $\dot{z}_t = \Omega z_t + b\nu_t$, where $z_t = V^{-1}Z_t$ and $b = V^{-1}B$.

Solving the equation with a positive eigenvalue forward and the one with a negative eigenvalue backward, and rotating the system back to the original coordinates, we obtain

$$y_t = V_{12}\left(V^{21}y_0 + V^{22}\pi_0\right)e^{\underline{\omega}t} - V_{11}V^{11}\int_t^\infty e^{-\overline{\omega}(z-t)}\nu_z dz + V_{12}V^{21}\int_0^t e^{\underline{\omega}(t-z)}\nu_z dz$$
$$\pi_t = V_{22}\left(V^{21}y_0 + V^{22}\pi_0\right)e^{\underline{\omega}t} - V_{21}V^{11}\int_t^\infty e^{-\overline{\omega}(z-t)}\nu_z dz + V_{22}V^{21}\int_0^t e^{\underline{\omega}(t-z)}\nu_z dz,$$

where $V^{i,j}$ is the (i,j) entry of matrix V^{-1}. Integrating $e^{-\rho t}y_t$, we obtain $\Omega_0 = V_{12}\left(V^{21}y_0 + V^{22}\pi_0\right)\frac{1}{\rho-\underline{\omega}} - \frac{1}{\rho-\overline{\omega}}V_{11}V^{11}\int_0^\infty\left(e^{-\overline{\omega}t} - e^{-\rho t}\right)\nu_t dt + \frac{1}{\rho-\underline{\omega}}V_{12}V^{21}\int_0^\infty e^{-\rho t}\nu_t dt$. Rearranging the above expression, we obtain $V_{12}\left(V^{21}y_0 + V^{22}\pi_0\right) = (\rho-\underline{\omega})\Omega_0 + \frac{\rho-\underline{\omega}}{\rho-\overline{\omega}}V_{11}V^{11}\int_0^\infty e^{-\overline{\omega}t}\nu_t dt$, where we used the fact $\frac{V_{11}V^{11}}{\rho-\overline{\omega}} + \frac{V_{12}V^{21}}{\rho-\underline{\omega}} = 0$. Output is then given by $y_t = \tilde{y}_t + (\rho-\underline{\omega})e^{\underline{\omega}t}\Omega_0$, where $\tilde{y}_t = -\frac{\overline{\omega}-\rho}{\overline{\omega}-\underline{\omega}}\int_t^\infty e^{-\overline{\omega}(z-t)}\nu_z dz + \frac{\overline{\omega}-\delta}{\overline{\omega}-\underline{\omega}}\int_0^t e^{\underline{\omega}(t-z)}\nu_z dz - \frac{\rho-\underline{\omega}}{\overline{\omega}-\underline{\omega}}e^{\underline{\omega}t}\int_0^\infty e^{-\overline{\omega}z}\nu_z dz$. Inflation is given by $\pi_t = \tilde{\pi}_t + \kappa e^{\underline{\omega}t}\Omega_0$, where $\tilde{\pi}_t = \frac{\kappa}{\overline{\omega}-\underline{\omega}}\int_t^\infty e^{-\overline{\omega}(z-t)}\nu_z dz + \frac{\kappa}{\overline{\omega}-\underline{\omega}}\int_0^t e^{\underline{\omega}(t-z)}\nu_z dz - \frac{\kappa}{\overline{\omega}-\underline{\omega}}e^{\underline{\omega}t}\int_0^\infty e^{-\overline{\omega}z}\nu_z dz$.

If $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$, then $\nu_t = \tilde{\sigma}^{-1}e^{-\psi_m t}(i_0 - r_n) + \chi_{p_d}\epsilon_\lambda e^{-\psi_\lambda t}(i_0 - r_n)$. Then, $\tilde{y}_t = \tilde{\sigma}^{-1}\hat{y}_{m,t} + \chi_\lambda \hat{y}_{\lambda,t}$ and $\tilde{\pi}_t = \tilde{\sigma}^{-1}\hat{\pi}_{m,t} + \chi_\lambda \hat{\pi}_{\lambda,t}$, where $\chi_\lambda \equiv \chi_{p_d}\epsilon_\lambda$, $\hat{y}_{k,t} = \frac{(\rho-\underline{\omega})e^{\underline{\omega}t} - (\rho+\psi_k)e^{-\psi_k t}}{(\psi_k+\overline{\omega})(\psi_k+\underline{\omega})}(i_0 - r_n)$, and $\hat{\pi}_{k,t} = \frac{\kappa(e^{\underline{\omega}t} - e^{-\psi_k t})}{(\underline{\omega}+\psi_k)(\overline{\omega}+\psi_k)}(i_0 - r_n)$. Note that $\int_0^\infty e^{-\rho t}\hat{y}_{k,t}dt = 0$, $\frac{\partial \hat{y}_{k,0}}{\partial i_0} = -\frac{1}{\psi_k + \overline{\omega}} < 0$, and $\lim_{t\to\infty}\hat{y}_{k,t} = 0$. Moreover, $\hat{\pi}_0 = 0$, $\frac{\partial \hat{\pi}_{k,t}}{\partial i_0} \geq 0$ with strict inequality if $t > 0$.

$\square$

Proof of Proposition 8. The workers' financial wealth in the no-disaster state evolves according to $\dot{B}_{w,t} = (i_t - \pi_t + r_{P,t})B_{w,t} + W_t N_{w,t} + T_{w,t} - C_{w,t}$. Using the fact that $B_{w,t} = -Q_{P,t}\overline{F}$ and $q_{P,t} = -\frac{i_{P,t} - i_P}{i_P + \psi_P}$, we obtain equation (30). From the market clearing condition for goods, we obtain savers' consumption: $c_{s,t} = \frac{1-\mu_w\chi_y}{1-\mu_w}y_t + \frac{\mu_w \overline{d}_P}{1-\mu_w}\left(\frac{\psi_P}{i_P+\psi_P}(i_{P,t} - i_P) - \pi_t\right)$. Assuming exponentially decaying interest rates, and using the yield on the private bond $i_{P,t} - i_P = \frac{i_P+\psi_P}{\rho+\psi_P+\psi_m}(i_t - r_n) + \frac{i_P+\psi_P}{\rho+\psi_P+\psi_\lambda}r_P\hat{\lambda}_t$, we can write savers' consumption as follows

$$c_{s,t} = \frac{1-\mu_w\chi_y}{1-\mu_w}y_t + \frac{\mu_w\overline{d}_P}{1-\mu_w}\left[\frac{\psi_P}{\rho+\psi_P+\psi_m}(i_t - r_n) + \frac{\psi_P r_P}{\rho+\psi_P+\psi_\lambda}\hat{\lambda}_t - \pi_t\right]. \tag{31}$$

The Euler equation for savers can be written as

$$\dot{c}_{s,t} = \sigma^{-1}(i_t - \pi_t - r_n) + \lambda\left(\frac{C_s}{C_s^*}\right)^\sigma\left[c_{s,t} + \sigma^{-1}\hat{\lambda}_t\right]. \tag{32}$$

Combining equations (31) and (32), we obtain

$$\dot{y}_t = \left[\tilde{\sigma}^{-1} - \frac{\mu_w\overline{d}_P}{1-\mu_w\chi_y}r_n\right](i_t - \pi_t - r_n) + \left[\lambda\left(\frac{C_s}{C_s^*}\right)^\sigma - \frac{\mu_w\overline{d}_P}{1-\mu_w\chi_y}\kappa\right]y_t$$
$$+\left[\chi_{p_d} + \frac{\mu_w\overline{d}_P}{1-\mu_w\chi_y}\frac{\psi_P r_P(\rho - r_n + \psi_\lambda)}{\rho+\psi_P+\psi_\lambda}\right]\hat{\lambda}_t + \frac{\mu_w\overline{d}_P}{1-\mu_w\chi_y}\left[r_n + \frac{\psi_P(\rho - r_n + \psi_m)}{\rho+\psi_P+\psi_m}\right](i_t - r_n).$$

The aggregate Euler equation is given by $\dot{y}_t = -\hat{\sigma}^{-1}\pi_t + \hat{\delta}y_t + \hat{v}_t$, where $\hat{\sigma}^{-1} \equiv \tilde{\sigma}^{-1} - \frac{\mu_w\overline{d}_P r_n}{1-\mu_w\chi_y}$, $\hat{\delta} \equiv \lambda\left(\frac{C_s}{C_s^*}\right)^\sigma - \frac{\mu_w\overline{d}_P\kappa}{1-\mu_w\chi_y}$, and $\hat{v}_t \equiv \left[\chi_{p_d} + \frac{\mu_w\overline{d}_P}{1-\mu_w\chi_y}\frac{\psi_P r_P\tilde{\psi}_\lambda}{\rho+\psi_P+\psi_\lambda}\right]\hat{\lambda}_t + \left[\tilde{\sigma}^{-1} + \frac{\mu_w\overline{d}_P}{1-\mu_w\chi_y}\frac{\psi_P\hat{\psi}_m}{\rho+\psi_P+\psi_m}\right](i_t - r_n)$, where $\tilde{\psi}_k \equiv \psi_k + \rho - r_n$ for $k \in \{m, \lambda\}$. Therefore, following a derivation analogous to the one in Proposition 6, output is given by $y_t = \tilde{\sigma}^{-1}\hat{y}_{m,t} + \chi_\lambda \hat{y}_{\lambda,t} + \frac{\mu_w \overline{d}_P}{1-\mu_w \chi_y}\left[\frac{\psi_P \tilde{\psi}_m \hat{y}_{m,t}}{\rho+\psi_P+\psi_m} + \frac{r_P \epsilon_\lambda \tilde{\psi}_\lambda \hat{y}_{\lambda,t}}{\rho+\psi_P+\psi_\lambda}\right] + (\rho - \underline{\omega})e^{\underline{\omega}t}\Omega_0$, where the eigenvalues are given by $\overline{\omega} = \frac{\rho+\hat{\delta}+\sqrt{(\rho+\hat{\delta})^2+4(\hat{\sigma}^{-1}\kappa-\rho\hat{\delta})}}{2}$ and $\underline{\omega} = \frac{\rho+\hat{\delta}-\sqrt{(\rho+\hat{\delta})^2+4(\hat{\sigma}^{-1}\kappa-\rho\hat{\delta})}}{2}$.

□

Internet Appendix

A Derivations for Section 2

A.1 The non-linear model

Savers' problem. The HJB for the savers' problem is given by

$$\rho_{j,t} V_{j,t} = \max_{C_{j,t}, B^L_{j,t}, B^E_{j,t}} \frac{C_{j,t}^{1-\sigma}}{1-\sigma} + \frac{\partial V_{j,t}}{\partial t} + \lambda_j \left[V^*_{j,t} - V_{j,t} \right] + \frac{\partial V_{j,t}}{\partial B_{j,t}} \left[(i_t - \pi_t) B_{j,t} + r_{L,t} B^L_{j,t} + r_{E,t} B^E_{j,t} + T_{j,t} - C_{j,t} \right]. \tag{A.1}$$

where $V^*_{j,t}$ is evaluated at $B^*_{j,t} = B_{j,t} + B^L_{j,t} \frac{Q^*_{L,t} - Q_{L,t}}{Q_{L,t}} + B^E_{j,t} \frac{Q^*_{E,t} - Q_{E,t}}{Q_{E,t}}$ and $B_{j,t} > 0$.

The corresponding HJB in the disaster state is given by

$$\rho^*_{j,t} V^*_{j,t} = \max_{C^*_{j,t}, B^{L,*}_{j,t}, B^{E,*}_{j,t}} \frac{(C^*_{j,t})^{1-\sigma}}{1-\sigma} + \frac{\partial V^*_{j,t}}{\partial t} + \frac{\partial V^*_{j,t}}{\partial B^*_{j,t}} \left[(i^*_t - \pi^*_t) B_{j,t} + T^*_{j,t} - C^*_{j,t} \right],$$

where we imposed that $r^*_{L,t} = r^*_{E,t} = 0$, as there is no risk in the disaster state.

The first-order conditions are given by[1]

$$C_{j,t}^{-\sigma} = \frac{\partial V_{j,t}}{\partial B_{j,t}}, \qquad \frac{\partial V_{j,t}}{\partial B_{j,t}} r_{k,t} = \frac{\partial V^*_{j,t}}{\partial B^*_{j,t}} \frac{Q_{k,t} - Q^*_{k,t}}{Q_{k,t}}, \qquad (C^*_{j,t})^{-\sigma} = \frac{\partial V^*_{j,t}}{\partial B^*_{j,t}}, \tag{A.2}$$

for $k \in \{L, E\}$. Savers are indifferent about their portfolio composition in the disaster state. From the expressions above, we obtain eqn. (2) and (3). Differentiating the HJB

[1] Formally, the solution is also subject to a state-constraint boundary condition . See **?** for a discussion of such constraints in continuous-time savings problems.

equation in the no-disaster state with respect to $B_{j,t}$, we obtain the envelope condition:[2]

$$\rho_{j,t}\frac{\partial V_{j,t}}{\partial B_{j,t}} = \frac{\partial V_{j,t}}{\partial B_{j,t}}(i_t - \pi_t) + \frac{\mathbb{E}_{j,t}[d\left(\frac{\partial V_{j,t}}{\partial B_{j,t}}\right)]}{dt}. \tag{A.3}$$

Using the optimality condition for consumption and the condition above, we obtain:

$$i_t - \pi_t - \rho_{j,t} = -\frac{\mathbb{E}_t[dC_{j,t}^{-\sigma}]}{C_{j,t}^{-\sigma}dt} = \frac{\sigma C_{j,t}^{-\sigma-1}\dot{C}_{j,t} - \lambda_j\left[(C_{j,t}^{*})^{-\sigma} - C_{j,t}^{-\sigma}\right]}{C_{j,t}^{-\sigma}}, \tag{A.4}$$

using the fact that $dC_{j,t} = \dot{C}_{j,t}dt + [C_{j,t}^{*} - C_{j,t}]d\mathcal{N}_t$ and Ito's lemma. Rearranging the expression above, we obtain eqn. (1). A similar envelope condition holds in the disaster state, which gives the Euler equation for the disaster state

$$\frac{\dot{C}_{j,t}^{*}}{C_{j,t}^{*}} = \sigma^{-1}(i_t - \pi_t - \rho_{j,t}^{*}). \tag{A.5}$$

The relative net worth of optimistic and pessimistic savers evolves according to

$$\frac{\dot{B}_{o,t}}{B_{o,t}} - \frac{\dot{B}_{p,t}}{B_{p,t}} = \sum_{k\in\{L,E\}} r_{k,t}\left(\frac{B_{o,t}^{L}}{B_{o,t}} - \frac{B_{p,t}^{k}}{B_{p,t}}\right) - \left(\frac{C_{o,t} - T_{s,t}}{B_{o,t}} - \frac{C_{p,t} - T_{s,t}}{B_{p,t}}\right). \tag{A.6}$$

Workers' problem. The HJB for the workers' problem is given by

$$\rho_w V_{w,t} = \max_{\tilde{C}_{w,t},N_{w,t},B_{w,t}^{L}} \frac{\tilde{C}_{w,t}^{1-\sigma}}{1-\sigma} + \frac{\partial V_{w,t}}{\partial B_{w,t}}\left[(i_t - \pi_t)B_{w,t} + r_{L,t}B_{w,t}^{L} + \frac{W_t}{P_t}N_{w,t} + T_{w,t} - \tilde{C}_{w,t} - \frac{N_{w,t}^{1+\phi}}{1+\phi}\right].$$

$$+ \frac{\partial V_{w,t}}{\partial t} + \lambda_w\left[V_{w,t}^{*} - V_{w,t}\right] \tag{A.7}$$

[2] Here we used the fact that $\mathbb{E}_{j,t}[dF(B_{j,t},t)] = \left[F_t + \lambda_j[F^{*} - F] + F_B\left((i-\pi)B_j + r_L B_j^{L} + r_E B_j^{E} - C_j\right)\right]dt$ for any function $F(B_{j,t},t)$, according to Ito's lemma.

subject to the state-constraint boundary condition

$$\frac{\partial V_{w,t}(0)}{\partial B_{w,t}} \geq \left(\frac{W_t}{P_t} N_{w,t} - \frac{N_{w,t}^{1+\phi}}{1+\phi} + T_{w,t} \right)^{-\sigma}, \tag{A.8}$$

where we adopted the change of variables $\tilde{C}_{w,t} \equiv C_{w,t} - \frac{N_{w,t}^{1+\phi}}{1+\phi}$.

For simplicity, we have already imposed that $B_{w,t}^E = 0$. We show below that $B_{w,t}^L = 0$ and a similar argument shows that workers would be against the short-selling constraint for equities when $B_{w,t}^E$ is a choice variable.

The optimality condition for labor supply is given by

$$N_{w,t}^{\phi} = \frac{W_t}{P_t}. \tag{A.9}$$

We focus on an equilibrium where workers are always constrained. To derive the conditions that ensure this is indeed the case, we start by considering a stationary equilibrium where all variables are constant conditional on the state. If workers are constrained in the stationary equilibrium, then they will also be constrained if fluctuations are small enough.

In a stationary equilibrium, net consumption $\tilde{C}_w$ in the no-disaster state is given by

$$\tilde{C}_w = \frac{W}{P} N_w - \frac{N_w^{1+\phi}}{1+\phi} + T_w, \tag{A.10}$$

and an analogous expression holds in the disaster state. Notice that the expression above does not depend on ρ_w or λ_w.

For workers to be unconstrained, the following condition would have to hold:

$$\frac{\dot{\tilde{C}}_{w,t}}{\tilde{C}_{w,t}} = \sigma^{-1}(r_n - \rho_w) + \frac{\lambda_w}{\sigma} \left[\left(\frac{\tilde{C}_{w,t}}{\tilde{C}_{w,t}^*} \right)^{\sigma} - 1 \right]. \tag{A.11}$$

For ρ_w sufficiently large, workers would want a declining path of consumption, so current consumption would be above $\frac{W}{P}N_w - \frac{N_w^{1+\phi}}{1+\phi} + T_w$, which would violate the state-constraint. Hence, the constraint must be binding for ρ_w sufficiently large.

If the workers hold a positive amount of the long-term bonds, then the following condition must hold

$$r_L = \lambda_w \left(\frac{\tilde{C}_w}{\tilde{C}_w^*}\right)^\sigma \frac{Q_L - Q_L^*}{Q_L}. \tag{A.12}$$

As C_w and C_w^* are independent of λ_w, the equation above would hold only if λ_w equals the value $\overline{\lambda}_w \equiv \frac{r_L}{\left(\frac{C_w}{C_w^*}\right)^\sigma \frac{Q_L - Q_L^*}{Q_L}}$. For $\lambda_w > \overline{\lambda}_w$, borrowers would want a smaller dispersion between C_w and C_w^*, which requires holding less risky bonds, violating the non-negativity constraint on long-term bonds. Therefore, borrowers will hold zero long-term bonds for λ_w sufficiently large.

Firms' problem. The intermediate-goods producers' problem is given by

$$Q_{i,t}(P_i) = \max_{[\pi_{i,s}]_{s \geq t}} \mathbb{E}_t \left[\int_t^{t^*} \frac{\eta_s}{\eta_t}\left(\frac{P_{i,s}}{P_s}Y_{i,s} - \frac{W_s}{P_s}\frac{Y_{i,s}}{A_s} - \frac{\varphi}{2}\pi_s^2(j)\right) ds + \frac{\eta_{t^*}}{\eta_t} Q_{i,t^*}^*(P_{i,t^*})\right],$$

subject to $Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} Y_t$ and $\dot{P}_{i,t} = \pi_{i,t} P_{i,t}$, given $P_{i,t} = P_i$.

The HJB equation for this problem is

$$0 = \max_{\pi_{i,t}} \eta_t \left(\frac{P_{i,t}}{P_t}Y_{i,t} - \frac{W_t}{P_t}\frac{Y_{i,t}}{A} - \frac{\varphi}{2}\pi_{i,t}^2\right) dt + \mathbb{E}_t[d(\eta_t Q_{i,t})], \tag{A.13}$$

where $\frac{\mathbb{E}_t[d(\eta_t Q_{i,t})]}{\eta_t dt} = -(i_t - \pi_t) Q_{i,t} + \frac{\partial Q_{i,t}}{\partial P_{i,t}} \pi_{i,t} P_{i,t} + \frac{\partial Q_{i,t}}{\partial t} + \lambda_t \frac{\eta_t^*}{\eta_t}\left[Q_{i,t}^* - Q_{i,t}\right]$.

The first-order condition is given by

$$\frac{\partial Q_{i,t}}{\partial P_i} P_{i,t} = \varphi \pi_{i,t}.$$

The change in π_t conditional on no disaster is then given by

$$\left(\frac{\partial^2 Q_{i,t}}{\partial t \partial P_i} + \frac{\partial^2 Q_{i,t}}{\partial P_i^2}\pi_{i,t}P_{i,t}\right)P_{i,t} + \frac{\partial Q_{i,t}}{\partial P_i}\pi_{i,t}P_{i,t} = \varphi\dot{\pi}_{i,t}. \tag{A.14}$$

The envelope condition with respect to $P_{i,t}$ is given by

$$0 = \left((1-\epsilon)\frac{P_{i,t}}{P_t} + \epsilon\frac{W_t}{P_t A}\right)\left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon}\frac{Y_t}{P_{i,t}} + \frac{\partial^2 Q_{i,t}}{\partial t \partial P_i} + \frac{\partial^2 Q_{i,t}}{\partial P_i^2}\pi_{i,t}P_{i,t} + \frac{\partial Q_{i,t}}{\partial P_i}\pi_{i,t} - (i_t - \pi_t)\frac{\partial Q_{i,t}}{\partial P_i} + \lambda_t\frac{\eta_t^*}{\eta_t}\left(\frac{\partial Q_{i,t}^*}{\partial P_i} - \frac{\partial Q_{i,t}}{\partial P_i}\right). \tag{A.15}$$

Multiplying the expression above by $P_{i,t}$ and using eqn. (A.14), we obtain

$$0 = \left((1-\epsilon)\frac{P_{i,t}}{P_t} + \epsilon\frac{W_t}{P_t A}\right)\left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} Y_t + \varphi\dot{\pi}_t - (i_t - \pi_t)\varphi\pi_{i,t} + \lambda_t\varphi\frac{\eta_t^*}{\eta_t}\left(\pi_{i,t}^* - \pi_{i,t}\right).$$

Rearranging the expression above, we obtain the non-linear New Keynesian Phillips curve

$$\dot{\pi}_t = \left(i_t - \pi_t + \lambda_t\frac{\eta_t^*}{\eta_t}\right)\pi_t - \frac{\epsilon\varphi^{-1}}{A}\left(\frac{W_t}{P_t} - (1-\epsilon^{-1})A\right)Y_t,$$

where we have assumed that $P_{i,t} = P_t$ for all $i \in [0,1]$ and that $\pi_t^* = 0$.

A.2 The stationary equilibrium

Aggregate output. Consider a stationary equilibrium with zero inflation. From the New Keynesian Phillips curve, we obtain

$$\frac{W}{P} = (1-\epsilon^{-1})A, \qquad \frac{W^*}{P} = (1-\epsilon^{-1})A^*. \tag{A.16}$$

Combining the expressions above with the labor supply condition, we obtain

$$Y = \mu_w (1-\epsilon^{-1})^{\frac{1}{\phi}} A^{\frac{1+\phi}{\phi}}, \qquad Y^* = \mu_w (1-\epsilon^{-1})^{\frac{1}{\phi}} (A^*)^{\frac{1+\phi}{\phi}}. \tag{A.17}$$

Disaster state. From the Euler equation for short-term bonds, an allocation with constant consumption must satisfy $r_n^* = \rho_j^*$. Uzawa preferences implies that this condition is eventually satisfied. For simplicity, we assume that $\rho_j^*(\cdot)$ is constant and $\rho_o^* = \rho_p^*$. This is assumption is not necessary for our results, but it simplifies presentation, as it ensures that allocations are constant as the economy switches to the disaster state. We set $\rho_j^* = \rho_s$, so there is no jump in the discount rate of the representative saver. In this case, the real interest rate in the disaster state is given by $i_t^* - \pi_t^* = r_n^* = \rho_s$.

The excess return on long-terms bonds and equity are equal to zero, $r_L^* = r_E^* = 0$, so the price of the long-term bond is given by

$$Q_L^* = \frac{1}{r_n^* + \psi_L}, \tag{A.18}$$

and the equity price is given by $Q_E^* = \frac{\Pi^*}{r_n^*}$.

The consumption of borrowers is given by

$$C_w^* = (1-\epsilon^{-1})\frac{Y^*}{\mu_w} + T_w^*. \tag{A.19}$$

We assume that the government chooses fiscal transfers so workers have a given share $0 < \mu_{Y,w} < 1$ of output, so $C_w^* = \mu_{Y,w}\frac{Y^*}{\mu_w}$. The parameter $\mu_{Y,w}$ captures the government's preference for redistribution. This requires that the government sets $T_w^* = \left[\frac{\mu_{Y,w}}{\mu_w} - \frac{1-\epsilon^{-1}}{\mu_w}\right] Y^*$. In the main text, we focus on the case $\mu_{Y,w} = \mu_w$.

Savers' consumption is given by

$$C_j^* = r_n^* B_j^* + T_j^*, \tag{A.20}$$

where $B_j^* = B_j + B_j^L \frac{Q_L^* - Q_L}{Q_L} + B_j^E \frac{Q_E^* - Q_E}{Q_E}$.

Aggregate consumption of savers is given by

$$C_s^* = r_n^* \frac{\overline{D}_G^*}{\mu_s} + \frac{\Pi^*}{\mu_s} + T_s. \tag{A.21}$$

Transfers to savers must satisfy $T_s = (1 - \mu_{Y,w} - \epsilon^{-1})\frac{Y^*}{\mu_s} - r_n^* \frac{\overline{D}_G^*}{\mu_s}$ such that the government's budget constraint is satisfied. This implies that the aggregate consumption of savers is given by $C_s^* = (1 - \mu_{Y,w})\frac{Y^*}{\mu_s}$.

We focus on a symmetric allocation in the disaster state, so we assume that $T_{o,t}^* - T_{p,t}^* = -r_n^*(B_o^* - B_p^*)$, for $t \geq t^*$. This implies that $C_j^* = C_s^*$.

No-disaster state. The consumption of workers is given by

$$C_w = \left[(1 - \epsilon^{-1})A\right]^{\frac{1+\phi}{\phi}} + T_w. \tag{A.22}$$

As in the disaster state, the government chooses fiscal transfers so workers have a given share $0 < \mu_{Y,w} < 1$ of output, so $C_w = \mu_{Y,w}\frac{Y}{\mu_w}$ and $C_s = (1 - \mu_{Y,w})\frac{Y}{\mu_s}$. This requires that the government sets $T_w = \left[\frac{\mu_{Y,w}}{\mu_w} - \frac{1-\epsilon^{-1}}{\mu_w}\right] Y$.

From the market clearing condition for assets, we obtain

$$B_s = \frac{\overline{D}_G + Q_E}{1 - \mu_w}, \qquad B_s^L = \frac{\overline{D}_G}{1 - \mu_w}, \qquad B_s^E = \frac{Q_E}{1 - \mu_w}. \tag{A.23}$$

The consumption of individual savers is given by

$$C_j = r_n B_j + r_L B_j^L + r_E B_j^E - T_j \tag{A.24}$$

From the Euler equation for short-term bonds to be satisfied for both types of savers, the following condition must be satisfied: $\rho_o - \rho_p = \lambda_p - \lambda_o$, where ρ_j is an increasing function of $\frac{C_j}{C_s}$. As the consumption of type-j savers is increasing in B_j, $\rho_o - \rho_p$ is increasing in B_o. Hence, there is a unique value of B_o such that $\rho_o - \rho_p = \lambda_p - \lambda_o$. We assume the function $\rho_j(\cdot)$ is such that this equality is achieved when $B_o = B_p$.

Using the fact that $B_o = B_p$ and $T_o = T_p$ in a stationary equilibrium, we can write the consumption of optimistic and pessimistic savers as follows:

$$C_o = C_s + r_L \frac{\mu_p}{\mu_o + \mu_p}(B_o^L - B_p^L) + r_E \frac{\mu_p}{\mu_o + \mu_p}(B_o^E - B_p^E) \tag{A.25}$$

$$C_p = C_s - r_L \frac{\mu_o}{\mu_o + \mu_p}(B_o^L - B_p^L) - r_E \frac{\mu_o}{\mu_o + \mu_p}(B_o^E - B_p^E). \tag{A.26}$$

We can use the Euler equations for risky assets to eliminate r_L and r_E from the expressions above, which gives us

$$C_o = C_s \left[1 + \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} \frac{\mu_p}{\mu_o + \mu_p} \mathcal{R}_o\right], \qquad C_o^* = C_s^*, \tag{A.27}$$

$$C_p = C_s \left[1 - \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} \frac{\mu_o}{\mu_o + \mu_p} \mathcal{R}_o\right], \qquad C_p^* = C_s^*, \tag{A.28}$$

where $\mathcal{R}_o \equiv \frac{Q_L - Q_L^*}{Q_L} \frac{B_o^L - B_p^L}{C_s} + \frac{Q_E - Q_E^*}{Q_E} \frac{B_o^E - B_p^E}{C_s}$ represents optimistic relative risk exposure.

From the optimality condition for risky assets, we obtain

$$\left(1 + \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} \frac{\mu_p}{\mu_o + \mu_p} \mathcal{R}_o\right)^{\sigma} = \frac{\lambda_p}{\lambda_o} \left(1 - \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} \frac{\mu_o}{\mu_o + \mu_p} \mathcal{R}_o\right)^{\sigma}. \tag{A.29}$$

Rearranging the expression above, we obtain

$$\lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} \mathcal{R}_0 = \frac{\lambda_p^{\frac{1}{\sigma}} - \lambda_o^{\frac{1}{\sigma}}}{\frac{\mu_o}{\mu_o + \mu_p} \lambda_p^{\frac{1}{\sigma}} + \frac{\mu_p}{\mu_o + \mu_p} \lambda_o^{\frac{1}{\sigma}}}, \tag{A.30}$$

which is positive if $\lambda_p > \lambda_o$. The value of $\mathcal{R}_o$ pins down only a linear combination of $B_o^L - B_p^L$ and $B_o^E - B_o^E$. For concreteness, we assume that $B_o^E = B_p^E$, so savers have different exposure to bonds in equilibrium.

Given $\mathcal{R}_o$, we can solve for the share of consumption of optimistic savers:

$$\frac{\mu_o C_o}{\mu_o C_o + \mu_p C_p} = \frac{\mu_o}{\mu_o + \mu_p}\left[1 + \frac{\mu_p(\lambda_o^{-\frac{1}{\sigma}} - \lambda_p^{-\frac{1}{\sigma}})}{\mu_o \lambda_o^{-\frac{1}{\sigma}} + \mu_p \lambda_p^{-\frac{1}{\sigma}}}\right]. \tag{A.31}$$

Given the expression above, we obtain the market-implied disaster probability:

$$\lambda = \left[\frac{\mu_o C_o}{\mu_p C_p + \mu_p C_p}\lambda_o^{\frac{1}{\sigma}} + \frac{\mu_p C_p}{\mu_p C_p + \mu_p C_p}\lambda_p^{\frac{1}{\sigma}}\right]^{\sigma} \Rightarrow \lambda = \left[\frac{\mu_o \lambda_o^{-\frac{1}{\sigma}}}{\mu_o + \mu_p} + \frac{\mu_p \lambda_p^{-\frac{1}{\sigma}}}{\mu_o + \mu_p}\right]^{-\sigma}. \tag{A.32}$$

From the Euler equations for short-term and long-term bonds, we obtain

$$r_n = \rho_j - \lambda_j\left[\left(\frac{C_j}{C_j^*}\right)^{\sigma} - 1\right], \qquad r_k = \lambda_j \left(\frac{C_j}{C_j^*}\right)^{\sigma}\frac{Q_k - Q_k^*}{Q_k}, \tag{A.33}$$

for $k \in \{L, E\}$, where $r_L = \frac{1}{Q_L} - \psi_L - r_n$, $r_E = \frac{\Pi}{Q_E} - r_n$, and $\Pi = \epsilon^{-1} Y$.

Using the fact that $\lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma} = \lambda_j\left(\frac{C_j}{C_j^*}\right)^{\sigma}$, we can write the Euler equations in terms of aggregate savers' consumption:

$$r_n = \rho_s - \lambda\left[\left(\frac{C_s}{C_s^*}\right)^{\sigma} - 1\right], \qquad r_k = \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma}\frac{Q_k - Q_k^*}{Q_k}, \tag{A.34}$$

for $k \in \{L, E\}$, where ρ_s satisfy the condition $\rho_s + \lambda = \rho_j + \lambda_j$ for $j \in \{o, p\}$.

We solve next for the price of risky assets. Given r_L, we can solve for Q_L:

$$\frac{1}{Q_L} - \psi_L - r_n = \lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma}\left(1 - \frac{Q_L^*}{Q_L}\right) \Rightarrow Q_L = Q_L^* \frac{r_n^* + \psi_L + \lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma}}{r_n + \psi_L + \lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma}}, \tag{A.35}$$

where $Q_L > Q_L^*$, as $r_n < r_n^*$ due to the precautionary motive in the no-disaster state.

The loss in long-term bonds in the disaster state is given by

$$\frac{Q_L - Q_L^*}{Q_L} = \frac{r_n^* - r_n}{r_n^* + \psi_L + \lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma}}, \tag{A.36}$$

which is positive as $r_n^* > r_n$. Long-term interest rates are higher than short-term interest rates in the stationary equilibrium, i.e., the yield curve is upward sloping in this economy.

The equity price is given by

$$\frac{\Pi}{Q_E} - r_n = \lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma}\left(1 - \frac{Q_E^*}{Q_E}\right) \Rightarrow Q_E = \frac{\Pi + \lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma} Q_E^*}{r_n + \lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma}}, \tag{A.37}$$

so the loss on equity in the disaster state is given by

$$\frac{Q_E - Q_E^*}{Q_E} = \frac{\Pi - r_n Q_E^*}{\Pi + \lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma} Q_E^*} = \frac{\rho_s \zeta_\Pi + \lambda\left[\left(\frac{C_s}{C_s^*}\right)^{\sigma} - 1\right](1 - \zeta_\Pi)}{\rho_s + \lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma}(1 - \zeta_\Pi)}, \tag{A.38}$$

where $\zeta_\Pi \equiv 1 - \frac{\Pi^*}{\Pi}$ is the size of the drop in profits. As the expression above is positive, the equity premium is positive in the stationary equilibrium.

A.3 Log-linear approximation

We consider next the effects of an unexpected monetary shock for an economy starting at the stationary equilibrium described above.

Disaster state. As there is no monetary shock in the disaster state, inflation is equal to zero, $\pi_t^* = 0$, and output is kept at the stationary-equilibrium level, $y_t^* = 0$. Wages and hours are unchanged, so $c_{w,t}^* = 0$. Savers' aggregate consumption is also the same as in the stationary equilibrium, $c_{s,t}^* = 0$. Savers' flow budget constraint is given by

$\mu_s C^*_{s,t} = r^*_{n,t}(D_{G,t}\frac{Q^*_{L,t}}{Q_{L,t}} + Q^*_{E,t}) + T^*_{s,t}$. Notice that $r^*_{n,t} = r^*_n$, $Q^*_{L,t} = Q^*_L$, and $Q^*_{E,t} = Q^*_E$. For simplicity, we further assume that the government chooses transfers in the no-disaster state such that $D_{G,t} = D_G q_{L,t}$, so transfers must satisfy $T^*_{s,t} = T^*_s$. Consumption of type-j saver is then given by $\frac{C^*_j}{B^*_j} c^*_{j,t} = r^*_n b^*_{j,t}$.

Market-based disaster probability. Linearizing eqn. (4) around the stationary equilibrium, we obtain

$$\frac{\lambda^{\frac{1}{\sigma}}}{\sigma}\hat{\lambda}_t = \mu_{c,o}\mu_{c,p}\left(\lambda_p^{\frac{1}{\sigma}} - \lambda_o^{\frac{1}{\sigma}}\right)\left[c_{p,t} - c_{o,t}\right], \tag{A.39}$$

where $\mu_{c,j} \equiv \frac{\mu_j C_j}{\mu_o C_o + \mu_p C_p}$ and $c_{j,t} \equiv \log C_{j,t}/C_j$, for $j \in \{o, p\}$.

Euler equation for short-term bonds. Using the fact that $\lambda_j\left(\frac{C_{j,t}}{C^*_{j,t}}\right)^\sigma = \lambda_t\left(\frac{C_{s,t}}{C^*_{s,t}}\right)^\sigma$, we can write the Euler equation for short-term bonds as follows

$$\dot{c}_{j,t} = \sigma^{-1}\left(i_t - \pi_t - (\rho_{j,t} + \lambda_j)\right) + \frac{\lambda_t}{\sigma}\left(\frac{C_{s,t}}{C^*_{s,t}}\right)^\sigma. \tag{A.40}$$

Linearizing the discount-rate function, we obtain $\rho_{j,t} = \rho_j + \sigma\xi(c_{j,t} - c_{s,t})$, where we assumed a common slope for both types $\sigma\xi$, so we obtain the linearized Euler equation

$$\dot{c}_{j,t} = \sigma^{-1}(i_t - \pi_t - r_n) + \frac{\lambda}{\sigma}\left(\frac{C_s}{C^*_s}\right)^\sigma\left(\hat{\lambda}_t + \sigma c_{s,t}\right) - \xi(c_{j,t} - c_{s,t}). \tag{A.41}$$

Aggregating the expression above, and using $c_{s,t} = \sum_{j\in\{o,p\}}\mu_{c,j}c_{j,t}$, we obtain

$$\dot{c}_{s,t} = \sigma^{-1}(i_t - \pi_t - r_n) + \frac{\lambda}{\sigma}\left(\frac{C_s}{C^*_s}\right)^\sigma\left(\hat{\lambda}_t + \sigma c_{s,t}\right). \tag{A.42}$$

Relative consumption. From the optimality condition for risky assets, we obtain

$$\lambda_o^{\frac{1}{\sigma}} \frac{C_{o,t}}{C_{o,t}^*} = \lambda_p^{\frac{1}{\sigma}} \frac{C_{p,t}}{C_{p,t}^*} \Rightarrow c_{p,t} - c_{o,t} = c_{p,t}^* - c_{o,t}^* \tag{A.43}$$

Relative consumption in the no-disaster evolves according to

$$\dot{c}_{p,t} - \dot{c}_{o,t} = -\xi(c_{p,t} - c_{o,t}). \tag{A.44}$$

Relative net worth. Linearizing eqn. (A.6), we obtain

$$\begin{aligned}\dot{b}_{p,t} - \dot{b}_{o,t} = &\sum_{k\in\{L,E\}} r_k \left[\hat{r}_{k,t}\left(\frac{b_p^k}{b_p} - \frac{b_o^k}{b_o}\right) + \frac{b_p^k}{b_p}(b_{p,t}^k - b_{p,t}) - \frac{B_o^k}{B_o}(b_{o,t}^k - b_{o,t})\right] \\ &- \left(\frac{C_p}{B_p}c_{p,t} - \frac{C_o}{B_o}c_{o,t}\right) + \frac{C_p - T_p}{B_p}b_{p,t} - \frac{C_o - T_o}{B_o}b_{o,t},\end{aligned} \tag{A.45}$$

where $\hat{r}_{k,t} = \hat{\lambda}_t + \sigma c_{s,t} + \frac{Q_k^*}{Q_k - Q_k^*}q_{k,t}$. Using the fact that $\frac{C_j - T_j}{B_j} = r_n + \sum_{k\in\{L,E\}} r_k \frac{B_j^k}{B_j}$, we can write the expression above as follows

$$\begin{aligned}\dot{b}_{p,t} - \dot{b}_{o,t} = &\sum_{k\in\{L,E\}} r_k \left[\hat{r}_{k,t}\left(\frac{B_p^k}{B_p} - \frac{B_o^k}{B_o}\right) + \frac{B_p^k}{B_p}b_{p,t}^k - \frac{B_o^k}{B_o}b_{o,t}^k\right] - \left(\frac{C_p}{B_p}c_{p,t} - \frac{C_o}{B_o}c_{o,t}\right) \\ &+ r_n(b_{p,t} - b_{o,t}).\end{aligned} \tag{A.46}$$

The relative net worth in the disaster state at $t = t^*$ is given by

$$\frac{B_p^*}{B_p}b_{p,t^*}^* - \frac{B_o^*}{B_o}b_{o,t^*}^* = b_{p,t^*} - b_{o,t^*} - \sum_{k\in\{L,E\}}\left[\left(\frac{B_p^k}{B_p} - \frac{B_o^k}{B_o}\right)\frac{Q_k^*}{Q_k}q_{k,t^*} + \frac{Q_k - Q_k^*}{Q_k}\left(\frac{B_p^k}{B_p}b_{p,t^*}^k - \frac{B_o^k}{B_o}b_{o,t^*}^k\right)\right]. \tag{A.47}$$

Relative risk exposure. Consumption of savers in the disaster state is given by $c_{j,t}^* = \frac{r_n^* B_j^*}{C_s^*}b_{j,t}^*$, so we obtain that $c_{p,t}^* - c_{o,t}^* = \frac{r_n^*}{C_s^*}(B_p^* b_{p,t}^* - B_o^* b_{o,t}^*)$. Using this expression and the fact that $c^*_{p,t} - c^*_{o,t} = c_{p,t} - c_{o,t}$, we can solve for the relative risk exposure:

$$\sum_{k\in\{L,E\}} \frac{Q_k - Q^*_k}{Q_k} \left(\frac{B^k_p}{B_p} b^k_{p,t} - \frac{B^k_o}{B_o} b^k_{o,t} \right) = b_{p,t} - b_{o,t} - \frac{C^*_s}{r^*_n B_s}(c_{p,t} - c_{o,t}) - \sum_{k\in\{L,E\}} \left(\frac{B^k_p}{B_p} - \frac{B^k_o}{B_o} \right) \frac{Q^*_k}{Q_k} q_{k,t}. \tag{A.48}$$

The dynamic system. Using the expression above to eliminate the relative risk exposure, the relative net worth at the no-disaster state is given by

$$\begin{aligned} \dot{b}_{p,t} - \dot{b}_{o,t} &= (\hat{\lambda}_t + (\sigma-1)c_{s,t}) \sum_{k\in\{L,E\}} r_k \left(\frac{B^k_p}{B_p} - \frac{B^k_o}{B_o} \right) + \rho(b_{p,t} - b_{o,t}) \\ &\quad - \left(r_n + \frac{T_s}{B_s} + \frac{C^*_s(\rho - r_n)}{r^*_n B_s} \right)(c_{p,t} - c_{o,t}) - \sum_{k\in\{L,E\}} r_k \left(\frac{B^k_p}{B_p}(c_{p,t} - c_{s,t}) - \frac{B^k_o}{B_o}(c_{o,t} - c_{s,t}) \right), \end{aligned} \tag{A.49}$$

using $\hat{r}_{k,t} = \hat{\lambda}_t + \sigma c_{s,t} + \frac{Q^*_k}{Q_k - Q^*_k} q_{k,t}$, $\frac{C_j}{B_j} = r_n + \frac{T_j}{B_j} + \sum_{k\in\{L,E\}} r_k \frac{B^k_j}{B_j}$, and $\lambda \left(\frac{C_s}{C^*_s}\right)^\sigma = \rho - r_n$.

The deviation of consumption from average can be written as

$$c_{p,t} - c_{s,t} = \mu_{c,o}(c_{p,t} - c_{o,t}), \qquad c_{o,t} - c_{s,t} = -\mu_{c,p}(c_{p,t} - c_{o,t}). \tag{A.50}$$

Combining the expressions above, we can write $\dot{b}_{p,t} - \dot{b}_{o,t}$ as follows

$$\dot{b}_{p,t} - \dot{b}_{o,t} = \rho(b_{p,t} - b_{o,t}) - \chi_{b,c}(c_{p,t} - c_{o,t}) + \chi_{b,c_s} c_{s,t}, \tag{A.51}$$

where $\chi_{b,c_s} \equiv (\sigma - 1)\sum_{k\in\{L,E\}} r_k \left(\frac{B^k_p}{B_p} - \frac{B^k_o}{B_o} \right)$, and

$$\begin{aligned} \chi_{b,c} &\equiv \sigma \mu_{c,o}\mu_{c,p} \left(\frac{\lambda_p^{\frac{1}{\sigma}} - \lambda_o^{\frac{1}{\sigma}}}{\lambda^{\frac{1}{\sigma}}} \right) \sum_{k\in\{L,E\}} r_k \left(\frac{B^k_o}{B_o} - \frac{B^k_p}{B_p} \right) + \left(r_n + \frac{T_s}{B_s} + \frac{C^*_s(\rho - r_n)}{r^*_n B_s} \right) \\ &\quad + \sum_{k\in\{L,E\}} r_k \left(\mu_{c,o} \frac{B^k_p}{B_p} + \mu_{c,p} \frac{B^k_o}{B_o} \right). \end{aligned} \tag{A.52}$$

Note that $r_n + \frac{T_s}{B_s} = \frac{C_j}{B_j} - \sum_{k \in \{L,E\}} r_k \frac{B_j^k}{B_j}$, so $r_n + \frac{T_s}{B_s} = \mu_{c,p} \frac{C_o}{B_o} + \mu_{c,o} \frac{C_p}{B_p} - \sum_{k \in \{L,E\}} r_k \left(\mu_{c,p} \frac{B_o^k}{B_o} + \mu_{c,o} \frac{B_p^k}{B_p} \right)$. We can then write $\chi_{b,c}$ as follows:

$$\chi_{b,c} = \sigma \mu_{c,o} \mu_{c,p} \left(\frac{\lambda_p^{\frac{1}{\sigma}} - \lambda_o^{\frac{1}{\sigma}}}{\lambda^{\frac{1}{\sigma}}} \right) \sum_{k \in \{L,E\}} r_k \left(\frac{B_o^k}{B_o} - \frac{B_p^k}{B_p} \right) + \mu_{c,p} \frac{C_o}{B_o} + \mu_{c,o} \frac{C_p}{B_p} + \frac{C_s^*(\rho - r_n)}{r_n^* B_s}, \tag{A.53}$$

so $\chi_{b,c} > 0$, as $r_n < \rho$.

In general, we would have to simultaneously solve for the aggregate variables and the relative net worth and relative consumption of pessimistic savers, which would increase the dimensionality of the problem relative to the standard New Keynesian. We assume that $r_k c_{s,t} = \mathcal{O}(||i_t - r_n||^2)$, so this term is small and can be ignored in our approximate solution. This implies that the system is now *block recursive*, where we can solve for the dynamics of relative consumption and relative net worth before fully characterizing the behavior of other aggregate variables. Under this assumption, we can write the joint dynamics of $b_{p,t} - b_{o,t}$ and $c_{p,t} - c_{o,t}$ as follows:

$$\begin{bmatrix} \dot{c}_{p,t} - \dot{c}_{o,t} \\ \dot{b}_{p,t} - \dot{b}_{o,t} \end{bmatrix} = \begin{bmatrix} -\xi & 0 \\ -\chi_{b,c} & \rho \end{bmatrix} \begin{bmatrix} c_{p,t} - c_{o,t} \\ b_{p,t} - b_{o,t} \end{bmatrix}. \tag{A.54}$$

Persistence of $\hat{\lambda}_t$. The system above has a positive and a negative eigenvalue, so there is a unique bounded solution given by

$$\begin{bmatrix} c_{p,t} - c_{o,t} \\ b_{p,t} - b_{o,t} \end{bmatrix} = \begin{bmatrix} \frac{\rho + \xi}{\chi_{b,c}} \\ 1 \end{bmatrix} e^{-\psi_\lambda t} (b_{p,0} - b_{o,0}) \tag{A.55}$$

where $\psi_\lambda = \xi$.

We can then write the market-implied disaster probability as follows:

$$\hat{\lambda}_t = e^{-\psi_\lambda t}\hat{\lambda}_0, \tag{A.56}$$

where

$$\hat{\lambda}_0 \equiv \sigma\mu_{c,o}\mu_{c,p}\left(\frac{\lambda_p^{\frac{1}{\sigma}} - \lambda_o^{\frac{1}{\sigma}}}{\lambda^{\frac{1}{\sigma}}}\right)\frac{\rho+\xi}{\chi_{b,c}}(b_{p,0} - b_{o,0}). \tag{A.57}$$

Hence, ψ_λ captures the persistence of $\hat{\lambda}_t$. If $\xi = 0$, then $\psi_\lambda = 0$ and changes in λ_t are permanent. For high values of ψ_λ, the effects on λ_t are transitory and ψ_λ controls the speed of the convergence.

Wealth revaluation and $\hat{\lambda}_0$. The revaluation of the relative net worth is given by

$$b_{p,0} - b_{o,0} = \sum_{k\in\{L,E\}}\left(\frac{B_p^k}{B_p} - \frac{B_o^k}{B_o}\right)q_{k,0}. \tag{A.58}$$

The price of the long-term bond satisfies the condition

$$-\frac{1}{Q_L}q_{L,t} + \dot{q}_{L,t} - (i_t - r_n) = r_L\left[\hat{\lambda}_t + \sigma c_{s,t} + \frac{Q_L^*}{Q_L - Q_L^*}q_{L,t}\right] \tag{A.59}$$

Rearranging the expression above, we obtain

$$\dot{q}_{L,t} - (\rho + \psi_L)q_{L,t} = (i_t - r_n) + r_L(\hat{\lambda}_t + \sigma c_{s,t}). \tag{A.60}$$

Solving the differential equation above, we obtain

$$q_{L,0} = -\int_0^\infty e^{-(\rho+\psi_L)t}(i_t - r_n)dt - \int_0^\infty e^{-(\rho+\psi_L)t}r_L(\hat{\lambda}_t + \sigma c_{s,t})dt. \tag{A.61}$$

Suppose $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$ and $r_L \sigma c_{s,t} = \mathcal{O}(||i_t - r_n||^2)$, then

$$q_{L,0} = -\frac{i_0 - r_n}{\rho + \psi_L + \psi_m} - \frac{r_L \hat{\lambda}_0}{\rho + \psi_L + \psi_\lambda}. \tag{A.62}$$

We focus on the case $\frac{B_p^E}{B_p} = \frac{B_o^E}{B_o}$, so the initial relative wealth revaluation is given by

$$b_{p,0} - b_{o,0} = -\left(\frac{B_p^L}{B_p} - \frac{B_o^L}{B_o}\right)\left[\frac{i_0 - r_n}{\rho + \psi_L + \psi_m} + \frac{r_L \hat{\lambda}_0}{\rho + \psi_L + \psi_\lambda}\right]. \tag{A.63}$$

Plugging the expression above into the expression for $\hat{\lambda}_0$

$$\hat{\lambda}_0 \equiv \frac{\sigma \mu_{c,o} \mu_{c,p} \left(\frac{\lambda_p^{\frac{1}{\sigma}} - \lambda_o^{\frac{1}{\sigma}}}{\lambda^{\frac{1}{\sigma}}}\right) \frac{\rho + \xi}{\chi_{b,c}} \left(\frac{B_o^L}{B_o} - \frac{B_p^L}{B_p}\right)}{1 - \sigma \mu_{c,o} \mu_{c,p} \left(\frac{\lambda_p^{\frac{1}{\sigma}} - \lambda_o^{\frac{1}{\sigma}}}{\lambda^{\frac{1}{\sigma}}}\right) \frac{\rho + \xi}{\chi_{b,c}} \left(\frac{B_o^L}{B_o} - \frac{B_p^L}{B_p}\right) \frac{r_L}{\rho + \psi_L + \psi_\lambda}} \frac{i_0 - r_n}{\rho + \psi_L + \psi_m}. \tag{A.64}$$

Notice that there is an amplification mechanism between the price of the long-term bond and the change in disaster probability. A wealth redistribution towards pessimistic investors tends to increase $\hat{\lambda}_0$. An increase in $\hat{\lambda}_0$ depresses the value of long-term bonds, redistributing towards pessimistic investors, further increasing $\hat{\lambda}_t$.

Workers' consumption. Log-linearizing workers' budget constraint, we obtain

$$c_{w,t} = \frac{WN_w}{PC_w}(w_t - p_t + n_{w,t}) + \frac{Y}{C_w} T_w'(Y) y_t. \tag{A.65}$$

Using the fact that $w_t - p_t + n_{w,t} = (1 + \phi) y_t$, we can write the expression above as follows

$$c_{w,t} = \chi_y y_t. \tag{A.66}$$

where $\chi_y \equiv \frac{WN_w}{PC_w}(1 + \phi) + \frac{Y}{C_w} T_w'(Y)$.

Phillips curve. Linearizing the Phillips curve, we obtain

$$\dot{\pi}_t = \rho \pi_t - \kappa y_t, \tag{A.67}$$

where $\kappa \equiv \frac{\phi \epsilon}{\varphi} \frac{WN}{P}$.

Stock prices. Linearizing the expression for $r_{E,t}$, we obtain

$$\frac{\Pi}{Q_E}(\hat{\Pi}_t - q_{E,t}) + \dot{q}_{E,t} - (i_t - \pi_t - r_n) = r_E \left[\hat{\lambda}_t + \sigma c_{s,t} + \frac{Q_E^*}{Q_E - Q_E^*} q_{E,t} \right]. \tag{A.68}$$

Rearranging the expression above, we obtain

$$\dot{q}_{E,t} - \rho q_{E,t} = -\frac{1}{Q_E}\hat{\Pi}_t + (i_t - \pi_t - r_n) + r_E \left[\hat{\lambda}_t + \sigma c_{s,t} \right], \tag{A.69}$$

Solving the differential equation above, we obtain

$$q_{E,t} = \frac{1}{Q_E} \int_t^\infty e^{-\rho(s-t)} \hat{\Pi}_s ds - \int_t^\infty e^{-\rho(s-t)} \left[(i_s + \pi_s - r_n) + r_E(\hat{\lambda}_t + \sigma c_{s,t}) \right] ds. \tag{A.70}$$

A.4 The approximation in the price of risk

Propostion 3 shows that an approximate block recursivity property holds when $r_k \sigma c_{s,t} = \mathcal{O}(\|i_t - r_n\|^2)$, for $k \in \{L, E\}$. The term premium at $t = 0$ is given by $\int_0^\infty e^{-(\rho+\psi_L)t} r_L(\sigma c_{s,t} + \hat{\lambda}_t)dt$, so this assumption implies that we can approximate the term premium, up to first order, by the expression $\int_0^\infty e^{-(\rho+\psi_L)t} r_L \hat{\lambda}_t dt$. Similarly, the drop in the stock price caused by changes in risk premia is given by $\int_0^\infty e^{-\rho t} r_E(\sigma c_{s,t} + \hat{\lambda}_t)dt \approx \int_0^\infty e^{-\rho t} r_E \hat{\lambda}_t dt$ under our assumption about $r_k \sigma c_{s,t}$. To assess the quantitative importance of this assumption, we compare the discount rate effect on long-term bonds and equities when we include the term $r_k \sigma c_{s,t}$ to the corresponding solution when this term is ignored.

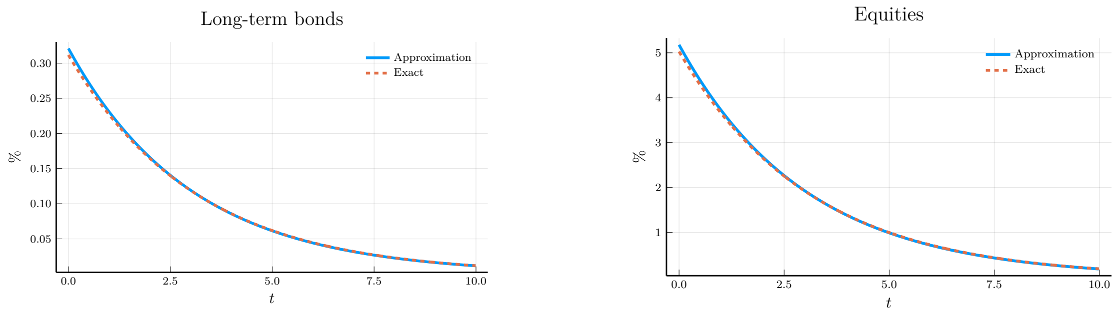


Figure A.1: Risk premium effect on long-term bonds and stocks.

Note: The left panel shows the term premium when the term $r_L \sigma c_{s,t}$ is included in the calculation (exact) and when this term is omitted (approximation). The right panel shows the drop on the stock price due to changes in the price of risk when the term $r_L \sigma c_{s,t}$ is included in the calculation (exact) and when this term is omitted (approximation).

Figure A.1 shows the effect of this approximation on the pricing of stocks and bonds.[3] The left panel shows that the response of the yield on the long-term bond when we ommit the term $r_L \sigma c_{s,t}$ is nearly identical to the one when this term is included. A similar pattern emerges for stocks. The right panel shows the magnitude of the drop in the stock price caused by movements in the price of risk. The solid line represent the calculation when the term $r_E \sigma c_{s,t}$ is included, and the dashed line shows the calculation when this term is omitted. Once again the approximate solution is nearly identical to the exact one.

B Derivations for Section 3

B.1 Trading in stocks

We consider next an extension where investors trade in stocks in the stationary equilibrium. In this case, the wealth effect of individual investors depends on the amount they trade on short-term bonds, long-term bonds, and stocks. However, as in the baseline model, the aggregate wealth effect depends only on the amount of government bonds

[3]Notice that we are only assessing the role of the assumption $\mathcal{O}(\|i_t - r_n\|^2)$. The lines we refer as "Exact" in Figure A.1 still corresponds to a linearized solution.

traded, as the household sector as a whole act as buy-and-hold investors on stocks.

The replicating portfolio. Let $i \in \mathcal{I}_j$ denote saver i of type j and assume that saver i receives real income $I_{j,t}(i) = a_j(i)e^{-\psi_E t}\Pi_t$. We assume that $\int_{i\in\mathcal{I}_j} a_j(i)di = 0$ and that the following condition is satisfied in a stationary equilibrium:

$$B_{j,0}(i) + \mathbb{E}\left[\int_0^\infty \frac{\eta_t}{\eta_0} I_{j,t}(i)dt\right] = B_{j,0}, \tag{B.1}$$

where $B_{j,0}(i)$ is the initial wealth of saver i and $B_{j,0}$ is the average wealth of type-j savers. This implies that the consumption of all savers is the same in the stationary equilibrium. Let $B^S_{j,t}(i) = B^S_j + \tilde{B}^S_{j,t}(i)$ and $B^E_{j,t}(i) = B^E_j + \tilde{B}^E_{j,t}(i)$, then

$$\tilde{B}^S_{j,t} + \tilde{B}^E_{j,t} + Q_{I_j(i),t} = 0, \qquad \tilde{B}^S_{j,t} + \tilde{B}^E_{j,t}\frac{Q^*_E}{Q_E} + Q^*_{I_j(i),t} = 0. \tag{B.2}$$

We can then solve for the portfolio of individual i:

$$\tilde{B}^S_{j,t}(i) = Q_{I_j(i),t}\frac{Q^*_E}{Q_E - Q^*_E} - Q^*_{I_j(i),t}\frac{Q_E}{Q_E - Q^*_E}, \tag{B.3}$$

$$\tilde{B}^E_{j,t}(i) = Q^*_{I_j(i),t}\frac{Q_E}{Q_E - Q^*_E} - Q_{I_j(i),t}\frac{Q_E}{Q_E - Q^*_E}. \tag{B.4}$$

Pricing. Notice that we can write the expression for $\tilde{B}^E_{j,t}(i)$ as follows:

$$\frac{Q_E - Q^*_E}{Q_E}\tilde{B}^E_{j,t}(i) = -\frac{Q_{I_j(i),t} - Q^*_{I_j(i),t}}{Q_{I_j(i),t}}Q_{I_j(i),t}, \tag{B.5}$$

so $r_E\tilde{B}^E_{j,t}(i) = -r_{I_j(i)}Q_{I_j(i),t}$. Assuming the economy is in the stationary equilibrium, the value of the income claim in the disaster state is given by

$$Q^*_{I_j(i),t} = a_j(i)\frac{e^{-\psi_E t}\Pi^*}{r^*_n + \psi_E}, \tag{B.6}$$

and the value of the income claim in the no-disaster state is given by

$$Q_{I_j(i),t} = \frac{a_j(i)\Pi e^{-\psi_E t} + \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} Q^*_{I_j(i),t}}{r_n + \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} + \psi_E}. \tag{B.7}$$

We can then write the portfolio holdings of investor i as follows:

$$\tilde{B}^E_{j,t}(i) = -a_j(i)e^{-\psi_E t}\frac{Q_E}{Q_E - Q_E^*}\frac{\Pi - \frac{r_n+\psi_E}{r_n^*+\psi_E}\Pi^*}{r_n + \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} + \psi_E} \tag{B.8}$$

$$\tilde{B}^S_{j,t}(i) = a_j(i)e^{-\psi_E t}\frac{Q_E}{Q_E - Q_E^*}\left[\frac{\Pi + \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma}\frac{\Pi^*}{r_n^*+\psi_E}}{r_n + \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} + \psi_E}\frac{Q_E^*}{Q_E} - \frac{\Pi^*}{r_n^* + \psi_E}\right]. \tag{B.9}$$

Notice that $r_{I_j(i)}$ is given by

$$r_{I_j(i)} = \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} \frac{\Pi - \frac{r_n+\psi_E}{r_n^*+\psi_E}\Pi^*}{\Pi + \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma}\frac{\Pi^*}{r_n^*+\psi_E}}. \tag{B.10}$$

Linearizing the pricing condition for the income claim, we obtain

$$q_{I_j,0} = \frac{a_j(i)Y}{Q_{I_j,0}}\int_0^\infty e^{-(\rho+\psi_E)t}\hat{\Pi}_t dt - \int_0^\infty e^{-(\rho+\psi_E)t}\left(i_t - \pi_t - r_n + r_{I_j(i)}p_{d,t}\right)dt. \tag{B.11}$$

Wealth effects. The intertemporal budget constraint for household i is given by

$$\mathbb{E}_0\left[\int_0^\infty \frac{\eta_t}{\eta_0}C_{j,t}(i)dt\right] = B_{j,0}(i) + \mathbb{E}\left[\int_0^\infty \frac{\eta_t}{\eta_0}\left(I_{j,t}(i) + T_{j,t}\right)dt\right]. \tag{B.12}$$

Linearizing the equation above, we obtain

$$\Omega_{j,0}(i) = \frac{1}{C_j}\left[B_j^L q_{L,0} + B^E_{j,0}(i)q_{E,0} + Q_{T_j}q_{T_j,0} + Q_{I_j(i),0}q_{I_j(i),0}\right] + \frac{Q_{C_j}}{C_j}\int_0^\infty e^{-\rho t}\left(i_t - \pi_t - r_n + r_{C_j}p_{d,t}\right)dt, \tag{B.13}$$

where $Q_{I_j(i),0}$ is the value at 0 of a claim on $I_{j,t}(i)$ for all $t \geq 0$.

Using the fact that $Q_{C_j} = B^S_{j,0}(i) + B^L_j + B^E_{j,0}(i) + Q_{I_j(i),0} + Q_{T_j}$ and $Q_C r_{C_j} = B^L_j r_L + B^E_{j,0}(i) r_E + Q_{I_j(i),0} r_{I_j(i)} + Q_{T_j} r_{T_j}$, we can write the wealth effect as follows:

$$\Omega_{j,0}(i) = \Omega_{j,0} + \frac{Y}{C_j}\int_0^\infty e^{-\rho t}\left(\frac{B^E_{j,0}(i)}{Q_E} + e^{-\psi_E t} a_j(i)\right)\hat{\Pi}_t dt$$
$$+ \frac{\tilde{B}^S_{j,0}(i)}{C_j}\int_0^\infty e^{-\rho t}(i_t - \pi_t - r_n) dt$$
$$+ \frac{Q_{I_j(i),0}}{C_j}\int_0^\infty e^{-\rho t}\left(1 - e^{-\psi_E t}\right)\left(i_t - \pi_t - r_n + r_{I_j(i)} p_{d,t}\right) dt \tag{B.14}$$

Notice that $(1 - e^{-\psi_E t}) Q_{I_j(i),0} = Q_{I_j(i),0} - Q_{I_j(i),t}$, $Q_{I_j(i),t} = -\tilde{B}^S_{j,t}(i) - \tilde{B}^E_{j,t}(i)$, and $r_{I_j} Q_{I_j(i),t} = r_E \tilde{B}^E_{j,t}(i)$. We can then write the expression above as follows:

$$\Omega_{j,0}(i) = \Omega_{j,0} + \frac{Y}{C_j}\int_0^\infty e^{-\rho t}\left(\frac{\tilde{B}^E_{j,0}(i)}{Q_E} + e^{-\psi_E t} a_j(i)\right)\hat{\Pi}_t dt$$
$$+ \frac{1}{C_j}\int_0^\infty e^{-\rho t}\Delta B^S_{j,t}(i_t - \pi_t - r_n) dt$$
$$+ \frac{1}{C_j}\int_0^\infty e^{-\rho t}\Delta B^E_{j,t}(i_t - \pi_t - r_n + r_E p_{d,t}) dt, \tag{B.15}$$

where $\Delta B^E_{j,t} = \tilde{B}^E_{j,t}(i) - \tilde{B}^E_{j,0}(i)$ and $\Delta B^S_{j,t} = \tilde{B}^S_{j,t}(i)$. Notice that as $\int_{i \in I_j} a_j(i) di = 0$, then $\frac{1}{\mu_j}\int_{i \in \mathcal{I}_j} \Omega_{j,0}(i) di = \Omega_{j,0}$.

The equation above express the wealth effect in terms of cumulative purchases of assets. We can equivalently write the expression above in terms of instantaneous net purchases of assets, as in Fagereng, Gomez, Gouin-Bonenfant, Holm, Moll and Natvik (2022). For simplicity, assume there is no cash-flow effect. We can then write the integral above involving equities as follows:

$$\int_0^\infty e^{-\rho t}\Delta B_{j,t}^E(i_t-\pi_t-r_n+r_E p_{d,t})dt = \int_0^\infty e^{-\rho t}(1-e^{-\psi_E t})B_{j,0}^E(\dot{q}_{E,t}-\rho q_{E,t})dt$$

$$= B_{j,0}^E\left[\int_0^\infty d(e^{-\rho t}q_{E,t}) - \int_0^\infty d(e^{-(\rho+\psi_E)t}q_{E,t})\right] + \int_0^\infty e^{-\rho t}N_{j,t}^E q_{E,t}dt \tag{B.16}$$

$$= -\int_0^\infty e^{-\rho t}N_{j,t}^E q_{E,t}dt. \tag{B.17}$$

where $N_{j,t}^E = -\psi_E B_{j,t}^E$ denotes the net purchases at period t, using the fact that $i_t - \pi_t - r_n + r_E p_{d,t} = \dot{q}_{E,t} - \rho q_{E,t}$

B.2 Wealth effects and Hicksian compensation

Hicksian compensation. We show next that $\Omega_{j,0}$ corresponds to (minus) the *Hicksian wealth compensation* for each household. Let $e_j(\eta, U)$ define the expenditure function

$$e_j(\eta, U) = \min_{\{C_j\}} \mathbb{E}_{j,0}\left[\int_0^{t^*}\frac{\eta_{j,t}}{\eta_{j,0}}C_{j,t}dt + \int_{t^*}^\infty \frac{\eta_{j,t}^*}{\eta_{j,0}}C_{j,t}^*dt\right], \tag{B.18}$$

subject to $\mathbb{E}_{j,0}\left[\int_0^{t^*} e^{-\int_0^t \rho_{j,s}ds}\frac{C_{j,t}^{1-\sigma}-C_j^{1-\sigma}}{1-\sigma}dt + \int_{t^*}^\infty e^{-\int_0^t \rho_{j,s}ds}\frac{(C_{j,t}^*)^{1-\sigma}-(C_j^*)^{1-\sigma}}{1-\sigma}dt\right] = U$. We subtracted the utility of the stationary-equilibrium consumption bundle, so $U = 0$ corresponds to the utility obtained in the stationary equilibrium. The solution to this problem is the Hicksian demand $C_{j,t}^h(\eta_j, U)$ and $C_{j,t}^{h,*}(\eta_j, U)$ in the no-disaster and disaster states.

Let η' denote an alternative price process and U' the corresponding utility under the new equilibrium. Mas-Colell et al. (1995) (see page 62) defines the Hicksian wealth compensation as $e_j(\eta_j', U) - e_j(\eta_j', U')$. We focus on a first-order approximation, that is, $\eta_t'/\eta_0' = \eta_t/\eta_0 + \tilde{\eta}_t$, where $\tilde{\eta}_t$ is small. Let $\tilde{c}_{j,t} \equiv \log C_{j,t}^h(\eta', U)/C_{j,t}^h(\eta, U)$. Plugging the expression for $C^h_{j,t}(\eta', U)$ into the constraint and linearizing, we obtain

$$\mathbb{E}_{j,0}\left[\int_0^{t^*} e^{-\rho_j t} C^h_{j,t}(\eta, U)^{1-\sigma}\tilde{c}_{j,t}dt + e^{-\rho_j t^*}\int_{t^*}^{\infty} e^{-\rho_j^*(t-t^*)} C^{h,*}_{j,t}(\eta, U)^{1-\sigma}\tilde{c}^*_{j,t}dt\right] = 0. \quad \text{(B.19)}$$

Notice this implies that $\mathbb{E}_{j,0}\left[\int_0^{t^*} \frac{\eta_{j,t}}{\eta_{j,0}} C^h_{j,t}(\eta, U)\tilde{c}_{j,t}dt + \int_{t^*}^{\infty} \frac{\eta^*_{j,t}}{\eta_{j,0}} C^{h,*}_{j,t}(\eta, U)\tilde{c}^*_{j,t}dt\right] = 0$. As workers do not engage in intertemporal substitution, we set $\tilde{c}_{w,t} = \tilde{c}^*_{w,t} = 0$, so this equation would hold for them as well. We can then write $e_j(\eta', U)$ up to first order as follows

$$\begin{aligned}
e_j(\eta', U) &= \mathbb{E}_0\left[\int_0^{t^*} \frac{\eta'_t}{\eta'_0} C^h_{j,t}(\eta, U)dt + \int_{t^*}^{\infty} \frac{\eta'_t}{\eta'_0} C^{*,h}_{j,t}(\eta, U)dt + \int_0^{t^*} \frac{\eta_t}{\eta_0} C^h_{j,t}(\eta, U)\tilde{c}_{j,t}dt + \int_{t^*}^{\infty} \frac{\eta^*_t}{\eta_0} C^{h,*}_{j,t}(\eta, U)\tilde{c}^*_{j,t}dt\right], \\
&= \mathbb{E}_0\left[\int_0^{t^*} \frac{\eta'_t}{\eta'_0} C^h_{j,t}(\eta, U)dt + \int_{t^*}^{\infty} \frac{\eta'_t}{\eta'_0} C^{*,h}_{j,t}(\eta, U)dt\right]. \qquad \text{(B.20)}
\end{aligned}$$

We assume that the initial equilibrium corresponds to the stationary equilibrium, so $C^h_{j,t}(\eta, U) = C_j$ and $C^{h,*}_{j,t}(\eta, U) = C^*_j$. Let η'_j denote the SDF after the monetary shock and U' the corresponding utility level. Therefore, the Hicksian wealth compensation is given by

$$e_j(\eta'_j, U) - e_j(\eta'_j, U') = \mathbb{E}_{j,0}\left[\int_0^{t^*} \frac{\eta'_{j,t}}{\eta'_{j,0}} C_j dt + \int_{t^*}^{\infty} \frac{\eta'_{j,t}}{\eta'_{j,0}} C^*_j dt\right] - \mathbb{E}_{j,0}\left[\int_0^{t^*} \frac{\eta'_{j,0}}{\eta'_{j,0}} C_{j,t}dt + \int_{t^*}^{\infty} \frac{\eta'_{j,t}}{\eta'_{j,0}} C^*_{j,t}dt\right], \quad \text{(B.21)}$$

which corresponds to $-\Omega_{j,0}C_j$ as defined in the text.

Compensating and equivalent variation. From the derivation above, we obtain that $\Omega_{j,0}C_j = e_j(\eta'_j, U') - e_j(\eta'_j, U)$, which corresponds to the compensating variation. We show next that $\Omega_{j,0}C_j$ also coincides with the equivalent variation up to first order. The EV is given by $e_j(\eta_j, U') - e_j(\eta_j, U)$. Up to first order, $C^h_{j,t}(\eta'_j, U') - C^h_{j,t}(\eta_j, U) = C^h_{j,t}(\eta_j, U') - C^h_{j,t}(\eta_j, U) + C^h_{j,t}(\eta'_j, U) - C^h_{j,t}(\eta_j, U)$. As the present discounted value of $C^h_{j,t}(\eta'_j, U) - C^h_{j,t}(\eta_j, U)$ is equal to zero, evaluated at the initial SDF, then the present discounted value of the left-hand side, $C^h_{j,t}(\eta'_j, U') - C^h_{j,t}(\eta_j, U)$, equals the present discounted value of $C^h_{j,t}(\eta_j, U') - C^h_{j,t}(\eta_j, U)$. The present discounted value of $C^h_{j,t}(\eta'_j, U') - C^h_{j,t}(\eta_j, U)$ evaluated at η (or η') corresponds to $C_j\Omega_{j,0}$. The present discounted value of $C^h_{j,t}(\eta_j, U') - C^h_{j,t}(\eta_j, U)$ evaluated at η equals the equivalent varation, so $e_j(\eta_j, U') - e_j(\eta_j, U) = C_j\Omega_{j,0}$.

B.3 Consumption decomposition

We show next that the consumption response to a shock can be decomposed into the response of a compensated (Hicksian) demand, which captures intertemporal subsitution and precautionary effects, and a wealth effect. Consumption taxes only affect the compensated demand, so the wealth effect is independent of taxes. We also provide a comparison with standard results with log utility. To better compare with previous results in the literature, we focus on the case of a constant subjective discount rate, so we abstract from Uzawa preferences here.

Household's problem. Consider an exogenously given SDF:

$$\frac{d\eta_t}{\eta_t} = -r_t dt + \xi_t(d\mathcal{N}_t - \lambda_t dt), \tag{B.22}$$

where the probability of a Poisson event is λ_t.

The household's problem with consumption taxes is given by

$$V(X, B) = \max_{C_t} \mathbb{E}_0\left[\int_0^\infty e^{-\rho t} u(C_t) dt\right], \;\; s.t. \;\; \mathbb{E}_0\left[\int_0^\infty \frac{\eta_t}{\eta_0}\left(C_t(1+\tau^c_t) - T_t\right) dt\right] = B, \tag{B.23}$$

where $X = \{r_t, \xi_t, \lambda_t, \tau^c_t\}$ denote the path of interest rate, the jump in marginal utilities in a disaster, the disaster probability, and the consumption tax.

The optimality condition for this problem is $C_t = e^{-\frac{\rho}{\sigma}t}\left(\frac{\eta_t}{\eta_0}(1+\tau^c_t)\right)^{-\frac{1}{\sigma}} \mu^{-\frac{1}{\sigma}}$, where μ is the Lagrange multiplier on the budget constraint (IBC). Plugging the expression above into the IBC, and using the fact that $T_t = \tau_t^c C_t$, we obtain

$$C_t(X, B) = \frac{e^{-\frac{\rho}{\sigma}t}\left(\frac{\eta_t}{\eta_0}\frac{1+\tau_t^c}{1+\tau_0^c}\right)^{-\frac{1}{\sigma}}}{\mathbb{E}_0\left[\int_0^\infty e^{-\frac{\rho}{\sigma}s}\left(\frac{\eta_s}{\eta_0}\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{1+\tau_s^c}{1+\tau_0^c}\right)^{-\frac{1}{\sigma}} ds\right]} B. \tag{B.24}$$

Compensated demand. The expenditure minimization problem can be written as

$$e(X, U) = \min_{C_t} \mathbb{E}_0\left[\int_0^\infty \frac{\eta_t}{\eta_0}\left(C_t(1+\tau_t^c) - T_t\right) dt\right], \;\; s.t. \;\; \mathbb{E}_0\left[\int_0^\infty e^{-\rho t} u(C_t) dt\right] = U. \tag{B.25}$$

The first-order condition is given by

$$\frac{\eta_t}{\eta_0}(1+\tau_t^c) = \frac{1}{\mu} e^{-\rho t} u'(C_t) \Rightarrow C_t = e^{-\frac{\rho}{\sigma}t}\left(\frac{\eta_t}{\eta_0}(1+\tau_t^c)\right)^{-\frac{1}{\sigma}} \mu^{-\frac{1}{\sigma}}. \tag{B.26}$$

Plugging the expression above into the expression for utility, we obtain

$$C_t^h(X, U) = \frac{e^{-\frac{\rho}{\sigma}t}\left(\frac{\eta_t}{\eta_0}(1+\tau_t^c)\right)^{-\frac{1}{\sigma}}}{\mathbb{E}_0\left[\int_0^\infty e^{-\frac{\rho}{\sigma}s}\left(\frac{\eta_s}{\eta_0}(1+\tau_s^c)\right)^{\frac{\sigma-1}{\sigma}} ds\right]^{\frac{1}{1-\sigma}}} g(U). \tag{B.27}$$

where $g(U) \equiv ((1-\sigma)U)^{\frac{1}{1-\sigma}}$. The expenditure function is given by

$$e(X, U) = \frac{\mathbb{E}_0\left[\int_0^\infty e^{-\frac{\rho}{\sigma}t}\left(\frac{\eta_t}{\eta_0}\right)^{\frac{\sigma-1}{\sigma}}(1+\tau_t^c)^{-\frac{1}{\sigma}} dt\right]}{\mathbb{E}_0\left[\int_0^\infty e^{-\frac{\rho}{\sigma}t}\left(\frac{\eta_t}{\eta_0}(1+\tau_t^c)\right)^{\frac{\sigma-1}{\sigma}} dt\right]^{\frac{1}{1-\sigma}}} g(U). \tag{B.28}$$

Initial equilibrium. Let $(\overline{X}, \overline{B})$ denote the state in an initial equilibrium. In the initial equilibrium, all variables are constant conditional on no switching, and the consumption tax is set to zero. The SDF on the no-disaster state is given by $\overline{\eta}_t = e^{-(\overline{r}+\overline{\xi}\overline{\lambda})t}\overline{\eta}_0$. The SDF on the disaster state is given by $\overline{\eta}_t^* = e^{-\overline{r}^*(t-\tau)}(1+\overline{\xi})\overline{\eta}_\tau$, where τ is the (random) date of switch. In equilibrum, we must have $\overline{r} = \rho - \overline{\xi\lambda}$ and $\overline{r}^* = \rho$.

Consumption in the no-disaster and disaster states are given by

$$C_t(\overline{X}, \overline{B}) = \frac{\rho(\rho + \overline{\lambda})}{\rho + \overline{\lambda}(1+\overline{\xi})^{\frac{\sigma-1}{\sigma}}}\overline{B}, \qquad C_t^*(\overline{X}, \overline{B}) = \frac{\rho(\rho + \overline{\lambda})(1+\overline{\xi})^{-\frac{1}{\sigma}}}{\rho + \overline{\lambda}(1+\overline{\xi})^{\frac{\sigma-1}{\sigma}}}\overline{B}. \tag{B.29}$$

Initial wealth is given by

$$\overline{B} = \int_0^\infty e^{-(\rho+\overline{\lambda})t}\left[\overline{D}_t + \overline{\lambda}(1+\overline{\xi})\overline{B}_t^*\right] dt, \tag{B.30}$$

where $\overline{B}_t^* = \int_t^\infty \frac{\eta_s^*}{\eta_t^*}\overline{D}_s^* ds$. We assume that $D_t = e^{-\psi_D t}D_0$ and $D_t^* = (1-\zeta_D)e^{-\psi_D(t-\tau)}D_\tau$, which nests the case of long-term bonds and stocks. The value of a consumption claim is

$$Q_C = \frac{\rho + \lambda(1+\xi)^{\frac{\sigma-1}{\sigma}}}{\rho(\rho+\lambda)}C, \qquad Q_C^* = \frac{(1+\xi)^{-\frac{1}{\sigma}}}{\rho}C \Rightarrow \frac{Q_C - Q_C^*}{Q_C} = 1 - \frac{(\rho+\lambda)(1+\xi)^{-\frac{1}{\sigma}}}{\rho + \lambda(1+\xi)^{\frac{\sigma-1}{\sigma}}}. \tag{B.31}$$

Economy of interest. Consider the economy of interest with state (X, B). We consider the effect of a shock to the interest rate r_t, the disaster probability λ_t, and the consumption tax τ_t^c. The SDF in the economy of interest is given by $\eta_t = e^{-\int_0^t(\rho + r_t - r + \xi(\lambda_z - \lambda))dz}\eta_0$ in the no-disaster state and $\eta_t^* = e^{-\rho(t-\tau)}(1+\overline{\xi})\eta_\tau$.

Initial wealth. B_t denotes the household's wealth at t. The pricing condition for B_t is

$$\frac{\dot{B}_t}{B_t} = r_t - \frac{D_t}{B_t} + \lambda_t(1+\xi_t)\xi_{B,t}, \tag{B.32}$$

where $\xi_{B,t} = \frac{B_t - B_t^*}{B_t}$. Linearizing the expression above, we obtain

$$\dot{b}_t = (\rho + \overline{\lambda} + \psi_D)b_t + r_t - \rho + r_B\hat{\lambda}_t, \tag{B.33}$$

where $r_B = \overline{\lambda}(1+\overline{\xi})\overline{\xi}_B$ and $\hat{\lambda}_t = \frac{\lambda_t - \overline{\lambda}}{\overline{\lambda}}$. The initial wealth is then given by

$$b_0 = -\int_0^\infty e^{-(\rho+\overline{\lambda}+\psi_D)t}\left[r_t - \rho + r_B\hat{\lambda}_t\right]dt. \tag{B.34}$$

Consumption claim. The consumption claim is given by

$$Q_{C,0} = \mathbb{E}_0\left[\int_0^\infty \frac{\eta_s}{\eta_s} C_s ds\right]. \tag{B.35}$$

The pricing condition for $Q_{C,0}$ is given by

$$\frac{\dot{Q}_{C,t}}{Q_{C,t}} = -\frac{C_t}{Q_{C,t}} + r_t + \lambda_t(1+\xi_t)\frac{Q_{C,t} - Q^*_{C,t}}{Q_{C,t}}. \tag{B.36}$$

Let $q_{C,t} \equiv \log Q_{C,t}/Q_C$ and $q^*_{C,t} \equiv \log Q^*_{C,t}/Q^*_C$. Then,

$$\dot{q}_{C,t} = (\rho+\overline{\lambda})q_{C,t} + r_t - \overline{r} + r_C\hat{\lambda}_t - \frac{\overline{C}}{Q_C}c_t - \overline{\lambda}(1+\overline{\xi})\frac{Q^*_C}{Q_C}q^*_{C,t}. \tag{B.37}$$

where $c_t \equiv \log C_t/\overline{C}$. Solving the expression above forward, we obtain

$$q_{C,0} = (\rho+\overline{\lambda})\int_0^\infty e^{-(\rho+\overline{\lambda})t}c_t dt - \int_0^\infty e^{-(\rho+\overline{\lambda})t}[r_t - \overline{r} + r_C\hat{\lambda}_t]dt. \tag{B.38}$$

using $q^*_{C,t} = c^*_t$, $c^*_t = c_t$, and $\frac{\overline{C}}{Q_C} + \overline{\lambda}(1+\overline{\xi})\frac{Q^*_C}{Q_C} = \rho + \overline{\lambda}$.

IBC and wealth effects. Linearizing the intertemporal budget constraint, we obtain

$$Q_C q_{C,0} = Bb_0 \Rightarrow (\rho+\overline{\lambda})\int_0^\infty e^{-(\rho+\overline{\lambda})t}c_t dt = \frac{C}{B}\Omega_0, \tag{B.39}$$

where Ω_0 denotes the wealth effect:

$$\Omega_0 \equiv \frac{B}{C}\left[b_0 + \int_0^\infty e^{-(\rho+\overline{\lambda})t}(r_t - \overline{R} + r_C\hat{\lambda}_t)dt\right]. \tag{B.40}$$

Consumption. Denote the denominator of the expression for consumption as follows:

$$Q_{\mathcal{C},0} = \mathbb{E}_0\left[\int_0^\infty \frac{\eta_s}{\eta_s}\mathcal{C}_s ds\right], \tag{B.41}$$

where $\mathcal{C}_t = \left(e^{\rho t}\frac{\eta_t}{\eta_0}(1+\tau_t^c)\right)^{-\frac{1}{\sigma}}$. Notice that $\mathcal{C}_t$ is proportional to consumption. A similar derivation as above yields

$$q_{\mathcal{C},0} = (\rho+\lambda)\int_0^\infty e^{-(\rho+\lambda)t}\hat{\mathcal{C}}_t dt - \int_0^\infty e^{-(\rho+\lambda)t}[r_t - \overline{r} + r_C\hat{\lambda}_t]dt. \tag{B.42}$$

using $q^*_{\mathcal{C},t} = \hat{\mathcal{C}}_t$ and $\frac{\mathcal{C}}{Q_\mathcal{C}} + \overline{\lambda}(1+\overline{\xi})\frac{Q^*_\mathcal{C}}{Q_\mathcal{C}} = \rho + \overline{\lambda}$. Notice that $\hat{\mathcal{C}}_t = \frac{1}{\sigma}\int_0^t(r_z - \overline{r} + \overline{\xi\lambda}\hat{\lambda}_z - \dot{\hat{\tau}}_z^c)dz$.

Consumption is then given by

$$c_t = \hat{\mathcal{C}}_t + b_0 - q_{\mathcal{C},0} = \hat{\mathcal{C}}_t - (\rho+\overline{\lambda})\int_0^\infty e^{-(\rho+\overline{\lambda})t}\hat{\mathcal{C}}_t dt + \mathcal{M}\Omega_0 \tag{B.43}$$

Compensated demand. We evaluate the compensated demand at the utility level:

$$U_0 = \mathbb{E}_t\left[\int_0^\infty e^{-\rho t}u(\overline{C}_t)dt\right]. \tag{B.44}$$

Notice that U_0 responds to changes in λ_t even if $\overline{C}_t$ does not change. The utility level satisfies the HJB equation:

$$\rho U_t = u(\overline{C}_t) + \dot{U}_t + \lambda_t(U_t^* - U_t). \tag{B.45}$$

Let $u_t = \log U_t / U$. Then, u_t satisfies the condition

$$\dot{u}_t - (\rho + \lambda) u_t = \lambda \left(1 - \frac{U^*}{U}\right) \hat{\lambda}_t \Rightarrow u_0 = -\lambda \left(1 - \frac{U^*}{U}\right) \int_0^\infty e^{-(\rho+\lambda)t} \hat{\lambda}_t dt, \tag{B.46}$$

where $\frac{U^*}{U} = \frac{(\rho+\lambda)(1+\xi)^{\frac{\sigma-1}{\sigma}}}{\rho+\lambda(1+\xi)^{\frac{\sigma-1}{\sigma}}}$. The expenditure function is given by

$$\hat{e}_0 = \frac{\sigma}{\sigma - 1} \left[(\rho + \lambda) \int_0^\infty e^{-(\rho+\lambda)t} \hat{C}_t dt - \int_0^\infty e^{-(\rho+\lambda)t} [r_t - \rho + r_C \hat{\lambda}_t] dt \right] - \int_0^\infty e^{-(\rho+\lambda)t} \frac{\dot{\hat{\tau}}_t^c}{1-\sigma} dt + \frac{u_0}{1-\sigma}. \tag{B.47}$$

We can write the expression above as follows:

$$\hat{e}_0 = \frac{\sigma}{\sigma - 1} \left[\frac{1}{\sigma} \int_0^\infty e^{-(\rho+\lambda)t} (r_t - \rho + \overline{\xi\lambda} \hat{\lambda}_t) dt - \int_0^\infty e^{-(\rho+\lambda)t} [r_t - \rho + r_C \hat{\lambda}_t] dt \right] + \frac{u_0}{1-\sigma}, \tag{B.48}$$

where $\hat{\tau}_t^c$ cancels out from the previous expression.

We can write the expression above as follows:

$$\hat{e}_0 = \frac{\lambda \left(1 - \frac{U^*}{U}\right) - (r_C - \overline{\xi\lambda})}{\sigma - 1} \int_0^\infty e^{-(\rho+\lambda)t} \hat{\lambda}_t dt - \int_0^\infty e^{-(\rho+\lambda)t} [r_t - \rho + r_C \hat{\lambda}_t] dt, \tag{B.49}$$

Notice that $r_C - \xi\lambda$ can be written as follows:

$$\lambda \left[1 + \xi - \frac{(\rho+\lambda)(1+\xi)^{\frac{\sigma-1}{\sigma}}}{\rho + \lambda(1+\xi)^{\frac{\sigma-1}{\sigma}}} \right] - \xi\lambda = \lambda \left(1 - \frac{U^*}{U}\right). \tag{B.50}$$

We can then write the expenditure function as follows:

$$\hat{e}_0 = -\int_0^\infty e^{-(\rho+\lambda)t} [r_t - \rho + r_C \hat{\lambda}_t] dt. \tag{B.51}$$

The compensated demand is given by

$$c_t^h = \hat{\mathcal{C}}_t - \frac{1}{1-\sigma} q_{\mathcal{C},0} - \frac{1}{1-\sigma}\int_0^\infty e^{-(\rho+\lambda)t}\dot{\hat{\tau}}_t^c dt + \frac{u_0}{1-\sigma} = \mathcal{C}_0 - q_{\mathcal{C},0} + \hat{e}_0. \tag{B.52}$$

We can write the expression above as follows:

$$c_t^h = \frac{1}{\sigma}\int_0^t (r_z - \overline{r} + \overline{\xi\lambda}\hat{\lambda}_z - \dot{\hat{\tau}}_z^c) dz - \frac{1}{\sigma}\int_0^\infty e^{-(\rho+\lambda)z}(r_z - \overline{r} + \overline{\xi\lambda}\hat{\lambda}_z - \dot{\hat{\tau}}_z^c) dz. \tag{B.53}$$

Decomposition. We can write consumption as follows:

$$c_t = \hat{\mathcal{C}}_t - q_{\mathcal{C},0} + e_0 + b_0 - e_0 = c_t^h + \mathcal{M}\Omega_0. \tag{B.54}$$

The role of taxes. The decomposition above sheds light on the role of the consumption taxes in Proposition 4. Notice that the consumption tax only affects the compensated demand. Moreover, if $\dot{\hat{\tau}}_t^c = \xi\lambda$, then c_t^h is independent of $\hat{\lambda}_t$. In this case, the only channel that changes in $\hat{\lambda}_t$ can affect consumption is through the wealth effect. Consistent with the discussion in Section 3, the wealth effect is equal to zero in response to a discount rate shock if $\overline{C}_t = \overline{D}_t$.

Comparison with log utility. If $\sigma = 1$, so the household has log utility, then consumption is given by

$$C_0 = \mathbb{E}_0\left[\int_0^\infty e^{-\int_0^t(\rho + \dot{\hat{\tau}}_z^c)dz} dt\right]^{-1} B_0. \tag{B.55}$$

We recover the standard log-utility result, $C_0 = \rho B_0$, when $\hat{\tau}_t^c = 0$. In this case, log consumption is given by $c_0 = b_0$. We have seen above that $c_0 = c_0^h + \mathcal{M}\Omega_0$ and $\Omega_0 = 0$ when $\overline{C}_t = \overline{D}_t$. This implies that $c_t = c_t^h$, which holds for any value of σ. The log-utility case is special as we have the following equalities:

$$c_0 = c_0^h = b_0, \tag{B.56}$$

as $r_C = \overline{\xi\lambda}$ when $\sigma = 1$. In this case, the consumption of a log-utility investor coincides with the response of the compensated demand.

B.4 iMPCs

The problem of saver j can be written as

$$V(\eta, \omega) = \mathbb{E}_{j,0}\left[\int_0^\infty e^{-\int_0^t \rho_{j,s}ds}\frac{\tilde{C}_{j,t}^{1-\sigma}}{1-\sigma}dt\right], \tag{B.57}$$

subject to

$$\mathbb{E}_{j,0}\left[\int_0^\infty \frac{\tilde{\eta}_{j,t}}{\eta_{j,0}}\tilde{C}_{j,t}dt\right] = \omega, \tag{B.58}$$

where $\eta_{j,t}$ denotes the SDF under saver j's beliefs, which evolves according to $\frac{d\eta_{j,t}}{\eta_{j,t}} = -\left[i_t - \pi_t + \lambda_j \frac{\eta_{j,t}^* - \eta_{j,t}}{\eta_{j,t}}\right]dt + \frac{\eta_{j,t}^* - \eta_{j,t}}{\eta_{j,t}}d\mathcal{N}_t$, $\tilde{C}_{j,t} = C_{j,t}$ if the economy is in the no-disaster state, and $\tilde{C}_{j,t} = C_{j,t}^*$ if the economy is in the disaster state. The SDF satisfies the change of measure conditions: $\lambda_j \frac{\eta_{j,t}^*}{\eta_{j,t}} = \lambda_t \frac{\eta_t^*}{\eta_t}$ and $\frac{\eta_{j,t}}{\eta_{0,t}} = e^{-\int_0^t(\lambda_s - \lambda_j)ds}\frac{\eta_t}{\eta_0}$.

The first-order conditions for this problem are given by

$$e^{-\int_0^t \rho_{j,s}ds}\tilde{C}_{j,t}^{-\sigma} = \Lambda\frac{\tilde{\eta}_{j,t}}{\eta_{j,0}}, \tag{B.59}$$

where Λ is the Lagrange multiplier on the intertemporal budget constraint.

The intertemporal budget constraint can be written as

$$\int_0^\infty e^{-\lambda_j t} \frac{\eta_{j,t}}{\eta_{j,0}} \left[C_{j,t} + \lambda_j \frac{\eta^*_{j,t}}{\eta_{j,t}} Q^*_{C_j,t} \right] dt = \omega, \tag{B.60}$$

where $Q^*_{C_j,t} = \int_t^\infty \frac{\eta^*_{j,s}}{\eta^*_{j,t}} C^*_{j,s} ds$ is the value of a consumption claim for an economy that switches to the disaster state at time t. Applying a change of measure, we can write the equation above as follows:

$$\int_0^\infty e^{-\int_0^t \lambda_s ds} \frac{\eta_t}{\eta_0} \left[C_{j,t} + \lambda_t \frac{\eta^*_t}{\eta_t} Q^*_{C_j,t} \right] dt = \omega. \tag{B.61}$$

The stationary equilibrium. In a stationary equilibrium, we have that $\eta^*_{j,t} = e^{-r^*_n(t-t^*)} \eta^*_{j,t^*}$. Given our assumption that $\rho^*_j = r^*_n$, then $C^*_{j,t} = C^*_{j,t^*}$, so $Q^*_{C_j,t^*} = \frac{C^*_{j,t^*}}{r^*_n}$. From the optimality condition for risky assets, $\lambda \left(\frac{C_s}{C^*_s} \right)^\sigma = \lambda_j \left(\frac{C_j}{C^*_j} \right)^\sigma$, we obtain

$$C^*_j = \frac{\lambda_j^{\frac{1}{\sigma}}}{\lambda^{\frac{1}{\sigma}}} \frac{C^*_s}{C_s} C_j. \tag{B.62}$$

Plugging the condition above, and using the fact that consumption in the no-disaster state is constant, we obtain

$$\int_0^\infty e^{-\rho t} \left[C_j + \lambda \left(\frac{C_s}{C^*_s} \right)^{\sigma - 1} \frac{\lambda_j^{\frac{1}{\sigma}}}{\lambda^{\frac{1}{\sigma}}} \frac{C_j}{r^*_n} \right] dt = \omega. \tag{B.63}$$

Rearranging the expression above, we obtain

$$C_j = \underbrace{\frac{\rho}{1+\overline{\chi}\lambda_j^{\frac{1}{\sigma}}}}_{\text{MPC}_j}\omega, \qquad C_j^* = \underbrace{\frac{\rho\overline{\chi}^*\lambda_j^{\frac{1}{\sigma}}}{1+\overline{\chi}\lambda_j^{\frac{1}{\sigma}}}}_{\text{MPC}_j^*}\omega \tag{B.64}$$

where $\overline{\chi} \equiv \frac{\lambda^{\frac{\sigma-1}{\sigma}}}{r_n^*}\left(\frac{C_s}{C_s^*}\right)^{\sigma-1}$ and $\overline{\chi}^* \equiv \lambda^{-\frac{1}{\sigma}}\frac{C_s^*}{C_s}$. The expressions above show that savers have heterogeneous MPCs. Optimistic investors have higher MPCs in the no-disaster state, while they have lower MPCs (out of initial wealth) in the disaster state.

Perturbation. Consider a perturbation of the environment above, where wealth and the SDF are subject to small shocks. From the Euler equations, we obtain

$$\dot{c}_{j,t} = \sigma^{-1}(i_t - \pi_t - r_n) + \frac{\lambda}{\sigma}\left(\frac{C_s}{C_s^*}\right)^{\sigma}(\hat{\lambda}_t + \sigma c_{s,t}) - \xi(c_{j,t} - c_{s,t}), \tag{B.65}$$

and

$$\hat{\lambda}_t + \sigma(c_{s,t} - c_{s,t}^*) = \sigma(c_{j,t} - c_{j,t}^*). \tag{B.66}$$

We can write the equations above as follows:

$$c_{j,t} = c_{s,t} + e^{-\xi t}(c_{j,0} - c_{s,0}), \qquad c_{j,t}^* = c_{j,t} - c_{s,t} - \frac{1}{\sigma}\hat{\lambda}_t. \tag{B.67}$$

Linearizing the intertemporal budget constraint, we obtain

$$\int_0^\infty e^{-\rho t}\left[c_{j,t} + \chi_{c_j^*}c_{j,t}^*\right]dt = \Omega_{j,0}, \tag{B.68}$$

where $\chi_{c_j^*} = \frac{\lambda}{\rho_s}\left(\frac{C_s}{C_s^*}\right)^{\sigma}\frac{C_j^*}{C_j} = \overline{\chi}\lambda_j^{\frac{1}{\sigma}}$.

Combining the expressions for consumption with the intertemporal budget constraint,

we obtain
$$\frac{1+\chi_{c_j^*}}{\rho+\xi}(c_{j,0}-c_{s,0}) = \Omega_{j,0}-\Omega_{s,0}+\frac{\chi_{c_j^*}}{\sigma}\frac{\hat{\lambda}_0}{\rho+\xi}. \tag{B.69}$$

Rearranging the expression above, we obtain

$$c_{j,0} = c_{s,0}+\frac{\rho+\xi}{1+\overline{\chi}\lambda_j^{\frac{1}{\sigma}}}(\Omega_{j,0}-\Omega_{s,0})+\frac{\overline{\chi}\lambda_j^{\frac{1}{\sigma}}}{1+\overline{\chi}\lambda_j^{\frac{1}{\sigma}}}\frac{\hat{\lambda}_0}{\sigma}. \tag{B.70}$$

Consumption at date t in the no-disaster state is given by

$$c_{j,t} = c_{s,t}+\frac{(\rho+\xi)e^{-\xi t}}{1+\overline{\chi}\lambda_j^{\frac{1}{\sigma}}}(\Omega_{j,0}-\Omega_{s,0})+\frac{\overline{\chi}\lambda_j^{\frac{1}{\sigma}}}{1+\overline{\chi}\lambda_j^{\frac{1}{\sigma}}}\frac{\hat{\lambda}_t}{\sigma}. \tag{B.71}$$

Consumption at date t in the disaster state is given by

$$c_{j,t}^* = \frac{(\rho+\xi)e^{-\xi t}}{1+\overline{\chi}\lambda_j^{\frac{1}{\sigma}}}(\Omega_{j,0}-\Omega_{s,0})-\frac{1}{1+\overline{\chi}\lambda_j^{\frac{1}{\sigma}}}\frac{\hat{\lambda}_t}{\sigma}. \tag{B.72}$$

An increase in $\Omega_{j,0}$ raises consumption in both states, while an increase in $\hat{\lambda}_t$ raises consumption in the no-disaster state and reduces consumption in the disaster state.

MPCs and iMPCs. Define the *intertemporal MPCs*, or *iMPCs*, for saver j in the no-disaster and disaster states as follows

$$\mathcal{M}_{j,t} \equiv \frac{\partial c_{j,t}}{\partial \Omega_{j,0}} = \frac{(\rho+\xi)}{1+\overline{\chi}\lambda_j^{\frac{1}{\sigma}}}e^{-\xi t}, \qquad \mathcal{M}_{j,t}^* \equiv \frac{C_j^*}{C_j}\frac{\partial c_{j,t}^*}{\partial \Omega_{j,0}} = \frac{(\rho+\xi)\overline{\chi}^*\lambda_j^{\frac{1}{\sigma}}}{1+\overline{\chi}\lambda_j^{\frac{1}{\sigma}}}e^{-\xi t}. \tag{B.73}$$

Optimistic investors have higher iMPCs than pessimistic investors in the no-disaster state, while pessimistic investors have higher iMPCs than optimistic investors in the disaster state. However, the average iMPC is the same for both types of savers:

$$\int_0^\infty e^{-\rho t}\left[\mathcal{M}_{j,t}+\frac{\lambda}{\rho_s}\left(\frac{C_s}{C_s^*}\right)^\sigma\mathcal{M}_{j,t}^*\right]dt=1. \tag{B.74}$$

Market-implied disaster probability. We can write $\hat{\lambda}_t$ as follows:

$$\hat{\lambda}_t=\underbrace{\frac{\sigma\mu_{c,o}\mu_{c,p}\left(\lambda_p^{\frac{1}{\sigma}}-\lambda_o^{\frac{1}{\sigma}}\right)}{\mu_{c,o}\lambda_o^{\frac{1}{\sigma}}+\mu_{c,p}\lambda_p^{\frac{1}{\sigma}}}}_{\equiv\chi_{\lambda,c}}(c_{p,t}-c_{o,t}) \tag{B.75}$$

The consumption share is given by

$$\mu_{c,j}=\frac{\frac{\rho\mu_j\omega_j}{1+\overline{\chi}\lambda_j^{\frac{1}{\sigma}}}}{\frac{\rho\mu_o\omega_o}{1+\overline{\chi}\lambda_o^{\frac{1}{\sigma}}}+\frac{\rho\mu_p\omega_p}{1+\overline{\chi}\lambda_p^{\frac{1}{\sigma}}}}=\frac{\mu_j}{1+\overline{\chi}\lambda_j^{\frac{1}{\sigma}}}\frac{(1+\overline{\chi}\lambda_o^{\frac{1}{\sigma}})(1+\overline{\chi}\lambda_p^{\frac{1}{\sigma}})}{\mu_o(1+\overline{\chi}\lambda_p^{\frac{1}{\sigma}})+\mu_p(1+\overline{\chi}\lambda_o^{\frac{1}{\sigma}})}, \tag{B.76}$$

using the fact that $\omega_o=\omega_p$, as $B_o=B_p$ and $T_o=T_p$ in the stationary equilibrium.

The coefficient on multiplying $c_{p,t}-c_{o,t}$ on the expression for $\hat{\lambda}_t$ can then be written as

$$\chi_{\lambda,c}=\frac{\sigma\mu_{c,0}\mu_{c,p}}{\left[\frac{\mu_o\lambda_o^{\frac{1}{\sigma}}}{1+\overline{\chi}\lambda_o^{\frac{1}{\sigma}}}+\frac{\mu_p\lambda_p^{\frac{1}{\sigma}}}{1+\overline{\chi}\lambda_p^{\frac{1}{\sigma}}}\right]\frac{(1+\overline{\chi}\lambda_o^{\frac{1}{\sigma}})(1+\overline{\chi}\lambda_p^{\frac{1}{\sigma}})}{\mu_o(1+\overline{\chi}\lambda_p^{\frac{1}{\sigma}})+\mu_p(1+\overline{\chi}\lambda_o^{\frac{1}{\sigma}})}}\left(\lambda_p^{\frac{1}{\sigma}}-\lambda_o^{\frac{1}{\sigma}}\right) \tag{B.77}$$

$$=\frac{\overline{\chi}^*}{\overline{\chi}}\frac{\sigma\mu_{c,0}\mu_{c,p}\left(\frac{\mu_o}{\mu_o+\mu_p}(1+\overline{\chi}\lambda_p^{\frac{1}{\sigma}})+\frac{\mu_p}{\mu_o+\mu_p}(1+\overline{\chi}\lambda_o^{\frac{1}{\sigma}})\right)}{\left[\frac{\mu_o}{\mu_o+\mu_p}\frac{(\rho+\xi)\overline{\chi}^*\lambda_o^{\frac{1}{\sigma}}}{1+\overline{\chi}\lambda_o^{\frac{1}{\sigma}}}+\frac{\mu_p}{\mu_o+\mu_p}\frac{(\rho+\xi)\overline{\chi}^*\lambda_p^{\frac{1}{\sigma}}}{1+\overline{\chi}\lambda_p^{\frac{1}{\sigma}}}\right]}\frac{(\rho+\xi)\overline{\chi}\left(\lambda_p^{\frac{1}{\sigma}}-\lambda_o^{\frac{1}{\sigma}}\right)}{(1+\overline{\chi}\lambda_o^{\frac{1}{\sigma}})(1+\overline{\chi}\lambda_p^{\frac{1}{\sigma}})} \tag{B.78}$$

$$=\frac{\overline{\chi}^*}{\overline{\chi}}\frac{\sigma\mu_{c,0}\mu_{c,p}\left(\frac{\mu_o}{\mu_o+\mu_p}(1+\overline{\chi}\lambda_p^{\frac{1}{\sigma}})+\frac{\mu_p}{\mu_o+\mu_p}(1+\overline{\chi}\lambda_o^{\frac{1}{\sigma}})\right)}{\left[\frac{\mu_o}{\mu_o+\mu_p}\mathcal{M}_{o,0}^*+\frac{\mu_p}{\mu_o+\mu_p}\mathcal{M}_{p,0}^*\right]}\left(\mathcal{M}_{o,0}-\mathcal{M}_{p,0}\right). \tag{B.79}$$

Heterogeneous MPCs and precautionary motive. The difference in consumption at date t is given by

$$c_{p,t} - c_{o,t} = \mathcal{M}_{p,t}(\Omega_{p,0} - \Omega_{s,0}) - \mathcal{M}_{o,t}(\Omega_{o,0} - \Omega_{s,0}) + \left(\mathcal{M}^*_{p,0} - \mathcal{M}^*_{o,0}\right) \frac{\overline{\chi}^*}{\overline{\chi}} \frac{\hat{\lambda}_t}{\sigma} \frac{1}{\rho + \xi}. \quad \text{(B.80)}$$

Using $\Omega_{s,0} = \mu_{c,o}\Omega_{o,0} + \mu_{c,p}\Omega_{p,0}$, we can write the expression above as follows:

$$c_{p,t} - c_{o,t} = \left[\mathcal{M}_{p,t}\mu_{c,o} + \mathcal{M}_{o,t}\mu_{c,p}\right] (\Omega_{p,0} - \Omega_{o,0}) + \left(\mathcal{M}^*_{p,0} - \mathcal{M}^*_{o,0}\right) \frac{\overline{\chi}^*}{\overline{\chi}} \frac{\hat{\lambda}_t}{\sigma} \frac{1}{\rho + \xi}. \quad \text{(B.81)}$$

As $\hat{\lambda}_t = \chi_{\lambda,c}(c_{p,t} - c_{o,t})$, then

$$c_{p,t} - c_{o,t} = \frac{\mathcal{M}_{p,t}\mu_{c,o} + \mathcal{M}_{o,t}\mu_{c,p}}{1 - \left(\mathcal{M}^*_{p,0} - \mathcal{M}^*_{o,0}\right) \frac{\overline{\chi}^*}{\overline{\chi}} \frac{\chi_{\lambda,c}}{\sigma} \frac{1}{\rho+\xi}} \left[\Omega_{p,o} - \Omega_{o,0}\right]. \quad \text{(B.82)}$$

Therefore, $\hat{\lambda}_t$ is given by

$$\hat{\lambda}_t = \frac{\chi_{\lambda,c}\left(\mathcal{M}_{p,t}\mu_{c,o} + \mathcal{M}_{o,t}\mu_{c,p}\right)}{1 - \left(\mathcal{M}^*_{p,0} - \mathcal{M}^*_{o,0}\right) \frac{\overline{\chi}^*}{\overline{\chi}} \frac{\chi_{\lambda,c}}{\sigma} \frac{1}{\rho+\xi}} \left[\Omega_{p,o} - \Omega_{o,0}\right].$$

Given the expression for $\chi_{\lambda,c}$, we can write $\hat{\lambda}_t$ as follows:

$$\hat{\lambda}_t = \frac{\overline{\chi}^*}{\overline{\chi}} \frac{\sigma(\rho + \xi)\mu_{c,o}\mu_{c,p}}{\frac{\mu_o}{\mu_o+\mu_p}\mathcal{M}^*_{o,0} + \frac{\mu_p}{\mu_o+\mu_p}\mathcal{M}^*_{p,0}} \frac{\left(\mathcal{M}_{o,t} - \mathcal{M}_{p,t}\right)\left[\Omega_{p,o} - \Omega_{o,0}\right]}{1 - \left(\mathcal{M}^*_{p,0} - \mathcal{M}^*_{o,0}\right) \frac{\overline{\chi}^*}{\overline{\chi}} \frac{\chi_{\lambda,c}}{\sigma} \frac{1}{\rho+\xi}}. \quad \text{(B.83)}$$

B.5 Minimum State Variable Solution

General formulation. Consider a general dynamic system involving the vector of endogeneous variables $Z_t = [K_t', Y_t']'$, where Y_t is a vector of non-predetermined variables and K_t a vector of predetermined variables. The dynamics of Z_t is given by

$$\dot{Z}_t = AZ_t + BV_t, \tag{B.84}$$

given K_0, where V_t is a vector of disturbances following the dynamics $\dot{V}_t = \Psi_v V_t$.

The minimum state-variable (MSV) solution takes the form:

$$Y_t = \Phi_{YK} K_t + \Phi_{YV} V_t, \qquad \dot{K}_t = \Phi_{KK} K_t + \Phi_{KV} V_t. \tag{B.85}$$

We can obtain the MSV solution using the method of undetermined coefficients. Importantly, the method produces a unique solution even when the number of negative eigenvalues exceed the number of predetermined variables.

MSV solution of baseline model. Consider the dynamic system given by (10) and (11), given a process for i_t and $\hat{\lambda}_t$. In particular, we assume that i_t follows the continuous-time analog of an AR(K) process: $i_t - r_n = \Gamma_i' V_t$, where $\dot{V}_t = \Psi_V V_t$, for Ψ_V diagonal.[4] We know that $\hat{\lambda}_t = e^{-\psi_\lambda t}\hat{\lambda}_0$, where $\hat{\lambda}_0$ is a function of the path for $i_t - r_n$. We assume that one of the variables in V_t decay at rate ψ_λ, so we can write $\hat{\lambda}_t = \Gamma_\lambda' V_t$. After replacing $i_t - r_n$ and $\hat{\lambda}_t$ for the appropriate linear functions of V_t, we obtain a dynamic system in $Z_t = [y_t, \pi_t]'$. The MSV solution is given by

$$y_t = \Phi_y' V_t, \qquad \pi_t = \Phi_\pi' V_t. \tag{B.86}$$

Using the method of undetermined coefficients, we obtain

$$\Phi_y' \Psi_V = \tilde{\sigma}^{-1}(\Gamma_i' - \Phi_\pi') + \delta \Phi_y' + \chi_{p_d} \Gamma_\lambda', \qquad \Phi_\pi' \Psi_v = \rho \Phi_\pi' - \kappa \Phi_y'. \tag{B.87}$$

[4] In discrete time, we can write an AR(K) as $(1 - a_1 L - \ldots a_K L^K) y_t = v_t$, so $y_t = \frac{v_t}{(1-\lambda_1 L)\ldots(1-\lambda_K L)} = \sum_{k=1}^K \Gamma_{ik} V_{k,t}$, assuming λ_i are distinct, where $V_{k,t} \equiv \frac{v_t}{1-\lambda_i L}$. Hence, y_t is a sum of K AR(1) variables.

Rearranging the expression above, we obtain the linear system

$$\begin{bmatrix} -\psi_k - \delta & \tilde{\sigma}^{-1} \\ \kappa & -\psi_k - \rho \end{bmatrix} \begin{bmatrix} \Phi_{yk} \\ \Phi_{\pi k} \end{bmatrix} = \begin{bmatrix} \tilde{\sigma}^{-1}\Gamma_{ik} + \chi_{pd}\Gamma_{\lambda k} \\ 0 \end{bmatrix}, \tag{B.88}$$

where $-\psi_k$ is the k-th element of the diagonal of Ψ_V. Solving the system above, we obtain

$$\begin{bmatrix} \Phi_{yk} \\ \Phi_{\pi k} \end{bmatrix} = -\frac{1}{(\overline{\omega} + \psi_k)(\underline{\omega} + \psi_k)} \begin{bmatrix} \rho + \psi_k \\ \kappa \end{bmatrix} \left(\tilde{\sigma}^{-1}\Gamma_{ik} + \chi_{pd}\Gamma_{\lambda k}\right), \tag{B.89}$$

assuming $\psi_k \neq -\underline{\omega}$.

We show next how to implement the MSV solution using a Taylor rule. Suppose $u_t = \sum_{k=1}^{K} \varphi_k u_{k,t}$, where $u_{k,t} = V_{k,t}$. We adopt the normalization $V_{k,0} = i_0 - r_n$. The nominal rate under the Taylor rule is given by

$$i_t - r_n = \sum_{k=1}^{K} \varphi_k \frac{(\overline{\omega} + \psi_k)(\underline{\omega} + \psi_k)}{(\omega_1 + \psi_k)(\omega_2 + \psi_k)} u_{k,t} - \frac{\phi_\pi \kappa \chi_\lambda}{(\omega_1 + \psi_\lambda)(\omega_2 + \psi_\lambda)} e^{-\psi_\lambda t}(i_0 - r_n).$$

In the case $\psi_k \neq \psi_\lambda$, the coefficient $\varphi_k = \Gamma_{ik} \frac{(\omega_1+\psi_k)(\omega_2+\psi_k)}{(\overline{\omega}+\psi_k)(\underline{\omega}+\psi_k)}$. In the case $\psi_k = \psi_\lambda$, the coefficient is given by $\varphi_k = \frac{\Gamma_{ik}(\omega_1+\psi_\lambda)(\omega_2+\psi_\lambda)+\phi_\pi\kappa\chi_\lambda}{(\overline{\omega}+\psi_\lambda)(\underline{\omega}+\psi_\lambda)}$.

Output is then given by

$$\begin{aligned} y_t &= -\sum_{k=1}^{K} \Gamma_{ik} \frac{\rho + \psi_k}{(\overline{\omega} + \psi_k)(\underline{\omega} + \psi_k)} \tilde{\sigma}^{-1} u_{k,t} - \frac{(\rho + \psi_\lambda)\tilde{\chi}_\lambda}{(\overline{\omega} + \psi_\lambda)(\underline{\omega} + \psi_\lambda)} e^{-\psi_\lambda t}(i_0 - r_n) \\ \pi_t &= -\sum_{k=1}^{K} \varphi_k \frac{\kappa}{(\overline{\omega} + \psi_k)(\underline{\omega} + \psi_k)} \tilde{\sigma}^{-1} u_{k,t} - \frac{\kappa\tilde{\chi}_\lambda}{(\overline{\omega} + \psi_\lambda)(\underline{\omega} + \psi_\lambda)} e^{-\psi_\lambda t}(i_0 - r_n), \end{aligned}$$

where

$$\tilde{\chi}_\lambda = \chi_\lambda \left[\frac{\phi_\pi \kappa \tilde{\sigma}^{-1}}{(\omega_1 + \psi_\lambda)(\omega_2 + \psi_\lambda)} + \frac{(\overline{\omega} + \psi_\lambda)(\underline{\omega} + \psi_\lambda)}{(\omega_1 + \psi_\lambda)(\omega_2 + \psi_\lambda)}\right] = \chi_\lambda. \tag{B.90}$$

Finally, the coefficient ϵ_λ is given by

$$\epsilon_\lambda = \frac{\chi_{\lambda,c}\frac{\rho+\xi}{\chi_{b,c}}\left(\frac{B_o^L}{B_o} - \frac{B_p^L}{B_p}\right)}{1 - \chi_{\lambda,c}\frac{\rho+\xi}{\chi_{b,c}}\left(\frac{B_o^L}{B_o} - \frac{B_p^L}{B_p}\right)\frac{r_L}{\rho+\psi_L+\psi_\lambda}} \sum_{k=1}^{K} \Gamma_{ik} \frac{i_0 - r_n}{\rho + \psi_L + \psi_k} = \sum_{k=1}^{K} \Gamma_{ik}\epsilon_{\lambda,k}. \tag{B.91}$$

Hence, given an interest rate $i_t - r_n = \sum_{k=1}^{K} \Gamma_{ik} e^{-\psi_k t}(i_0 - r_n)$, we can write the solution for output and inflation as $y_t = \sum_{k=1}^{K} \Gamma_{ik} y_{k,t}$ and $\pi_t = \sum_{k=1}^{K} \Gamma_{ik} \pi_{k,t}$, where $y_{k,t}$ and $\pi_{k,t}$ correspond to the solution when the interest rate follows the process $e^{-\psi_k t}(i_0 - r_n)$.

The case where $u_t = \varphi_1 e^{-\psi_m t}(i_0 - r_n)$, $\psi_m \neq \psi_\lambda$, corresponds to the coefficients:

$$\Gamma_{i1} = 1 + \frac{\phi_\pi \kappa \chi_\lambda}{(\overline{\omega} + \psi_\lambda)(\underline{\omega} + \psi_\lambda) + \tilde{\sigma}^{-1}\phi_\pi \kappa}, \qquad \Gamma_{i2} = -\frac{\phi_\pi \kappa \chi_\lambda}{(\overline{\omega} + \psi_\lambda)(\underline{\omega} + \psi_\lambda) + \tilde{\sigma}^{-1}\phi_\pi \kappa}, \tag{B.92}$$

where $\psi_1 = \psi_m$ and $\psi_2 = \psi_\lambda$.

In the case $\psi_m = \psi_\lambda$, we have $\Gamma_{i1} = 1$, which requires

$$\varphi_1 = 1 + \frac{(\tilde{\sigma}^{-1} + \chi_\lambda)\phi_\pi \kappa}{(\overline{\omega} + \psi_m)(\underline{\omega} + \psi_m)}. \tag{B.93}$$

B.6 Determinacy and implementability

We derive next the conditions for local determinacy in our D-HANK model. We also show that any path of the nominal interest rate and the fiscal backing can be obtained with the monetary rule (6) and an appropriately chosen path of the monetary shock, $[u_t]_0^\infty$.

Proposition 9 (Determinacy and implementability)**.** *Consider a given monetary shock* $[u_t]_0^\infty$.

i. (Determinacy) If $\phi_\pi \geq \overline{\phi}_\pi \equiv 1 - \frac{\rho\delta}{\tilde{\sigma}^{-1}\kappa}$, *then there exists a unique bounded solution to the system comprised of the Taylor rule* (6)*, the aggregate Euler equation* (10)*, the NKPC* (11)*, the market-implied disaster probability* (15)*, and the law of motion of relative con-*

sumption (16) and relative net worth (17). We denote this solution by $[i_t^\star, y_t^\star, \pi_t^\star, \hat{\lambda}_t^\star, c_{p,t}^\star - c_{o,t}^\star, b_{p,t}^\star - b_{o,t}^\star]$ *and the associated path of taxes by* $\tau_t^\star$.

ii. *(Implementability) For a given path of nominal interest rates* $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$, $\psi_m \neq -\underline{\omega}$, *and fiscal backing* $\int_0^\infty e^{-\rho t}\tau_t dt$, *let* $\hat{\lambda}_t$ *be given by (17),* y_t *be given by (26), and* π_t *be given by (28), where* Ω_0 *is given by (29). If the monetary shock* u_t *is given by*

$$u_t = i_t - r_n - \phi_\pi \pi_t, \tag{B.94}$$

then $i_t^\star = i_t$ *and* $\int_0^\infty e^{-\rho t}\tau_t^\star dt = \int_0^\infty e^{-\rho t}\tau_t dt$. *Moreover,* $y_t^\star = y_t$, $\pi_t^\star = \pi_t$, *and* $\hat{\lambda}_t^\star = \hat{\lambda}_t$.

The first part of Proposition 9 shows that there is a unique bounded equilibrium if $\phi_\pi \geq \overline{\phi}_\pi$. As in Acharya and Dogra (2020), the threshold for determinacy satisfies $\overline{\phi}_\pi < 1$, so uniqueness is obtained under a weaker condition than in the textbook model. The second part of Proposition 9 shows how to implement any given path of policy variables by appropriately chosing the monetary shock u_t. Combined with Propositions 6-7, this provides a complete characterization of how output and inflation respond to monetary policy.

Proof of Proposition 9. We divide this proof in three steps. First, we derive the condition for local uniqueness of the solution under the policy rule (6). Second, we derive the path of $[y_t, \pi_t, \hat{\lambda}_t, b_{p,t} - b_{o,t}, i_t]_0^\infty$ for a given path of monetary shocks. Third, we show how to implement a given path of nominal interest rates $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$ and a given value of fiscal backing $\int_0^\infty e^{-\rho t}\tau_t dt$.

Equilibrium determinacy. First, using the Taylor rule, we can write ν_t in Equation **??** as $\nu_t = \tilde{\sigma}^{-1}(\phi_\pi - 1)\pi_t + \chi_\lambda \hat{\lambda}_t + \tilde{\sigma}^{-1}u_t$. Combining the Phillips curve (11) with the system

([A.54](#)), we obtain a dynamic system in the variables $[y_t, \pi, \hat{\lambda}_t, b_{p,t} - b_{o,t}]$:

$$\begin{bmatrix} \dot{y}_t \\ \dot{\pi}_t \\ \dot{\hat{\lambda}}_t \\ \dot{b}_{p,t} - \dot{b}_{o,t} \end{bmatrix} = \begin{bmatrix} \delta & \tilde{\sigma}^{-1}(\phi_\pi - 1) & \chi_\lambda & 0 \\ -\kappa & \rho & 0 & 0 \\ 0 & 0 & 0 & -\tilde{\tilde{\xi}}\chi_{\lambda,\Delta c} \\ 0 & 0 & -\chi_{\Delta b,\lambda} & \chi_{\Delta b,\Delta b} \end{bmatrix} \begin{bmatrix} y_t \\ \pi_t \\ \hat{\lambda}_t \\ b_{p,t} - b_{o,t} \end{bmatrix} + \begin{bmatrix} \tilde{\sigma}^{-1} \\ 0 \\ 0 \\ 0 \end{bmatrix} u_t,$$

where $\hat{\lambda}_t = \chi_{\lambda,\Delta c}(c_{o,t} - c_{p,t})$ and $\chi_{\Delta b,\lambda} = \chi_{\Delta b,\Delta c}/\chi_{\lambda,\Delta c}$, given the boundary condition

$$b_{p,0} - b_{o,0} = -\left(\frac{B_p^L}{B_p} - \frac{B_o^L}{B_o}\right) \int_0^\infty e^{-(\rho+\psi_L)t}(\phi_\pi \pi_t + u_t + r_L \hat{\lambda}_t)dt.$$

There is a unique bounded solution of the system above if the matrix of coefficients above has three eigenvalues with positive real parts and one eigenvalue with a negative real part. Denote the matrix of coefficients by A and consider the eigendecomposition of the matrix $A = V\Omega V^{-1}$, where Ω is the diagonal matrix of eigenvalues and V the matrix of eigenvectors. The eigenvalues are given by

$$\omega_1 = \frac{\rho + \delta + \sqrt{(\rho+\delta)^2 - 4\left(\tilde{\sigma}^{-1}(\phi_\pi - 1)\kappa + \rho\delta\right)}}{2}, \qquad \omega_3 = \frac{\chi_{\Delta b,\Delta b} + \sqrt{\chi_{\Delta b,\Delta b}^2 + \tilde{\tilde{\xi}}\chi_{\lambda,\Delta c}\chi_{\Delta b,\lambda}}}{2}$$

$$\omega_2 = \frac{\rho + \delta - \sqrt{(\rho+\delta)^2 - 4\left(\tilde{\sigma}^{-1}(\phi_\pi - 1)\kappa + \rho\delta\right)}}{2}, \qquad \omega_4 = \frac{\chi_{\Delta b,\Delta b} - \sqrt{\chi_{\Delta b,\Delta b}^2 + \tilde{\tilde{\xi}}\chi_{\lambda,\Delta c}\chi_{\Delta b,\lambda}}}{2}.$$

Notice that $\omega_3 > 0$ and $\omega_4 < 0$. Therefore, equilibrium determinacy requires $\omega_1 > 0$ and $\omega_2 > 0$. A necessary condition is $\phi_\pi > 1 - \frac{\rho\delta}{\tilde{\sigma}^{-1}\kappa} \equiv \overline{\phi}_\pi$, as otherwise the first two eigenvalues are real-valued and $\omega_2 \leq 0$. For ϕ_π sufficiently large, the eigenvalues are complex, but their real part is still positive. So, the condition $\phi_\pi > \overline{\pi}_\pi$ is sufficient to ensure determinacy.

Solution of the dynamic system. In matrix form, the dynamic system is given by $\dot{Z}_t = AZ_t + Bu_t$, where $Z_t = [y_t, \pi_t, \hat{\lambda}_t, b_{p,t} - b_{o,t}]'$ and $B = [\tilde{\sigma}^{-1}, 0, 0, 0]'$. Let $z_t = V^{-1}Z_t$ and $b = V^{-1}B$, which gives the system $\dot{z}_t = \Omega z_t + bu_t$. For $i = 1, 2, 3$, we can solve the equation forward, $z_{i,t} = -b_i \int_t^\infty e^{-\omega_i(s-t)} u_s ds$, and for $i = 4$ we solve it backwards: $z_{4,t} = e^{\omega_4 t} z_{4,0} + b_4 \int_0^t e^{\omega_4(t-s)} u_s ds$. Rotating to the original coordinates, we obtain:

$$Z_t = v_4 e^{\omega_4 t} z_{4,0} - \sum_{i=1}^{3} v_i b_i \int_t^\infty e^{-\omega_i(s-t)} u_s ds + v_4 b_4 \int_0^t e^{\omega_4(t-s)} u_s ds,$$

where v_i denotes the ith eigenvector, which are given by

$$v_1 = \left[\kappa^{-1}(\rho - \omega_1), 1, 0, 0\right]', \qquad v_3 = \left[v_{3,1}, \frac{\kappa v_{3,1}}{\rho - \omega_3}, \frac{\chi_{\Delta b,\Delta b} - \omega_3}{\chi_{\Delta b,\lambda}}, 1\right]'$$
$$v_2 = \left[\kappa^{-1}(\rho - \omega_2), 1, 0, 0\right]', \qquad v_4 = \left[v_{4,1}, \frac{\kappa v_{4,1}}{\rho - \omega_4}, \frac{\chi_{\Delta b,\Delta b} - \omega_4}{\chi_{\Delta b,\lambda}}, 1\right]',$$

$v_{i,1} = -\frac{(\rho-\omega_i)\chi_\lambda}{(\delta-\omega_i)(\rho-\omega_i)+\kappa\tilde{\sigma}^{-1}(\phi_\pi-1)} \frac{\chi_{\Delta b,\Delta b}-\omega_i}{\chi_{\Delta b,\lambda}}$ for $i = 3, 4$, and $b = [-\frac{\kappa\tilde{\sigma}^{-1}}{\omega_1-\omega_2}, \frac{\kappa\tilde{\sigma}^{-1}}{\omega_1-\omega_2}, 0, 0]'$. Using the fact that $\psi_\lambda = -\omega_4$, we obtain $b_{p,t} - b_{o,t} = e^{-\psi_\lambda t}(b_{p,0} - b_{o,t})$ and $\hat{\lambda}_t = \frac{\chi_{\Delta b,\Delta b}+\psi_\lambda}{\chi_{\Delta b,\lambda}} e^{-\psi_\lambda t}(b_{p,0} - b_{o,0})$, which coincides with the results from Proposition 3. y_t and π_t are given by

$$y_t = \sum_{i=1}^{2} (-1)^i \frac{\tilde{\sigma}^{-1}(\omega_i - \rho)}{\omega_1 - \omega_2} \int_t^\infty e^{-\omega_i(s-t)} u_s ds - \frac{\chi_\lambda(\rho + \psi_\lambda)}{(\omega_1 + \psi_\lambda)(\omega_2 + \psi_\lambda)} \hat{\lambda}_t$$
$$\pi_t = \sum_{i=1}^{2} (-1)^{i-1} \frac{\tilde{\sigma}^{-1}}{\omega_1 - \omega_2} \int_t^\infty e^{-\omega_i(s-t)} u_s ds - \frac{\kappa\chi_\lambda}{(\omega_1 + \psi_\lambda)(\omega_2 + \psi_\lambda)} \hat{\lambda}_t.$$

If $u_t = \sum_{k=1}^K \varphi_k u_{k,t}$, where $u_{k,t} = e^{-\psi_k t} u_{k,0}$, then

$$y_t = -\sum_{k=1}^{K} \varphi_k \frac{\rho + \psi_k}{(\omega_1 + \psi_k)(\omega_2 + \psi_k)} \tilde{\sigma}^{-1} u_{k,t} - \frac{\chi_\lambda(\rho + \psi_\lambda)}{(\omega_1 + \psi_\lambda)(\omega_2 + \psi_\lambda)} \hat{\lambda}_t$$
$$\pi_t = -\sum_{k=1}^{K} \varphi_k \frac{\kappa}{(\omega_1 + \psi_k)(\omega_2 + \psi_k)} \tilde{\sigma}^{-1} u_{k,t} - \frac{\kappa\chi_\lambda}{(\omega_1 + \psi_\lambda)(\omega_2 + \psi_\lambda)} \hat{\lambda}_t.$$

The nominal interest rate is given by

$$i_t - r_n = \sum_{k=1}^{K} \varphi_k \frac{(\delta+\psi_k)(\rho+\psi_k) - \tilde{\sigma}^{-1}\kappa}{(\omega_1+\psi_k)(\omega_2+\psi_k)} u_{k,t} - \frac{\phi_\pi \kappa \chi_\lambda}{(\omega_1+\psi_\lambda)(\omega_2+\psi_\lambda)} \hat{\lambda}_t.$$

The initial value of $\hat{\lambda}_0$ satisfies the condition:

$$\hat{\lambda}_0 = -\frac{\chi_{\Delta b,\Delta b} + \psi_\lambda}{\chi_{\Delta b,\lambda}} \left(\frac{B_p^L}{B_p} - \frac{B_o^L}{B_o} \right) \left[\int_0^\infty e^{-(\rho+\psi_L)t} (i_t - r_n) dt + \frac{r_L}{\rho+\psi_L+\psi_\lambda} \hat{\lambda}_0 \right],$$

solving for $\hat{\lambda}_0$, we obtain

$$\hat{\lambda}_0 = \frac{\frac{\chi_{\Delta b,\Delta b}+\psi_\lambda}{\chi_{\Delta b,\lambda}} \left(\frac{B_o^L}{B_o} - \frac{B_p^L}{B_p} \right)}{1 - \frac{\chi_{\Delta b,\Delta b}+\psi_\lambda}{\chi_{\Delta b,\lambda}} \left(\frac{B_o^L}{B_o} - \frac{B_p^L}{B_p} \right) \frac{r_L}{\rho+\psi_L+\psi_\lambda}} \int_0^\infty e^{-(\rho+\psi_L)t} (i_t - r_n) dt.$$

Combining the expression above with the expression for i_t, we obtain

$$\hat{\lambda}_0 = \frac{\frac{\chi_{\Delta b,\Delta b}+\psi_\lambda}{\chi_{\Delta b,\lambda}} \left(\frac{B_o^L}{B_o} - \frac{B_p^L}{B_p} \right) \sum_{k=1}^K \varphi_k \frac{(\delta+\psi_k)(\rho+\psi_k)-\tilde{\sigma}^{-1}\kappa}{(\omega_1+\psi_k)(\omega_2+\psi_k)} \frac{u_{k,0}}{\rho+\psi_L+\psi_k}}{1 + \frac{\chi_{\Delta b,\Delta b}+\psi_\lambda}{\chi_{\Delta b,\lambda}} \left(\frac{B_o^L}{B_o} - \frac{B_p^L}{B_p} \right) \left[\frac{\phi_\pi \kappa \chi_\lambda}{(\omega_1+\psi_\lambda)(\omega_2+\lambda)} \frac{1}{\rho+\psi_L+\psi_\lambda} - \frac{r_L}{\rho+\psi_L+\psi_\lambda} \right]}.$$

Implementability condition. Take $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$ and $\int_0^\infty e^{-\rho t} \tau_t dt$ as given, let Ω_0 be given by (29), y_t be given by (26) and π_t be given by (28). Define u_t as follows:

$$u_t = i_t - r_n - \phi_\pi \pi_t. \tag{B.95}$$

Let $[y_t^\star, \pi_t^\star, \hat{\lambda}_t^\star, b_{p,t}^\star - b_{o,t}^\star]_0^\infty$ be the solution to the four-dimensional dynamic system discussed above and $[i_t^\star, \tau_t^\star]$ the associated interest rate and fiscal backing. We show next that $y_t^\star = y_t$, $\pi_t^\star = \pi_t$, $\hat{\lambda}_t^\star = \hat{\lambda}_t$ and $i_t^\star = i_t$. First, notice that $u_t = \sum_{i=1}^3 \varphi_k e^{-\psi_k t} u_{k,0}$, where

$u_{k,0} = i_0 - r_n$, and φ_k and ψ_k are given by

$$\varphi_1 = 1 + \frac{\phi_\pi \tilde{\sigma}^{-1}\kappa}{(\underline{\omega} + \psi_m)(\overline{\omega} + \psi_m)}, \qquad \psi_1 = \psi_m, \qquad \varphi_2 = \frac{\phi_\pi \frac{1-\mu_w}{1-\mu_w \chi_y}\chi_p \kappa}{(\underline{\omega} + \psi_\lambda)(\overline{\omega} + \psi_\lambda)}, \qquad \psi_2 = \psi_\lambda,$$
$$\varphi_3 = -\kappa\phi_\pi \left[\frac{\tilde{\sigma}^{-1}\kappa}{(\underline{\omega} + \psi_m)(\overline{\omega} + \psi_m)} + \frac{\frac{1-\mu_w}{1-\mu_w \chi_y}\chi_p \kappa}{(\underline{\omega} + \psi_\lambda)(\overline{\omega} + \psi_\lambda)} + \frac{\Omega_0}{i_0 - r_n}\right], \qquad \psi_3 = -\underline{\omega}.$$

As $u_{k,0}$ is proportional to $i_0 - r_n$, for $k = 1, 2, 3$, and $\hat{\lambda}^\star_0$ is proportional to a linear combination of the $u_{k,0}$, then $\hat{\lambda}^\star_t = \epsilon^\star_\lambda e^{-\psi_\lambda t}(i_0 - r_n) = \epsilon^\star_\lambda u_{2,t}$, for some constant $\epsilon^\star_\lambda$. If $i_t - r_n = e^{-\psi_m t}(i_0 - r_n) =$, then $\epsilon^\star_\lambda = \epsilon_\lambda$. We guess that $\epsilon^\star_\lambda = \epsilon_\lambda$ and verify that nominal interest rates are exponentially decaying with rate ψ_m. The nominal interest rate is given by

$$i^\star_t - r_n = \sum_{k=1}^{3} \frac{(\delta + \psi_k)(\rho + \psi_k) - \tilde{\sigma}^{-1}\kappa}{(\omega_1 + \psi_k)(\omega_2 + \psi_k)}\varphi_k u_{k,t} - \frac{\phi_\pi \kappa \chi_\lambda}{(\omega_1 + \psi_\lambda)(\omega_2 + \psi_\lambda)}\epsilon_\lambda u_{2,t}.$$

Notice that $(\delta + \psi_k)(\rho + \psi_k) - \tilde{\sigma}^{-1}\kappa = (\underline{\omega} + \psi_k)(\overline{\omega} + \psi_k)$, so the term multiplying $u_{3,t}$ is equal to zero, as $\psi_3 = -\underline{\omega}$. Using the fact that $\chi_\lambda \epsilon_\lambda = \frac{1-\mu_w}{1-\mu_w\chi_y}\chi_p$, the term multiplying $u_{2,t}$ is also equal to zero. Finally, the term multiplying $u_{1,t}$ is equal to one, so $i^\star_t - r_n = i_t - r_n$. From the Taylor rule we have that $u_t = i_t - r_n - \phi_\pi \pi_t = i^\star_t - r_n - \phi_\pi \pi^\star_t$, so $\pi^\star_t = \pi_t$. If the nominal interest rate and $\hat{\lambda}_t$ coincide in the two equilibria, then we must have $b^\star_{p,t} - b^\star_{o,t} = b_{p,t} - b_{o,t}$. From the aggregate Euler equation, we obtain

$$y^\star_t = -\int_0^\infty e^{-\delta(s-t)}(i^\star_s - r_n - \pi^\star_s + \chi_\lambda \hat{\lambda}^\star_s)ds = -\int_0^\infty e^{-\delta(s-t)}(i_s - r_n - \pi_s + \chi_\lambda \hat{\lambda}_s)ds = y_t,$$

so $\overline{y}_t = y_t$. Finally, if output, inflation, nominal interest rates, and the market-implied disaster probability coincide in the two equilibria, from the intertemporal budget constraint we must have $\int_0^\infty e^{-\rho t}\tau^\star_t dt = \int_0^\infty e^{-\rho t}\tau_t dt$. $\square$

C Derivations for Section 4

C.1 Bond pricing and forward curve

In this section, we solve for prices, yields, and forward rates of zero-coupon bonds of different maturity. While in the main text we focused on the price of a single bond with exponentially decaying coupons, we solve here for the entire yield and forward curves.

Let $Q_{B,t}(h)$ denote the period t price of a nominal zero-coupon bond maturing at period $t+h$, $y_{B,t}(h)$ denotes the corresponding yield on the bond, and $f_{B,t}(h)$ denotes the instantaneous forward rate. The bond price satisfy the standard pricing condition

$$Q_{B,t}(h) = \mathbb{E}_t\left[\frac{\eta_{t+h}}{\eta_t}\frac{P_t}{P_{t+h}}\right], \tag{C.1}$$

using the fact that η_t/P_t is the nominal SDF in this economy.

Stationary equilibrium. The price of the bond in the no-disaster state of the stationary equilibrium is given by

$$Q_B(h) = \int_h^\infty \lambda e^{-\lambda t^*} e^{-\rho_s h} dt^* + \int_0^h \lambda e^{-\lambda t^*} e^{-\rho_s t^*}\left(\frac{C_s}{C_s^*}\right)^\sigma e^{-r^*(h-t^*)} dt^* \tag{C.2}$$

$$= e^{-\rho h} + (1-e^{-\lambda h}) e^{-\rho_s h}\left(\frac{C_s}{C_s^*}\right)^\sigma. \tag{C.3}$$

while the price of the bond in the disaster state is simply $Q_B^*(h) = e^{-\rho_s h}$. Notice that $\int_0^\infty e^{-\psi_L h} P(h) dh = \frac{1+Q_L^*\lambda\left(\frac{C_s}{C_s^*}\right)^\sigma}{\rho+\psi_L} = Q_L$, so this is consistent with our derivation for Q_L.

The yield on the bond is given by

$$y_B(h) = \rho_s + \lambda - \frac{1}{h}\log\left[1 + \left(e^{\lambda h} - 1\right)\left(\frac{C_s}{C_s^*}\right)^\sigma\right]. \tag{C.4}$$

Notice that $\lim_{h\to 0} y_B(h) = r_n^*$ and $\lim_{h\to\infty} y_B(h) = \rho > r_n^*$, capturing the fact that the yield curve is upward-sloping.

The forward rate is given by

$$f_B(h) = -\frac{\partial \log Q_B(h)}{\partial h} = \rho_s - \frac{\lambda\left[\left(\frac{C_s}{C_s^*}\right)^{\sigma} - 1\right]}{\left(e^{\lambda h} - 1\right)\left(\frac{C_s}{C_s^*}\right)^{\sigma} + 1}. \tag{C.5}$$

The linearized PDE. Let $r_{B,t}(h)$ denote the excess holding-period return on a bond maturing h periods ahead conditional on no disaster:

$$r_{B,t}(h) \equiv \frac{1}{Q_{B,t}(h)}\left[-\frac{\partial Q_{B,t}(h)}{\partial h} + \frac{\partial Q_{B,t}(h)}{\partial t}\right] - i_t. \tag{C.6}$$

The Euler equation for the bond is given by

$$r_{B,t}(h) = \lambda_t \left(\frac{C_{s,t}}{C_{s,t}^*}\right)^{\sigma} \frac{Q_{B,t}(h) - Q_{B,t}^*(h)}{Q_{B,t}(h)}. \tag{C.7}$$

Let $q_{b,t}(h) \equiv \log Q_{B,t}(h) - \log Q_B(h)$, then linearizing the equation above we obtain

$$-\frac{\partial q_{B,t}(h)}{\partial h} + \frac{\partial q_{B,t}(h)}{\partial t} = i_t - r_n + r_B(h)\left[\hat{\lambda}_t + \frac{Q_B^*(h)}{Q_B(h) - Q_B(h)^*} q_{b,t}(h)\right], \tag{C.8}$$

where we used the assumption that $r_B(h)\sigma c_{s,t}$ is second-order.

From PDE to system of ODEs. Assuming that the nominal interest rate is exponentially decaying, $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$, we will guess-and-verify that the solution takes the form:

$$q_{B,t}(h) = \chi_{B,i}(h)(i_t - r_n) + \chi_{B,\lambda}(h)\hat{\lambda}_t, \tag{C.9}$$

where $\chi_{B,i}(0) = \chi_{B,\lambda}(0) = 0$. Plugging the expression above into the PDE, we obtain

$$-\chi'_{B,i}(h)(i_t - r_n) - \chi'_{B,\lambda}(h)\hat{\lambda}_t - \psi_m \chi_{B,i}(h)(i_t - r_n) - \psi_\lambda \chi_{B,\lambda}(h)\hat{\lambda}_t = \tag{C.10}$$

$$i_t - r_n + r_B(h)\hat{\lambda}_t + \lambda \left(\frac{C_s}{C_s^*}\right)^\sigma \frac{Q_B^*(h)}{Q_B(h)} \left[\chi_{B,i}(h)(i_t - r_n) + \chi_{B,\lambda}(h)\hat{\lambda}_t\right]. \tag{C.11}$$

The equation above has to hold for any values of $i_0 - r_n$ and $\hat{\lambda}_0$, then we obtain a system decoupled ODEs

$$-\chi'_{B,i}(h) - \psi_m \chi_{B,i}(h) = 1 + \lambda \left(\frac{C_s}{C_s^*}\right)^\sigma \frac{Q_B^*(h)}{Q_B(h)} \chi_{B,i}(h) \tag{C.12}$$

$$-\chi'_{B,\lambda}(h) - \psi_\lambda \chi_{B,\lambda}(h) = r_B(h) + \lambda \left(\frac{C_s}{C_s^*}\right)^\sigma \frac{Q_B^*(h)}{Q_B(h)} \chi_{B,i}(h), \tag{C.13}$$

given the initial conditions $\chi_{B,i}(0) = \chi_{B,\lambda}(0) = 0$, where

$$r_B(h) = \lambda \left(\frac{C_s}{C_s^*}\right)^\sigma \frac{\left(1 - e^{-\lambda h}\right)\left[\left(\frac{C_s}{C_s^*}\right)^\sigma - 1\right]}{e^{-\lambda h} + \left(1 - e^{-\lambda h}\right)\left(\frac{C_s}{C_s^*}\right)^\sigma}, \qquad \frac{Q_B^*(h)}{Q_B(h)} = \frac{1}{e^{-\lambda h} + \left(1 - e^{-\lambda h}\right)\left(\frac{C_s}{C_s^*}\right)^\sigma}. \tag{C.14}$$

We can write the ODEs above as follows:

$$\chi'_{B,i}(h) = -1 - \left[\psi_m + \lambda \left(\frac{C_s}{C_s^*}\right)^\sigma \frac{Q_B^*(h)}{Q_B(h)}\right] \chi_{B,i}(h) \tag{C.15}$$

$$\chi'_{B,\lambda}(h) = -r_B(h) - \left[\psi_\lambda + \lambda \left(\frac{C_s}{C_s^*}\right)^\sigma \frac{Q_B^*(h)}{Q_B(h)}\right] \chi_{B,i}(h). \tag{C.16}$$

The system above can easily solve numerically using a finite-differences scheme. Given the bond prices, we can find the yield $y_{B,t}(h) = -\frac{1}{h}\log Q_{B,t}(h) = -\frac{1}{h}\log Q_B(h) - \frac{1}{h}q_{B,t}(h)$. Let $\hat{y}_{B,t}(h)$ denote the deviation of the yield on the bond from its value in the stationary equilibrium. The forward rate is given by $f_{B,t}(h) = -\frac{\partial \log Q_{B,t}(h)}{\partial h} = -\frac{\log Q_B(h)}{\partial h} - \frac{\partial q_{B,t}(h)}{\partial h}$, so $\hat{f}_{B,t}(h) \equiv -\frac{\partial q_{B,t}(h)}{\partial h}$ denotes the deviation of the forward rate from its value in the stationary equilibrium.

C.2 Cyclicality of transfers

In this section, we discuss the empirical plausibility of the countercyclicality of transfers assumed in our baseline model. It is useful to map the response of transfers in our model to the retention function of Heathcote et al. (2017), which has been shown to be able to capture the degree of progressivity of the tax system observed in the United States.

Retention function. Let $Y_{w,t} = \frac{W_t}{P_t} N_{w,t}$ denote the pre-tax income of workers. Workers consume their after-tax income, which is given by

$$C_{w,t} = \zeta Y_{w,t}^{1-\tau}, \tag{C.17}$$

where ζ captures the intercept of the retention function, and τ the curvature.

Linearizing the expression above around the stationary equilibrium, we obtain

$$c_{w,t} = (1-\tau)(w_t - p_t + n_{w,t}) = (1-\tau)(1+\phi)y_t, \tag{C.18}$$

using the fact that $w_t - p_t = \phi y_t$ and $n_{w,t} = y_t$.

The expression above relies on the assumption of sticky prices and flexible wages. In the opposite case of sticky wages and flexible prices, consumption of workers would be given by (see Appendix D.2)

$$c_{w,t} = (1-\tau)y_t, \tag{C.19}$$

using the fact that $w_t - p_t = 0$ with sticky wages.

Numerical examples. Auclert, Rognlie and Straub (2018) adopt the value $\tau = 0.181$, in line with the estimates of Heathcote, Storesletten and Violante (2017). We have $\phi = 1$ in our baseline calibration, which gives $\chi_y = 1.64$ for $\tau = 0.181$. In contrast, in the version of the model with sticky wages, we would have $c_{w,t} = (1-\tau)y_t$. Given $\tau = 0.181$, this

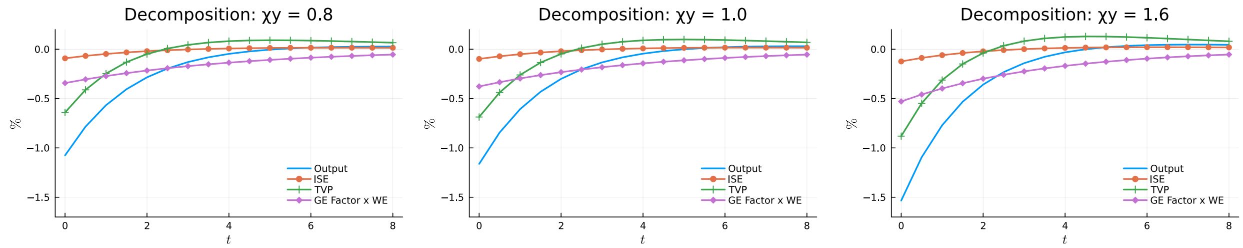


Figure C.1: Output decomposition for different values of χ_y.

implies $\chi_y = 0.82$. This gives us a range of alternative values of χ_y to consider. Figure C.1 shows the impact of assuming different values of χ_y. The case $\chi_y = 0.8$ gives very similar results to our baseline calibration of $\chi_y = 1.0$. Assuming $\chi_y = 1.6$ amplifies the response of output. The bulk of the amplification comes from the time-varying precautionary motive (52%) and the aggregate wealth effect (41%).

In the baseline model, the consumption of workers is given by

$$c_{w,t} = \left[\underbrace{\frac{WN_w}{PC_w}(1+\phi)}_{5.55} + \underbrace{\frac{T'_w(Y)Y}{C_w}}_{-4.55} \right] y_t = \chi_y y_t. \tag{C.20}$$

The numbers shown above correspond to the values under our baseline calibration. They imply that a drop in GDP of 1% leads to a drop in 5.55% in pre-tax income. Transfers increase by 4.55%, so overall consumption of workers drop by 1%. This strong countercyclicality of transfers is equivalent to a retention function with $\tau = 0.5$, a coefficient that is significantly larger than in standard calibrations. Assuming a weaker response of transfers, consistent with a value of τ closer to the empirically relevant case, would imply a larger drop in consumption, causing the amplification shown in Figure C.1. However, this amplification relies on the (counterfactual) strong pro-cyclicality of wages. In the more realistic case of sticky wages, we would obtain a value of χ_y less than one, which implies a small dampening of the overall effect. Therefore, a version of the model with sticky wages generates dynamics in line with our baseline model assuming an empirically plausible degree of countercyclicality of transfers.

C.3 Belief heterogeneity

Recall that Proposition 3 shows that the market-implied disaster probability is given by

$$\hat{\lambda}_t = e^{-\psi_\lambda t}\hat{\lambda}_0,$$

where

$$\hat{\lambda}_0 = \epsilon_\lambda (i_0 - r_n).$$

The persistence of the effect of monetary policy on the price of risk, ψ_λ, is governed by the Uzawa preference parameter ξ. This parameter is related not to the degree of belief heterogeneity but to the savers' iMPCs. In particular, our calibration implies a half-life for the iMPCs of four months, which is consistent with empirical values.

In contrast, ϵ_λ is indeed connected to the degree of belief heterogeneity. However, while belief heterogeneity is a necessary and sufficient condition for $\epsilon_\lambda > 0$, the actual relationship between the two depends on the interaction of other parameters. In Appendix A.3, we showed that

$$\hat{\lambda}_0 = \sigma \mu_{c,o}\mu_{c,p}\frac{\lambda_p^{\frac{1}{\sigma}} - \lambda_o^{\frac{1}{\sigma}}}{\lambda^{\frac{1}{\sigma}}}\frac{\rho+\xi}{\chi_{b,c}}(b_{p,0} - b_{o,0}),$$

where $\mu_{c,j}$ is the consumption share of savers of type $j \in \{o, p\}$, $b_{j,0}$ is the change in the value of their portfolio in period 0, and $\chi_{b,c}$ is a constant given by

$$\chi_{b,c} = \sigma \mu_{c,o}\mu_{c,p}\frac{\lambda_p^{\frac{1}{\sigma}} - \lambda_o^{\frac{1}{\sigma}}}{\lambda^{\frac{1}{\sigma}}}\sum_{k\in\{L,E\}} r_k \left(\frac{B_o^k}{B_o} - \frac{B_p^k}{B_p}\right) + \mu_{c,p}\frac{C_o}{B_o} + \mu_{c,o}\frac{C_p}{B_p} + \frac{C_s^*(\rho - r_n)}{r_n^* B_s}.$$

Moreover, we showed that the change in the initial value of the savers' portfolio is given by

$$b_{p,0} - b_{o,0} = \sum_{k \in \{L,E\}} \left(\frac{B_p^k}{B_p} - \frac{B_o^k}{B_o} \right) q_{k,0}.$$

Thus, besides the heterogeneity in beliefs between optimistic and pessimistic savers, there are two other important parameters that determine $\hat{\lambda}_0$: the mass of optimistic savers, μ_o, and the portfolio allocations in the stationary equilibrium. Because the stationary equilibrium features only one risk factor (the disaster shock), the composition of the savers' "risky" holdings, that is, B_j^L and B_j^E, are indeterminate. Naturally, optimistic agents are more exposed to disaster risk in equilibrium, but this only determines their *total* holdings of the aggregate risk factor, not the division between long-term bonds and stocks.

To simplify the calculations, we assume in the main text that optimistic and pessimistic agents hold the same fraction of equity in their portfolio, with optimistic savers holding proportionally more of the long-term bonds. This assumption implies that

$$b_{p,0} - b_{o,0} = \left(\frac{B_p^L}{B_p} - \frac{B_o^L}{B_o} \right) q_{L,0}.$$

Assuming that $r_L \sigma c_{s,t} = \mathcal{O}(||i_t - r_n||^2)$, we get

$$q_{L,0} = -\frac{i_0 - r_n}{\rho + \psi_L + \psi_m} - \frac{r_L \hat{\lambda}_0}{\rho + \psi_L + \psi_\lambda},$$

and

$$\frac{B_p^L}{B_p} - \frac{B_o^L}{B_o} = \frac{Q_L}{Q_L - Q_L^*} \frac{C_s}{B_s} \mathcal{R}_0,$$

where $\mathcal{R}_0 = \frac{1}{\lambda\left(\frac{C_s}{C_s^*}\right)} \frac{\lambda_p^{\frac{1}{\sigma}} - \lambda_o^{\frac{1}{\sigma}}}{\frac{\mu_o}{\mu_o + \mu_p} \lambda_p^{\frac{1}{\sigma}} + \frac{\mu_p}{\mu_o + \mu_p} \lambda_o^{\frac{1}{\sigma}}}$. Plugging these into the expression for $\hat{\lambda}_0$ above gives the expression for ϵ_λ found in Proposition 3. If, instead, we assumed that $\frac{B_p^E}{B_p} \neq \frac{B_o^E}{B_o}$, the calculations involved in solving for $\hat{\lambda}_0$ would be more complex. However, any combination of equity and long-term bonds resulting in the same *total* portfolio risk would yield the same *reduced-form* formulas. The only difference would be in the exact mapping between ϵ_λ and the degree of belief heterogeneity. Thus, to determine the degree of heterogeneity necessary to obtain our reduced-form calibration of ϵ_λ, we consider the more general case. In particular, we showed that, in general,

$$\mathcal{R}_0 = \frac{Q_L - Q_L^*}{Q_L}\frac{B_o^L - B_p^L}{C_s} + \frac{Q_E - Q_E^*}{Q_E}\frac{B_o^E - B_p^E}{C_s}.$$

Let $\alpha \equiv \frac{1}{\mathcal{R}_0}\frac{Q_L - Q_L^*}{Q_L}\frac{B_o^L - B_p^L}{C_s}$. When $\alpha = 1$, all the difference in the savers' portfolios is given by their holdings of long-term bonds. When $\alpha = 0$, all the difference in the savers' portfolios is given by their holdings of equity.

To discipline the free parameters, we consider the mapping of the model to the data. We interpret optimistic savers as representing the top 1% of the income distribution. This population owns roughly 50% of all the corporate equities in the U.S. and earn around 26% of all income. Choosing parameters to match these moments, we get that, in the model, the optimistic savers own 59% of the equities and earn 22% of all income. In terms of the implications for belief heterogeneity, we find that optimistic savers believe the disaster probability is less than 1bps per year, while pessimistic savers believe it is 7%.

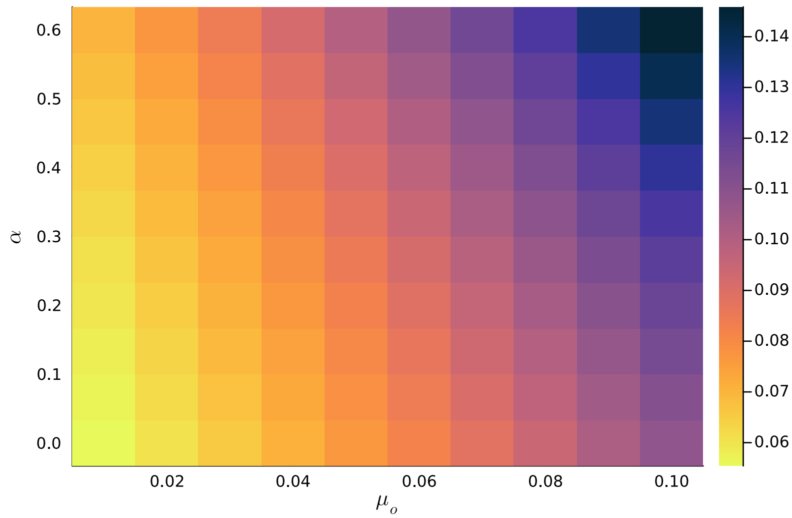
More generally, Figure C.2 shows the implied belief of pessimistic savers for different combinations of μ_o and α (the implied belief of the optimistic savers is always below 1bps). The values range from about 5.5% to more than 15%. For example, if we assumed, as in the text, that savers only differ in their holdings of long-term bonds, and we interpret the optimistic savers as the top 1% of the income distribution, the model implies that pessimistic savers believe that the probability is slightly below 9% per year. The plot shows that the belief of pessimistic savers is lowest when the difference in holdings of stocks is high, and when the fraction of optimistic savers in the economy is low. This result is intuitive. The determination of ϵ_λ depends on two variables: the degree of belief heterogeneity and the redistribution across types after the monetary shock. Fixing a target value for ϵ_λ (to match the VAR evidence), there is some degree of substitutability between these two dimensions. Since stock prices are more sensitive to a monetary shock than bond prices, a higher fraction of stocks owned by optimistic savers implies a larger redistribution after the shock. Hence, a lower degree of disagreement is necessary. Similarly, a lower fraction of optimistic agents implies that wealth is more concentrated, so, again, the same shock has a larger distributional impact.

Finally, it is worth noting that this exercise assumes that the entire movement in asset prices can be explained by our channel, which is an extreme assumption. To address this, we analyze next the sensitive of our results to changes in the calibrated value of ϵ_λ. For example, suppose we cut ϵ_λ in half. This reduces the impact of monetary policy on the 5-year yield and on stock prices by around 30%. In our preferred calibration (where optimistic savers represent the top 1% of the income distribution and own roughly 50% of the equities), we find an implied belief of pessimistic savers of 3.6% per year. Naturally, this reduction in ϵ_λ also implies a reduction in the effect of monetary policy on output. On impact, output responds 35% less when ϵ_λ is cut in half.

D Extensions

In this section, we discuss three different extensions of the baseline model. First, we introduce wealthy hand-to-mouth agents into the model to capture the evidence in Kaplan and Violante (2014). Second, we introduce capital into a simplified version of the model and study how the risk-premium neutrality extends to this setting. Third, we consider a version of the model with sticky wages instead of sticky prices.

Figure C.2: Implied belief of pessimistic savers for different values of μ_o and α



D.1 Wealthy hand-to-mouth

Consider an extension of the model with wealthy hand-to-mouth agents. In particular, we assume that there is a third type of savers who only holds stocks and never buys or sell shares.[5] This implies that we can write the amount invested in stocks as follows: $B^E_{r,t} = B^E_r \frac{Q_{E,t}}{Q_E}$, where we used r to index the rich hand-to-mouth. Plugging $B_{r,t} = B^E_{r,t}$ into the flow budget constraint, we obtain the consumption of wealthy hand-to-mouth agents:

$$C_{r,t} = \Pi_t \frac{B^E_r}{Q_E} + T_{h,t}, \tag{D.1}$$

where Π_t denotes real profits and $\frac{B^E_r}{Q_E}$ denotes the number of shares held by these savers. Analogous to workers, we assume that $T_{h,t} = T_h(Y_t)$. Linearizing the expression above:

$$c_{r,t} = \frac{Y}{C_r}\frac{B^E_r}{Q_E}\left[1-(1-\epsilon_p^{-1})(1+\phi)\right] y_t + \frac{T_r(Y)}{C_r}\frac{YT'_r(Y)}{T_r(Y)} y_t \equiv \chi^r_y y_t. \tag{D.2}$$

[5] In the context of models with a fixed cost to adjust the portfolio, this can be interpreted as the case where the monetary shock is not large enough to move these agents outside their inaction region.

Similarly, workers' consumption is given by

$$C_{w,t} = \frac{W_t}{P_t} N_{w,t} + T_{w,t} \Rightarrow c_{w,t} = \frac{W}{P}\frac{N_w}{C_w}(1+\phi)y_t + \frac{T_w(Y)}{C_w}\frac{YT_w'(Y)}{T_w(Y)}y_t \equiv \chi_y^w y_t. \tag{D.3}$$

Define the average consumption of hand-to-mouth agents as $C_{h,t} = \frac{\mu_r}{\mu_r+\mu_w}C_{r,t} + \frac{\mu_w}{\mu_r+\mu_w}C_{w,t}$. The market clearing condition for goods can be written as

$$\mu_o C_{o,t} + \mu_p C_{p,t} + \mu_h C_{h,t} = Y_t, \tag{D.4}$$

where $\mu_h \equiv \mu_r + \mu_h$. Linearized consumption of hand-to-mouth agents is given by:

$$c_{h,t} \equiv \frac{\mu_w C_w}{\mu_w C_w + \mu_r C_r} c_{w,t} + \frac{\mu_r C_r}{\mu_w C_w + \mu_r C_r} c_{r,t} = \chi_y y, \tag{D.5}$$

where $\chi_y \equiv \frac{\mu_w C_w}{\mu_w C_w + \mu_r C_r}\chi_y^w + \frac{\mu_r C_r}{\mu_w C_w + \mu_r C_r}\chi_y^r$.

The Euler equations for optimistic and pessimistic savers are unchanged, and the same is true for the Phillips curve. The market clearing condition for goods takes the same form as in the baseline model, with hand-to-moouth agents (rich and poor) playing the role of workers. Hence, the equilibrium conditions describing the aggregate dynamics are exactly the same as in the baseline model.

Introducing wealthy hand-to-mouth agents in the economy then changes the cyclicality of income of the hand-to-mouth agents, given the behavior of taxes. However, conditional on the value of χ_y, the economy behaves in the same way as our baseline economy. This shows our results are robust to the introduction of wealhy hand-to-mouth agents.

D.2 Sticky wages

In the baseline model, we assumed prices are sticky and wages are fully flexible. In this extension, we consider the opposite case where wages are sticky and prices are fully flexible. To keep the discussion brief, we focus on the aspects of the extension that are different from the baseline model.

D.2.1 Environment

Firms. To produce intermediate goods, firms combine a continuum of differentiated labor inputs, indexed by $k \in [0, \mu_w]$, where μ_w denotes the mass of workers in the economy. Firm $i \in [0,1]$ produces intermediate goods according to the production function:

$$Y_{i,t} = A_t \mu_w N_t(i), \tag{D.6}$$

where $N_t(i)$ is a CES aggregator of differentiated labor inputs:

$$N_t(i) = \left[\frac{1}{\mu_w} \int_0^{\mu_w} N_{k,t}(i)^{\frac{\epsilon_w - 1}{\epsilon_w}} dk \right]^{\frac{\epsilon_w}{\epsilon_w - 1}}, \tag{D.7}$$

and $N_{k,t}(i)$ denotes firm i's demand for labor variety $k \in [0, \mu_w]$.

Prices are fully flexible, so the problem of the firm is given by

$$\max_{P_{i,t}, [N_{k,t}(i)]_{j \in [0,1]}} P_{i,t} Y_{i,t} - \int_0^{\mu_w} W_{k,t} N_{k,t}(i) dk, \tag{D.8}$$

subject to Eq. (D.6), Eq. (D.7), and the demand for intermediate goods $Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon_p} Y_t$. The demand for labor variety k is given by

$$N_{k,t}(i) = \left(\frac{W_{k,t}}{W_t} \right)^{-\epsilon_w} N_t(i), \tag{D.9}$$

where W_t is given by

$$W_t = \left[\frac{1}{\mu_w} \int_0^{\mu_w} W_{k,t}^{1-\epsilon_w} dk \right]^{\frac{1}{1-\epsilon_w}}. \tag{D.10}$$

The price of intermediate good i is given by

$$P_{i,t} = \frac{\epsilon_p}{\epsilon_p - 1}\frac{W_t}{A_t}. \tag{D.11}$$

The aggregate demand for labor variety k is defined as $N_{k,t} = \int_0^1 N_{k,t}(i)di$. Given all firms solve identical problems, we have $P_{i,t} = P_t$ and $N_{k,t}(i) = N_{k,t}$.

Workers and unions. There a continuum of workers indexed by $k \in [0, \mu_w]$. Workers are subject to a borrowing constraint, $B^w_{k,t} \geq -\overline{D}_p$, where $B^w_{k,t}$ denotes the net worth of a type-k worker. Workers are hand-to-mouth and their consumption is given by

$$C_{k,t} = \frac{W_{k,t}}{P_t}N_{k,t} + T_{w,t}. \tag{D.12}$$

The wage $W_{k,t}$ and the hours worked $N_{k,t}$ are defined by a union. The union faces Rotemberg quadratic adjustment costs on wages. The union's problem can be written as follows

$$V^w_{k,t}(W_j) = \max_{[N_{k,z}, \pi^w_{k,t}]_{z\geq t}} \mathbb{E}_t \left[\int_t^{t^*} e^{-\rho_w(z-t)} \left(\frac{C^{1-\sigma}_{k,z}}{1-\sigma} - \frac{N^{1+\phi}_{k,z}}{1+\phi} - \frac{\varphi}{2}(\pi^w_{k,s})^2 \right) dz + e^{-\rho_w(t^*-t)} V^{*,w}_{k,t^*}(W^*_{k,t^*}) \right], \tag{D.13}$$

subject to

$$\dot{W}_{k,t} = \pi^w_{k,t} W_{k,t}, \qquad N_{k,t} = \left(\frac{W_{k,t}}{W_t}\right)^{-\epsilon_w} N_t, \qquad C_{k,t} = \frac{W_{k,t}}{P_t}N_{k,t} + T_{w,t}, \tag{D.14}$$

where $W^*_{k,t^*} = W_{k,t^*}$, and φ denotes the adjustment cost parameter.

The HJB equation for this problem is given by

$$\rho_w V^w_{k,t} = \frac{C^{1-\sigma}_{k,t}}{1-\sigma} - \frac{N^{1+\phi}_{k,t}}{1+\phi} - \frac{\varphi}{2}(\pi^w_{k,t})^2 + \frac{\mathbb{E}[dV^w_{k,t}]}{dt}, \tag{D.15}$$

where

$$\frac{\mathbb{E}[dV^w_{k,t}]}{dt} = \frac{\partial V^w_{k,t}}{\partial t} + \frac{\partial V^w_{k,t}}{\partial W_j}\pi^w_{k,t}W_{k,t} + \lambda_t\left[V^{*,w}_{k,t} - V^w_{k,t}\right]. \tag{D.16}$$

The first-order condition for this problem is

$$\varphi\pi^w_{k,t} = \frac{\partial V^w_{k,t}}{\partial W_{k,t}}W_{k,t}. \tag{D.17}$$

The wage Phillips curve. The envelope condition is given by

$$\begin{aligned}\rho_w\frac{\partial V^w_{k,t}}{\partial W_{k,t}} = C^{-\sigma}_{k,t}(1-\epsilon)\frac{W_t}{P_t}\frac{N_t}{W_{k,t}}\left(\frac{W_{k,t}}{W_t}\right)^{1-\epsilon} + \epsilon N^{\phi}_{k,t}\left(\frac{W_{k,t}}{W_t}\right)^{-\epsilon}\frac{N_t}{W_{k,t}} + \frac{\partial V^w_{k,t}}{\partial W_j}\pi^w_{k,t}\\ + \frac{\partial^2 V^w_{k,t}}{\partial t\partial W_{k,t}} + \frac{\partial^2 V^w_{k,t}}{\partial W^2_{k,t}}\pi^w_{k,t}W_{k,t} + \lambda_t\left[\frac{\partial V^{*,w}_{k,t}}{\partial W^*_{k,t}} - \frac{\partial V^w_{k,t}}{\partial W_{k,t}}\right].\end{aligned} \tag{D.18}$$

Differentiating the first-order condition for π^w_t with respect to time, we obtain

$$\varphi\dot{\pi}^w_{j,t} = \frac{\partial V^w_{k,t}}{\partial W_{k,t}}\pi^w_{k,t}W_{k,t} + \left[\frac{\partial^2 V^w_{k,t}}{\partial t\partial W_{k,t}} + \frac{\partial^2 V^w_{k,t}}{\partial W^2_{k,t}}\pi^w_{k,t}W_{k,t}\right]W_{k,t} \tag{D.19}$$

Multiplying the envelope condition by $W_{k,t}$ and using the expression above, we obtain

$$\rho_w\varphi\pi^w_{k,t} = C^{-\sigma}_{k,t}(1-\epsilon)\frac{W_t}{P_t}N_t\left(\frac{W_{k,t}}{W_t}\right)^{1-\epsilon} + \epsilon N^{\phi}_{k,t}\left(\frac{W_{k,t}}{W_t}\right)^{-\epsilon}N_t + \varphi\dot{\pi}^w_{j,t} + \lambda_t\varphi\left[\pi^{*,w}_{k,t} - \pi^w_{k,t}\right]. \tag{D.20}$$

Assuming all unions have the same initial condition, they will choose the same path of wages, so $W_{k,t} = W_t$ and $\pi^w_{k,t} = \pi^w_t$. This implies that the consumption and hours are equalized across workers: $C_{k,t} = C_{w,t}$ and $N_{k,t} = N_{w,t}$. Rearranging the expression above, we arrive at the New Keynesian Wage Phillips curve:

$$\dot{\pi}_t^w = (\rho_w + \lambda_t)\pi_t^w - \varphi^{-1}(\epsilon_w - 1)\left[\frac{\epsilon_w}{\epsilon_w - 1}\frac{C_{w,t}^{\sigma} N_{w,t}^{\phi}}{W_t/P_t} - 1\right] C_{w,t}^{-\sigma}\frac{W_t}{P_t}N_{w,t}, \tag{D.21}$$

where we assumed that the monetary authority implements $\pi_t^{*,w} = 0$.

In the no-disaster state, we have $\frac{W_t}{P_t} = (1 - \epsilon_p^{-1})A$, so wage inflation equals price inflation, $\pi_t^w = \pi_t$.

Savers, government, and market clearing. The savers' problem and the government policies are unchanged relative to the baseline model. The market clearing conditions for goods and labor are given by

$$\sum_{j\in\{o,p,w\}} \mu_j C_{j,t} = \int_0^1 Y_{i,t} di, \qquad \int_0^1 N_{k,t}(i) di = N_{k,t}, \tag{D.22}$$

for $k \in [0, \mu_w]$. The market clearing condition for assets are given by

$$\sum_{j\in\{o,p,w\}} \mu_j B_{j,t}^S = 0, \qquad \sum_{j\in\{o,p,w\}} \mu_j B_{j,t}^L = D_{G,t}, \qquad \sum_{j\in\{o,p,w\}} \mu_j B_{j,t}^E = Q_{E,t}. \tag{D.23}$$

Stationary equilibrium. In a stationary equilibrium, all variables are constant conditional on the aggregate state (disaster or no-disaster). We assume that T_w is such that $C_w = Y$. For wages and the price level to be constant, the following condition must be satisfied

$$\frac{\epsilon_w}{\epsilon_w - 1}\frac{Y^{\sigma} N_w^{\phi}}{(1 - \epsilon_p^{-1})A} = 1 \Rightarrow N_w = \left[(1 - \epsilon_w^{-1})(1 - \epsilon_p^{-1})\frac{A^{1-\sigma}}{\mu_w^{\sigma}}\right]^{\frac{1}{\sigma+\phi}}, \tag{D.24}$$

where we used the fact that $\frac{W}{P} = (1 - \epsilon_p^{-1})A$. Ouput is then given by

$$Y = \left[(1 - \epsilon_p^{-1})(1 - \epsilon_w^{-1})\right]^{\frac{1}{\sigma+\phi}} A^{\frac{1+\phi}{\sigma+\phi}} \mu_w^{\frac{\phi}{\sigma+\phi}}. \tag{D.25}$$

An analogous condition holds in the disaster state:

$$Y^* = \left[(1 - \epsilon_p^{-1})(1 - \epsilon_w^{-1})\right]^{\frac{1}{\sigma+\phi}} (A^*)^{\frac{1+\phi}{\sigma+\phi}} \mu_w^{\frac{\phi}{\sigma+\phi}}. \tag{D.26}$$

Worker's consumption is given by

$$C_w = \frac{(1 - \epsilon_p^{-1})}{\mu_w} Y + T_w. \tag{D.27}$$

Hence, $T_w > 0$ if and only if $1 - \epsilon_p^{-1} < \mu_w$. Profits are given by

$$\Pi = Y - \frac{W}{P} \mu_w N_w = \epsilon_p^{-1} Y, \tag{D.28}$$

and a similar condition holds in the disaster state. The determination of the savers' consumption, natural rate, term spread, and equity premium are similar to the one in the baseline model.

D.2.2 Wage Phillips Curve

Let's compute a first-order approximation around the stationary equilibrium. First, the log-linearized real wage is given by $w_t - p_t = 0$. The log-linearized production function gives us $y_t = n_{w,t}$. The consumption of workers is given by

$$c_{w,t} = \frac{W}{P} \frac{N_w}{C_w} y_t + \frac{Y}{C_w} T_w'(Y) y_t = \chi_y y_t, \tag{D.29}$$

where $\chi_y \equiv \frac{W}{P} \frac{N_w}{C_w} + \frac{Y}{C_w} T_w'(Y)$.

The New Keynesian Phillips curve is given by

$$\dot{\pi}_t = \rho\pi_t - \kappa y_t, \tag{D.30}$$

where $\kappa \equiv \varphi^{-1}(\epsilon_w - 1)(1 - \epsilon_p^{-1})\frac{AN_w}{C_w^\sigma}\left[\sigma\chi_y + \phi\right]$. We assumed that $\rho_w = \rho_s$, so $\rho_w + \lambda = \rho$.

Euler equations are the same as in the baseline model. Conditional on χ_y and κ, the aggregate implications of the model with sticky wages are the same as with sticky prices.

Cyclicality of profits. One important distinction between the model with sticky prices and sticky wages regards the cyclicality of profits. Profits are given by

$$\Pi_t = Y_t - \frac{W_t}{P_t}N_t \Rightarrow \hat{\Pi}_t = y_t - \frac{WN}{PY}(w_t - p_t + n_t). \tag{D.31}$$

Using the fact that $\frac{W}{P}N = (1 - \epsilon_p^{-1})Y$, we obtain that profits are given by

$$\hat{\Pi}_t = \epsilon_p^{-1} y_t. \tag{D.32}$$

In the baseline model, profits are instead given by

$$\hat{\Pi}_t = \left[1 - (1 - \epsilon_p^{-1})(1 + \phi)\right] y_t, \tag{D.33}$$

as $w_t - p_t = \phi n_t$ in constrast to the sticky-wages model. Hence, as long as $\phi > \frac{1}{\epsilon_p - 1}$, profits are countercyclical.

D.3 Risk-premium neutrality in a model with capital

We consider next a version of the model with endogenous investment in physical capital. To simplify the exposition, we consider a setting with a representative agent, but we allow the monetary authority to directly affect the subjective probability of disaster in a way analogous to the mechanism in our heterogeneous-agent economy. We also consider the effects of an uncertainty shock and monetary policy reacts endogenously to the shock. We will use this economy as a laboratory to study the extent our risk-premium neutrality result extends to an economy with capital.

D.3.1 Environment

Households. The household's problem can be written as follows

$$V_t(B_t) = \max_{[C_z, B_z^L, B_z^E]} \mathbb{E}_t \left[\int_t^{t^*} e^{-\rho_s(z-t)} \left(\frac{C_z^{1-\sigma}}{1-\sigma} - \frac{N_z^{1+\phi}}{1+\phi} \right) dz + e^{-\rho_s(t^*-t)} V_{t^*}^*(B_{t^*}^*) \right], \quad \text{(D.34)}$$

subject to

$$dB_t = \left[(i_t - \pi_t) B_t + r_{L,t} B_t^L + r_{E,t} B_t^E + T_t + (1+\tau_t^n) \frac{W_t}{P_t} N_t - (1+\tau_t^c) C_t \right] dt + [B_t^* - B_t] d\mathcal{N}_t, \quad \text{(D.35)}$$

where $B_t^* = B_t + B_t^L \frac{Q_{L,t}^* - Q_{L,t}}{Q_{L,t}} + B_t^E \frac{Q_{E,t}^* - Q_{E,t}}{Q_{E,t}}$.

The HJB equation for this problem is given by

$$\begin{aligned} \rho_s V_t = \max_{C_t, N_t, B_t^L, B_t^E} & \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi} + \frac{\partial V_t}{\partial t} + \lambda_t \left[V_t^* - V_t \right] \\ & + \frac{\partial V_t}{\partial B_t} \left[(i_t - \pi_t) B_t + r_{L,t} B_t^L + r_{E,t} B_t^E + T_t + (1+\tau_t^n) \frac{W_t}{P_t} N_t - (1+\tau_t^c) C_t \right]. \end{aligned} \quad \text{(D.36)}$$

The first-order conditions for this problem are given by

$$C_t^{-\sigma} = \frac{\partial V_t}{\partial B_t}(1+\tau_t^c), \qquad N_t^{\phi} = \frac{\partial V_t}{\partial B_t} \frac{W_t}{P_t}(1+\tau_t^n), \qquad \frac{\partial V_t}{\partial B_t} r_{k,t} = \frac{\partial V_t^*}{\partial B_t^*} \frac{Q_{k,t} - Q_{k,t}^*}{Q_{k,t}}, \quad \text{(D.37)}$$

for $k \in \{L, E\}$.

There is no uncertainty in a disaster state, so $r_{L,t}^* = r_{E,t}^* = 0$ in equilibrium. The HJB equation in the disaster equation is given by

$$\rho_s^* V_t^* = \max_{C_t^*, N_t^*} \frac{(C_t^*)^{1-\sigma}}{1-\sigma} - \frac{(N_t^*)^{1+\phi}}{1+\phi} + \frac{\partial V_t^*}{\partial t} + \frac{\partial V_t^*}{\partial B_t^*} \left[(i_t^* - \pi_t^*) B_t^* + T_t^* + (1 + \tau_t^{*,n}) \frac{W_t^*}{P_t^*} N_t^* - (1 + \tau_t^{c,*}) C_t^* \right].$$

The optimality conditions are given by

$$(C_t^*)^{-\sigma} = \frac{\partial V_t^*}{\partial B_t^*}(1 + \tau_t^{c,*}), \qquad (N_t^*)^{\phi} = \frac{\partial V_t^*}{\partial B_t^*} \frac{W_t^*}{P_t^*}(1 + \tau_t^{*,n}). \tag{D.38}$$

The envelope condition is given by

$$\rho_s \frac{\partial V_t}{\partial B_t} = \frac{\partial V_t}{\partial B_t}(i_t - \pi_t) + \frac{\mathbb{E}_t[d\left(\frac{\partial V_t}{\partial B_t}\right)]}{dt}. \tag{D.39}$$

Using the optimality condition for consumption, and assuming $\tau_t^c = \tau_t^{c,*}$ so there is no jump on the consumption tax rate, we obtain the Euler equation for short-term bonds:

$$\frac{\dot{C}_t}{C_t} = \sigma^{-1}(i_t - \pi_t - \dot{\tau}_t^c - \rho_s) + \frac{\lambda_t}{\sigma} \left[\left(\frac{C_t}{C_t^*} \right)^{\sigma} - 1 \right]. \tag{D.40}$$

Using the envelope condition in the disaster state, we obtain the corresponding Euler equation:

$$\frac{\dot{C}_t^*}{C_t^*} = \sigma^{-1}(i_t^* - \pi_t^* - \rho_s^*), \tag{D.41}$$

where we assumed that $\dot{\tau}_t^{c,*} = 0$.

The pricing condition for the long-term bonds and equities are given by

$$r_{k,t} = \lambda_t \left(\frac{C_t}{C_t^*} \right)^{\sigma} \frac{Q_{k,t} - Q_{k,t}^*}{Q_{k,t}}. \tag{D.42}$$

We will assume $\tau_t^c = \tau_t^n$ and $\tau_t^{c,*} = \tau_t^{*,n}$. The labor supply condition in the two state are then given by

$$\frac{W_t}{P_t} = C_t^{\sigma} N_t^{\phi}, \qquad \frac{W_t^*}{P_t^*} = (C_t^*)^{\sigma}(N_t^*)^{\phi}. \tag{D.43}$$

Firms. Firm i produces intermediate goods according to the production technology:

$$Y_{i,t} = A_t K_{i,t}^{\alpha} N_{i,t}^{1-\alpha}. \tag{D.44}$$

The firm is subject to quadratic price adjustment costs $0.5\varphi\pi_{i,s}^2$. The firm is also subject to investment adjustment costs. We assume that the firm pays an investment tax τ_t^K. We can then write the firm's problem as follows:

$$Q_{i,t}(P_i, K_i) = \max_{[\pi_{i,s}, \iota_{i,s}]_{s\geq t}} \mathbb{E}_t \left[\int_t^{t^*} \frac{\eta_s}{\eta_t} \left(\frac{P_{is}}{P_s} Y_{i,s} - \frac{W_s}{P_s} N_{i,s} - (1+\tau_t^K)\iota_{i,t} K_{i,t} - \frac{\varphi}{2}\pi_{i,s}^2 + T_{f,t} \right) ds + \frac{\eta_t^*}{\eta_t} Q_{i,t}^*(P_{i,t^*}^*, K_{i,t^*}^*) \right], \tag{D.45}$$

subject to

$$\dot{P}_{i,t} = \pi_{i,t} P_{i,t}, \qquad \frac{dK_{i,t}}{K_{i,t}} = [\Phi(\iota_{i,t}) - \delta]\, dt - \zeta_K d\mathcal{N}_t, \qquad Y_{i,t} = \left(\frac{P_{i,t}}{P_t} \right)^{-\epsilon} Y_t, \qquad N_{i,t} = \left(\frac{Y_{i,t}}{A_t K_{i,t}^{\alpha}} \right)^{\frac{1}{1-\alpha}}, \tag{D.46}$$

where $\Phi(\cdot)$ is an increasing and concave function, $P_{i,t^*}^* = P_{i,t^*}$, and $K_{i,t}^* = (1-\zeta_K)K_{i,t^*}$. Notice that, to achieve a production level $Y_{i,t}$ given the capital stock $K_{i,t}$, the firm needs $N_{i,t} = \left(\frac{Y_{i,t}}{A_t K_{i,t}^{\alpha}} \right)^{\frac{1}{1-\alpha}}$ units of labor. The lump-sum transfer $T_{f,t}$ corresponds to the value of the price adjustment costs plus the government's revenue from the investment tax. Therefore, the price adjustment costs does not represent a real resource cost. The lump-sum transfer also includes the revenue from the investment tax, so the investment tax allows the government to influence investment, but it does not represent a net source of fiscal revenue.

The HJB equation for this problem is given by

$$0 = \max_{\pi_{i,t}, \iota_{i,t}} \eta_t \left(\frac{P_{i,t}}{P_t} Y_{i,t} - \frac{W_t}{P_t} \left(\frac{Y_{i,t}}{A_t K_{i,t}^{\alpha}} \right)^{\frac{1}{1-\alpha}} - (1+\tau_t^K)\iota_{i,t} K_{i,t} - \frac{\varphi}{2}\pi_{i,t}^2 + T_{f,t} \right) + \mathbb{E}_t[d(\eta_t Q_{i,t})], \tag{D.47}$$

where

$$\frac{\mathbb{E}_t\left[d(\eta_t Q_{i,t})\right]}{\eta_t dt} = -(i_t - \pi_t)Q_{i,t} + \frac{\partial Q_{i,t}}{\partial P_{i,t}}\pi_{i,t}P_{i,t} + \frac{\partial Q_{i,t}}{\partial K_{i,t}}(\Phi(\iota_{i,t}) - \delta)K_{i,t} + \frac{\partial Q_{i,t}}{\partial t} + \lambda_t \frac{\eta_t^*}{\eta_t}\left[Q_{i,t}^* - Q_{i,t}\right]. \tag{D.48}$$

The first-order conditions for this problem are given by

$$\frac{\partial Q_{i,t}}{\partial K_{i,t}}\Phi'(\iota_{i,t}) = 1 + \tau_t^K, \qquad \varphi\pi_{i,t} = \frac{\partial Q_{i,t}}{\partial P_{i,t}}P_{i,t}. \tag{D.49}$$

New Keynesian Phillips Curve. The envelope condition with respect to $P_{i,t}$ is given by

$$\begin{aligned}
&\left[(1-\epsilon)\left(\frac{P_{i,t}}{P_t}\right)^{1-\epsilon} Y_t + \frac{\epsilon}{1-\alpha}\frac{W_t}{P_t}\left(\frac{P_{i,t}}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}}\left(\frac{Y_t}{A_t K_{i,t}^{\alpha}}\right)^{\frac{1}{1-\alpha}}\right]\frac{1}{P_{i,t}} - (i_t - \pi_t)\frac{\partial Q_{i,t}}{\partial P_{i,t}} + \frac{\partial Q_{i,t}}{\partial P_{i,t}}\pi_{i,t} \\
&\quad + \frac{\partial^2 Q_{i,t}}{\partial P_{i,t}^2}\pi_{i,t}P_{i,t} + \frac{\partial^2 Q_{i,t}}{\partial K_{i,t}\partial P_{i,t}}(\Phi(\iota_{i,t}) - \delta)K_{i,t} + \frac{\partial^2 Q_{i,t}}{\partial t \partial P_{i,t}} + \lambda_t\frac{\eta_t^*}{\eta_t}\left[\frac{\partial Q_{i,t}^*}{\partial P_{i,t}^*} - \frac{\partial Q_{i,t}}{\partial P_{i,t}}\right].
\end{aligned} \tag{D.50}$$

Differentiating the first-order condition for the price change with respect to time, we obtain

$$\varphi\dot{\pi}_{i,t} = \frac{\partial Q_{i,t}}{\partial P_{i,t}}\pi_{i,t}P_{i,t} + \left[\frac{\partial^2 Q_{i,t}}{\partial P_{i,t}^2}\pi_{i,t}P_{i,t} + \frac{\partial^2 Q_{i,t}}{\partial P_{i,t}\partial K_{i,t}}(\Phi(\iota_{i,t} - \delta)K_{i,t} + \frac{\partial Q_{i,t}}{\partial t \partial P_{i,t}}\right]P_{i,t} \tag{D.51}$$

Multiplying the envelope condition by $P_{i,t}$ and using the expression above, we obtain

$$\dot{\pi}_{i,t} = (i_t - \pi_t)\pi_{i,t} - \lambda_t \frac{\eta_t^*}{\eta_t}\left[\pi_{i,t}^* - \pi_{i,t}\right] - \varphi^{-1}\left[(1-\epsilon)\left(\frac{P_{i,t}}{P_t}\right)^{1-\epsilon} Y_t + \frac{\epsilon}{1-\alpha}\frac{W_t}{P_t}\left(\frac{P_{i,t}}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}}\left(\frac{Y_t}{A_t K_{i,t}^{\alpha}}\right)^{\frac{1}{1-\alpha}}\right]. \tag{D.52}$$

In a symmetric equilibrium, $\pi_{i,t} = \pi_t$, $P_{i,t} = P_t$, and $K_{i,t} = K_t$, which gives us the non-linear New Keynesian Phillips curve

$$\dot{\pi}_t = (i_t - \pi_t + \lambda_t \frac{\eta_t^*}{\eta_t})\pi_t - \varphi^{-1}(\epsilon - 1)\left[\frac{\epsilon}{\epsilon - 1}\frac{W_t/P_t}{1-\alpha}\frac{N_t}{Y_t} - 1\right] Y_t, \tag{D.53}$$

where we assumed that the central bank implements $\pi_t^* = 0$ for all t.

Optimal investment. The envelope condition for capital is given by

$$\frac{W_t}{P_t}\frac{\alpha K_{i,t}^{-1}}{1-\alpha}\left(\frac{Y_{i,t}}{A_t K_{i,t}^{\alpha}}\right)^{\frac{1}{1-\alpha}} - (1+\tau_t^K)\iota_{i,t} + q_{i,t}(\Phi(\iota_{i,t}) - \delta) - \zeta_K \lambda_t \frac{\eta_t^*}{\eta_t} q_{i,t}^* + \frac{\mathbb{E}[d(\eta_t q_{i,t})]}{\eta_t dt} = 0, \tag{D.54}$$

where $q_{i,t} \equiv \frac{\partial Q_{i,t}}{\partial K_{i,t}}$ corresponds to marginal q and $\frac{\mathbb{E}[d(\eta_t q_{i,t})]}{\eta_t dt}$ is given by

$$\frac{\mathbb{E}[d(\eta_t q_{i,t})]}{\eta_t dt} = -(i_t - \pi_t)q_{i,t} + \dot{q}_{i,t} + \lambda_t \frac{\eta_t^*}{\eta_t}[q_{i,t}^* - q_{i,t}]. \tag{D.55}$$

Investment is given by

$$\Phi'(\iota_{i,t}) = (1+\tau_t^K) q_{i,t}^{-1}. \tag{D.56}$$

Government. The government's flow budget constraint in the no-disaster is given by

$$dD_{G,t} = \left[(i_t - \pi_t + r_{L,t})D_{G,t} + T_t + \tau_t^n \frac{W_t}{P_t} N_t - \tau_t^c C_t\right] dt + \left[D_{G,t}^* - D_{G,t}\right] d\mathcal{N}_t, \tag{D.57}$$

where $D^*_{G,t} = D_{G,t}\frac{Q^*_{L,t}}{Q_{L,t}}$. The government is subject to a No-Ponzi condition $\lim_{T\to\infty} \mathbb{E}_t \left[\eta_T D_{G,T}\right] \leq 0$, where $D_{G,t}$ denotes the real value of government debt.

In the no-disaster state, the monetary rule is given by

$$i_t = r_n + \phi_n \pi_t + u_t, \tag{D.58}$$

and the monetary rule in the disaster state is given by $i^*_t = r^*_n + \phi_\pi \pi^*_t$.

Disaster probability. We assume that the disaster probability is given by $\lambda_t = \lambda e^{\hat{\lambda}_t}$, where $\hat{\lambda}_t$ is given by

$$\hat{\lambda}_t = e^{-\psi_\lambda t}\hat{\lambda}_0. \tag{D.59}$$

We will consider two different versions of the model. In the monetary shock version, we assume that u_t is exogenously given and $\hat{\lambda}_0$ reacts to the nominal interest rate:

$$\hat{\lambda}_0 = \epsilon_\lambda (i_0 - r_n). \tag{D.60}$$

This captures in reduced-form the main mechanism in our baseline model. In the uncertainty-shock version, we assume that $\hat{\lambda}_0$ is exogenously given and u_t reacts to the uncertainty shock.

Market clearing. The market clearing conditions are given by

$$C_t = \int_0^1 (Y_{i,t} - \iota_{i,t} K_{i,t}) di, \qquad \int_0^1 N_{i,t} di = N_t, \qquad B^S_t = 0, \qquad B^L_t = D_{G,t}, \qquad B^E_t = Q_{E,t}. \tag{D.61}$$

D.3.2 Stationary equilibrium

In a stationary equilibrium, all variables are constant conditional on the aggregate state (disaster or no-disaster). For the price level to be constant, the following condition must be satisfied

$$\frac{\epsilon}{\epsilon-1}\frac{W}{P}\frac{Y^{-1}}{1-\alpha}\left(\frac{Y}{AK^{\alpha}}\right)^{\frac{1}{1-\alpha}} = 1 \Rightarrow \frac{W}{P} = (1-\epsilon^{-1})(1-\alpha)AK^{\alpha}N^{-\alpha}. \tag{D.62}$$

The labor supply condition is given by $\frac{W}{P} = \nu\frac{1+\tau^c}{1+\tau^n}C^{\sigma}N^{\phi}$. We assume that $\tau^c = \tau^n = \tau^K = \tau^{c,*} = \tau^{*,n} = \tau^{k,*} = 0$. Consumption in the stationary equilibrium is given by

$$C = AK^{\alpha}N^{1-\alpha} - \delta K \tag{D.63}$$

Combining the labor supply condition with the labor demand derived above, we obtain

$$\left[AK^{\alpha}N^{1-\alpha} - \delta K\right]^{\sigma} N^{\phi} = (1-\epsilon^{-1})(1-\alpha)AK^{\alpha}N^{-\alpha}. \tag{D.64}$$

For capital to be constant, the following condition must be satisfied:

$$\Phi(\iota) = \delta \Rightarrow \iota = \Phi^{-1}(\delta) \tag{D.65}$$

The optimality condition for investment is given by

$$\Phi'(\iota) = 1/q \Rightarrow q = [\Phi'(\Phi^{-1}(\delta))]^{-1}. \tag{D.66}$$

Similarly, we have that $\iota^* = \iota$ and $q = q^*$.

The pricing condition for q is given by

$$\frac{\alpha}{1-\alpha}\frac{W}{P}\frac{N}{K} - \iota - r_n q - \zeta_K \lambda \frac{\eta^*}{\eta} q^* = 0 \tag{D.67}$$

In the disaster state, this condition simplifies to

$$\frac{\alpha}{1-\alpha}\frac{W^*}{P^*}\frac{N^*}{K^*} - \iota^* - r_n^* q^* = 0 \tag{D.68}$$

Disaster state. The natural rate in the disaster is given by $r_n^* = \rho_s^*$. We can then use the expression above to solve for N^*/K^*:

$$N^* = K^* \left[\frac{\iota^* + r_n^* q^*}{\alpha A^*(1-\epsilon^{-1})}\right]^{\frac{1}{1-\alpha}}. \tag{D.69}$$

We can then use the equation equation labor supply and labor demand to obtain K^*:

$$K^* = (A^*)^{\frac{(1+\phi)}{(1-\alpha)(\sigma+\phi)}} \left[\frac{(1-\epsilon^{-1})(1-\alpha)}{\left[\frac{(\iota^*+r_n^* q^*)}{\alpha(1-\epsilon^{-1})} - \delta\right]^{\sigma}}\right]^{\frac{1}{\phi+\sigma}} \left[\frac{\iota^* + r_n^* q^*}{\alpha(1-\epsilon^{-1})}\right]^{-\frac{\alpha+\phi}{(1-\alpha)(\phi+\sigma)}}. \tag{D.70}$$

Labor supply is given by

$$N^* = (A^*)^{\frac{(1-\sigma)}{(1-\alpha)(\sigma+\phi)}} \left[\frac{(1-\epsilon^{-1})(1-\alpha)}{\left[\frac{(\iota^*+r_n^* q^*)}{\alpha(1-\epsilon^{-1})} - \delta\right]^{\sigma}}\right]^{\frac{1}{\phi+\sigma}} \left[\frac{\iota^* + r_n^* q^*}{\alpha(1-\epsilon^{-1})}\right]^{\frac{\sigma-\alpha}{(1-\alpha)(\phi+\sigma)}}. \tag{D.71}$$

Output is given by

$$Y^* = (A^*)^{\frac{(1+\phi)}{(1-\alpha)(\sigma+\phi)}} \left[\frac{(1-\epsilon^{-1})(1-\alpha)}{\left[\frac{(\iota^*+r_n^* q^*)}{\alpha(1-\epsilon^{-1})} - \delta\right]^{\sigma}}\right]^{\frac{1}{\phi+\sigma}} \left[\frac{\iota^* + r_n^* q^*}{\alpha(1-\epsilon^{-1})}\right]^{\frac{\sigma-\alpha}{(\phi+\sigma)} - \frac{\alpha(\alpha+\phi)}{(1-\alpha)(\phi+\sigma)}} \tag{D.72}$$

Consumption in the disaster state is given by

$$C^* = K^* \left[\frac{\iota^* + r_n^* q^*}{\alpha(1-\epsilon^{-1})} - \delta\right]. \tag{D.73}$$

No-disaster state. The natural rate in the no-disaster state is given by

$$r_n = \rho_s - \lambda \left[\left(\frac{C}{C^*} \right)^{\sigma} - 1 \right]. \tag{D.74}$$

Labor is given by

$$N = K \left[\frac{\iota + r_q q}{\alpha A (1 - \epsilon^{-1})} \right]^{\frac{1}{1-\alpha}}, \tag{D.75}$$

using the fact that $q^* = q$, where r_q is given by

$$r_q \equiv r_n + \zeta_K \lambda \frac{\eta^*}{\eta} = \rho_s + \lambda \left[1 - (1 - \zeta_K) \left(\frac{C}{C^*} \right)^{\sigma} \right]. \tag{D.76}$$

We are going to construct an equilibrium where consumption drops by the same amount as capital in a disaster, so $C^* = (1 - \zeta_K)C$. In this case, the discount rate for q is given by

$$r_q = \rho_s + \lambda \left[1 - (1 - \zeta_K)^{1-\sigma} \right] \tag{D.77}$$

Consumption is given by

$$C = K \left[\frac{\iota + r_q q}{\alpha (1 - \epsilon^{-1})} - \delta \right] \tag{D.78}$$

Combining the expression for consumption with the expression for the r_n, we can solve for the real rate as a function of the capital stock $r_n(K)$. The capital stock is determined by the condition

$$K = A^{\frac{(1+\phi)}{(1-\alpha)(\sigma+\phi)}} \left[\frac{(1-\epsilon^{-1})(1-\alpha)}{\left[\frac{(\iota + r_q q)}{\alpha(1-\epsilon^{-1})} - \delta \right]^{\sigma}} \right]^{\frac{1}{\phi+\sigma}} \left[\frac{\iota + r_q q}{\alpha (1 - \epsilon^{-1})} \right]^{-\frac{\alpha+\phi}{(1-\alpha)(\phi+\sigma)}}. \tag{D.79}$$

Let $\zeta_A \equiv 1 - \frac{A^*}{A}$ and assume that $r_q = r_n^*$, so $K^* = (1 - \zeta_A)^{\frac{1+\phi}{(1-\alpha)(\sigma+\phi)}} K$. We assume that $1-\zeta_K = (1-\zeta_A)^{\frac{1+\phi}{(1-\alpha)(\sigma+\phi)}}$, so capital jumps immediately to its steady state level. Consumption and output drop by the same amount as capital in a disaster, while labor units will be constant if $\sigma = 1$.

In this scenario, equity prices also drop by the same amount as aggregate output. Equity prices in the disaster state are given by

$$Q_E^* = \frac{\Pi^*}{r_n^*} = \left[1 - (1-\epsilon^{-1})(1-\alpha) - \delta\right] \frac{Y^*}{r_n^*}. \tag{D.80}$$

Profits satisfy the relationship $\Pi^* = (1-\zeta_K)\Pi$. Then, $Q_E^* = (1-\zeta_K)\frac{\Pi}{r_n^*}$. If $Q_E^* = (1-\zeta_K)Q_E$, then Q_E satisfy the condition

$$\frac{\Pi}{Q_E} = r_n + \lambda(1-\zeta_K)^{-\sigma}\zeta_K \Rightarrow Q_E = \frac{\Pi}{r_q}. \tag{D.81}$$

Hence, $Q_E^* = (1-\zeta_K)Q_E$ if $r_q = r_n^*$, as assumed above. This implies that $r_E = \lambda(1-\zeta_K)^{-\sigma}\zeta_K$. The price of the long-term bond satisfies the conditinos:

$$Q_E^* = \frac{1}{r_n^* + \psi_L} = \frac{\rho + \psi_L}{\rho + \psi_L + \lambda(1-\zeta_K)^{-\sigma}\zeta_K} Q_E. \tag{D.82}$$

Then, the term spread is given by $r_L = \lambda(1-\zeta_K)^{-\sigma}\frac{\lambda(1-\zeta_K)^{-\sigma}\zeta_K}{\rho+\psi_L+\lambda(1-\zeta_K)^{-\sigma}\zeta_K}$

D.3.3 Log-linear dynamics

Wages and aggregate output. Let's compute a first-order approximation around the stationary equilibrium. First, the log-linearized labor supply condition can be written as

$$w_t - p_t = \phi n_t + \sigma c_t. \tag{D.83}$$

Log-linearizing the production function, we obtain

$$y_t = \alpha k_t + (1-\alpha) n_t. \tag{D.84}$$

Log-linearizing the market clearing condition for goods, we obtain

$$y_t = \varsigma_c c_t + \varsigma_i (\hat{\iota}_t + k_t), \tag{D.85}$$

given $\hat{\iota}_t \equiv \log \iota_t / \iota$, where $\varsigma_c \equiv \frac{C}{Y}$ and $\varsigma_i = \frac{\iota K}{Y}$.

Euler equations. The linearized Euler equation for short-term bonds can be written as follows:

$$\dot{c}_t = \sigma^{-1}(i_t - \pi_t - r_n - \dot{\hat{\tau}}_t^c) + \delta(c_t - c_t^*) + \chi_{c\lambda} \hat{\lambda}_t, \tag{D.86}$$

where $\chi_{c\lambda} \equiv \frac{\lambda}{\sigma}\left[\left(\frac{C}{C^*}\right)^{\sigma} - 1\right]$ and $\delta \equiv \lambda \left(\frac{C}{C^*}\right)^{\sigma}$, and $\delta = \sigma \chi_{c\tau}$. The corresponding equation in the disaster state is given by

$$\dot{c}_t^* = \sigma^{-1}(i_t^* - r_n^*) \tag{D.87}$$

Linearizing the Euler equation for the risky assets, we obtain

$$r_{k,t} - r_k = r_k \left[\hat{\lambda}_t + \sigma(c_t - c_t^*) - \frac{Q_k^*}{Q_k - Q_k^*}(q_{k,t}^* - q_{k,t})\right]. \tag{D.88}$$

The pricing condition for long-term bonds is given by

$$-\frac{1}{Q_L} q_{L,t} + \dot{q}_{L,t} - (i_t - r_n) = r_L \left[\hat{\lambda}_t + \sigma(c_t - c_t^*) - \frac{Q_L^*}{Q_L - Q_L^*}(q_{L,t}^* - q_{L,t})\right]. \tag{D.89}$$

Rearranging the expression above, we obtain

$$\dot{q}_{L,t} = (\rho + \psi_L) q_{L,t} + i_t - r_n + r_L \left[\hat{\lambda}_t + \sigma(c_t - c_t^*) - \frac{Q_L^*}{Q_L - Q_L^*} q_{L,t}^*\right]. \tag{D.90}$$

Similarly, the pricing condition for equities is given by

$$\frac{\Pi}{Q_E}(\hat{\Pi}_t - q_{E,t}) + \dot{q}_{E,t} - (i_t - \pi_t - r_n) = r_E\left[\hat{\lambda}_t + \sigma(c_t - c_t^*) - \frac{Q_E^*}{Q_E - Q_E^*}(q_{E,t}^* - q_{E,t})\right]. \tag{D.91}$$

Rearranging the expression above, we obtain

$$\dot{q}_{E,t} = \rho q_{E,t} - \frac{\Pi}{Q_E}\hat{\Pi}_t + i_t - \pi_t - r_n + r_E\left[\hat{\lambda}_t + \sigma(c_t - c_t^*) - \frac{Q_E^*}{Q_E - Q_E^*}q_{E,t}^*\right]. \tag{D.92}$$

Investment. Linearizing the optimality condition for investment, we obtain

$$\hat{\iota}_t = \chi_{\iota q}\hat{q}_t, \tag{D.93}$$

where $\chi_{\iota q} \equiv -\left[\frac{\Phi''(\iota)\iota}{\Phi'(\iota)}\right]^{-1} > 0$, and we define $\hat{q}_t \equiv \log\frac{q_t/q}{1+\tau_t^K}$. Notice that our definition of $\hat{q}_t$ includes the effect of the investment tax, which is the relevant variable to determine the investment rate.

Phillips curve. The NKPC can be written as

$$\dot{\pi}_t = \left(r_n + \lambda\frac{\eta^*}{\eta}\right)\pi_t - \varphi^{-1}(\epsilon - 1)\left[(\alpha + \phi)n_t + \sigma c_t - \alpha k_t\right]Y. \tag{D.94}$$

Combining the expression above with the production function, we obtain

$$\dot{\pi}_t = (\rho_s + \lambda)\pi_t - \varphi^{-1}(\epsilon - 1)\left[\frac{\alpha + \phi}{1 - \alpha}y_t + \sigma c_t - \frac{\alpha(1 + \phi)}{1 - \alpha}k_t\right]Y. \tag{D.95}$$

Using the market clearing condition for goods, we obtain

$$\dot{\pi}_t = (\rho_s + \lambda)\pi_t - \kappa\left[c_t + \omega_q\hat{q}_t - \omega_k k_t\right], \tag{D.96}$$

where $\kappa \equiv (\epsilon - 1)\frac{\varsigma_c(\alpha+\phi)+\sigma(1-\alpha)}{1-\alpha}\frac{Y}{\varphi}$, $\omega_q \equiv \frac{\varsigma_i(\alpha+\phi)\chi_{\iota q}}{\varsigma_c(\alpha+\phi)+\sigma(1-\alpha)}$, and $\omega_k \equiv \frac{\alpha\varsigma_c+(\alpha-\varsigma_i)\phi}{\varsigma_c(\alpha+\phi)+\sigma(1-\alpha)}$.

We assume the monetary authority implements zero inflation in the disaster state, so the following condition must be satisfied:

$$c_t^* = \omega_k k_t^* - \omega_q \hat{q}_t^*. \tag{D.97}$$

Marginal q. The pricing condition for q_t is given by

$$\frac{\alpha}{1-\alpha}\frac{W_t}{P_t}\frac{N_t}{K_t}\frac{1}{q_t} - (1+\tau_t^K)\frac{\iota_t}{q_t} + \Phi(\iota_t) - \delta - (i_t - \pi_t) + \frac{\dot{q}_t}{q_t} + \lambda_t \frac{\eta_t^*}{\eta_t}\frac{q_t^*(1-\zeta_K) - q_t}{q_t} = 0. \tag{D.98}$$

Linearizing the expression above, we obtain

$$\frac{\alpha}{1-\alpha}\frac{WN}{PKq}(w_t - p_t + n_t - k_t - (\hat{q}_t + \hat{\tau}_t^K)) - \frac{\iota}{q}(\hat{\iota}_t - \hat{q}_t) + \Phi'(\iota)\iota\hat{\iota}_t - (i_t - \pi_t - r_n) + \dot{\hat{q}}_t + \dot{\hat{\tau}}_t^K \tag{D.99}$$

$$-\lambda\left(\frac{C}{C^*}\right)^{\sigma}\left[\zeta_K\left(\hat{\lambda}_t + \sigma(c_t - c_t^*)\right) + (1-\zeta_K)(\hat{q}_t + \hat{\tau}_t^K - \hat{q}_t^*)\right] = 0 \tag{D.100}$$

Rearranging the expression above, and using the optimality condition for investment, we obtain

$$\dot{\hat{q}}_t = (i_t - \pi_t - r_n - \dot{\hat{\tau}}_t^K) + \chi_{qq}\hat{q}_t + \chi_{qc}c_t + \tilde{\chi}_{qk}k_t + \chi_{q\lambda}\hat{\lambda}_t + \chi_{q\tau}\hat{\tau}_t^K + \chi_{qq^*}\hat{q}_t^* + \chi_{qc^*}c_t^*, \tag{D.101}$$

where

$$\chi_{qq} = \lambda \left(\frac{C}{C^*}\right)^{\sigma} (1-\zeta_K) + \frac{\iota}{q}(\chi_{\iota q} - 1) + \frac{\alpha}{1-\alpha}\frac{WN}{PKq}\left[1 - \frac{\varsigma_i(1+\phi)}{1-\alpha}\chi_{\iota q}\right] - \Phi'(\iota)\iota\chi_{\iota q} \tag{D.102}$$

$$\chi_{qc} = -\frac{\alpha}{1-\alpha}\frac{WN}{PKq}\left[\sigma + \frac{\varsigma_c(1+\phi)}{1-\alpha}\right] + \lambda\left(\frac{C}{C^*}\right)^{\sigma}\zeta_K\sigma \tag{D.103}$$

$$\tilde{\chi}_{qk} = -\frac{\alpha}{1-\alpha}\frac{WN}{PKq}\left[\frac{\varsigma_i(1+\phi)}{1-\alpha} - 1\right] \tag{D.104}$$

$$\chi_{q\lambda} = \lambda\left(\frac{C}{C^*}\right)^{\sigma}\zeta_K \tag{D.105}$$

$$\chi_{q\tau} = \frac{\alpha}{1-\alpha}\frac{WN}{PKq} + \lambda\left(\frac{C}{C^*}\right)^{\sigma}(1-\zeta_K) \tag{D.106}$$

$$\chi_{qq^*} = -\lambda\left(\frac{C}{C^*}\right)^{\sigma}(1-\zeta_K) \tag{D.107}$$

$$\chi_{qc^*} = -\lambda\left(\frac{C}{C^*}\right)^{\sigma}\zeta_K\sigma. \tag{D.108}$$

The corresponding equation in the disaster state is given by

$$\dot{\hat{q}}_t^* = (i_t^* - r_n^*) + \chi_{qc}c_t^* + \tilde{\chi}_{qk}k_t^* + \chi_{q^*q^*}\hat{q}_t^*, \tag{D.109}$$

where $\chi_{q^*q^*} = \chi_{qq} - \lambda\left(\frac{C}{C^*}\right)^{\sigma}(1-\zeta_K)$.

The law of motion of capital is given by

$$\dot{\hat{k}}_t = \chi_{kq}\hat{q}_t, \qquad \dot{\hat{k}}_t^* = \chi_{kq}\hat{q}_t^*, \tag{D.110}$$

where $\chi_{kq} \equiv \Phi'(\iota)\iota\chi_{\iota q}$.

Using the expression for c_t^* and for i_t^*, we obtain

$$\dot{\hat{q}}_t^* = \sigma(\omega_k\chi_{kq}\hat{q}_t^* - \omega_q\dot{\hat{q}}_t^*) + \chi_{qc}(\omega_k k_t^* - \omega_q\hat{q}_t^*) + \tilde{\chi}_{qk}k_t^* + \chi_{q^*q^*}\hat{q}_t^*. \tag{D.111}$$

Rearranging the expression above, we obtain

$$\begin{bmatrix} \dot{\hat{q}}_t^* \\ \dot{k}_t^* \end{bmatrix} = \begin{bmatrix} \tilde{\chi}_{q^*q^*} & \tilde{\chi}_{q^*k^*} \\ \tilde{\chi}_{k^*q^*} & 0 \end{bmatrix} \begin{bmatrix} \hat{q}_t^* \\ k_t^* \end{bmatrix}, \tag{D.112}$$

given $k_{t_0}^* = k_{t_0}$, where

$$\tilde{\chi}_{q^*q^*} \equiv \frac{\chi_{q^*q^*} - \chi_{qc}\omega_q}{1 + \sigma(\omega_q - \omega_k\chi_{kq})}, \qquad \tilde{\chi}_{q^*k^*} \equiv \frac{\chi_{qc}\omega_k + \tilde{\chi}_{qk}}{1 + \sigma(\omega_q - \omega_k\chi_{kq})}. \tag{D.113}$$

Assuming that the matrix above has a positive and a negative eigenvalue, the dynamic system above has a unique bounded solution given by $k_t^* = e^{-\psi_k(t-t_0)}k_{t_0}^*$ and $\hat{q}_t^* = \omega_{q^*k^*}e^{-\psi_k(t-t_0)}k_{t_0}^*$, where $\omega_{q^*k^*}$ is a constant that can be derived from the eigenvector associated with the negative eigenvalue. Similarly, we can write consumption in the disaster state as $c_t^* = \omega_{c^*k^*}k_t^*$.

We can then write the dynamics of $\hat{q}_t$ in the no-disaster state as follows:

$$\dot{\hat{q}}_t = (i_t - \pi_t - r_n - \dot{\hat{\tau}}_t^K) + \chi_{qq}\hat{q}_t + \chi_{qc}c_t + \chi_{qk}k_t + \chi_{q\lambda}\hat{\lambda}_t + \chi_{q\tau}\hat{\tau}_t^K, \tag{D.114}$$

where $\chi_{qk} \equiv \tilde{\chi}_{qk} + \chi_{qq^*}\omega_{q^*k^*} + \chi_{qc^*}\omega_{c^*k^*}$, using the fact that $k_{t^*}^* = k_{t^*}$.

D.3.4 Risk-premium neutrality

The dynamic system describing the evolution of the equilibrium variables in the no-disaster state is given by

$$\begin{bmatrix} \dot{c}_t \\ \dot{\pi}_t \\ \dot{\hat{q}}_t \\ \dot{k}_t \end{bmatrix} = \begin{bmatrix} \delta & \sigma^{-1}(\phi_\pi - 1) & 0 & \omega_{ck} \\ -\kappa & \rho & -\kappa\omega_q & \kappa\omega_k \\ \chi_{qc} & \phi_\pi - 1 & \chi_{qq} & \chi_{qk} \\ 0 & 0 & \chi_{kq} & 0 \end{bmatrix} \begin{bmatrix} c_t \\ \pi_t \\ \hat{q}_t \\ k_t \end{bmatrix} + \begin{bmatrix} \chi_{c\lambda}\hat{\lambda}_t + \sigma^{-1}(u_t - \dot{\hat{\tau}}_t^c) \\ 0 \\ \chi_{q\lambda}\hat{\lambda}_t + \chi_{q\tau}\hat{\tau}_t^K + u_t - \dot{\hat{\tau}}_t^K \\ 0 \end{bmatrix}, \tag{D.115}$$

given k_0, where $\omega_{ck} \equiv -\delta\omega_{c^*k^*}$. Notice that the system is independent of $\hat{\lambda}_t$ if the following conditions are satified:

$$\dot{\hat{\tau}}_t^c = \sigma\chi_{c\lambda}\hat{\lambda}_t, \qquad \dot{\hat{\tau}}_t^K = \chi_{q\lambda}\hat{\lambda}_t + \chi_{q\tau}\hat{\tau}_t^K. \tag{D.116}$$

Flexible-price allocation. Consider next the flexible-price allocation. We focus on the case where consumption and investment taxes are set to zero. Under flexible prices, consumption is given by

$$c_t = \omega_k k_t - \omega_q \hat{q}_t. \tag{D.117}$$

The real rate is given by

$$i_t - \pi_t - r_n = \sigma\left(\omega_k \dot{k}_t - \omega_q \dot{\hat{q}}_t - \delta(\omega_k k_t - \omega_q \hat{q}_t) - \omega_{ck} k_t - \chi_{c\lambda}\hat{\lambda}_t\right) \tag{D.118}$$

In this case, marginal q evolves according to

$$\begin{aligned} \dot{\hat{q}}_t &= (i_t - \pi_t - r_n) + \chi_{qq}\hat{q}_t + \chi_{qc}c_t + \chi_{qk}k_t + \chi_{q\lambda}\hat{\lambda}_t \\ &= \chi_{qq}^{FP}\hat{q}_t + \chi_{qk}^{FP}k_t + \left[\chi_{q\lambda} - \sigma\chi_{c\lambda}\right]\hat{\lambda}_t, \end{aligned} \tag{D.119}$$

where χ_{qq}^{FP} and χ_{qk}^{FP} capture the dependence of $\hat{q}_t$ on $\hat{q}_t$ and k_t under flexible prices. The coefficient on $\hat{\lambda}_t$ is given by

$$\chi_{q\lambda} - \sigma\chi_{c\lambda} = \lambda(1-\zeta_K)^{-\sigma}\zeta_K - \lambda\left[(1-\zeta_K)^{-\sigma} - 1\right] \tag{D.120}$$

$$= \lambda\left[1 - (1-\zeta_K)^{1-\sigma}\right]. \tag{D.121}$$

In the case of a unit EIS, $\sigma = 1$, the coefficient on $\hat{\lambda}_t$ is equal to zero. Therefore, the flexible price solution to capital and marginal q are simply $k_t = \hat{q}_t = 0$. In this case, the real rate is given by

$$i_t - r_n = -\lambda(1-\zeta_K)^{-1}\zeta_K\hat{\lambda}_t. \tag{D.122}$$

Equity prices are constant, $q_{E,t} = 0$, so the equity premium is given by

$$r_{E,t} - r_E = \lambda(1-\zeta_K)^{-1}\zeta_K\hat{\lambda}_t. \tag{D.123}$$

The price of long-term bonds satisfy the condition:

$$q_{L,0} = [r_E - r_L]\int_0^\infty e^{-(\rho+\psi_L)t}\hat{\lambda}_t dt, \tag{D.124}$$

where it can be shown that $r_E > r_L$.

The role of interest rate rule. We consider next the role of the interest rate rule in more detail. Assume that the monetary policy rule now responds to changes in $\hat{\lambda}_t$, that, it is now given by

$$i_t = r_n + \phi_\pi \pi_t + \phi_\lambda \hat{\lambda}_t + u_t, \tag{D.125}$$

for some $\phi_\lambda \in \mathbb{R}$. We assume that ϕ_π is such that the equilibrium is locally unique.

Consider again the system (D.115), setting all taxes to zero. The system of equations can then be written as

$$\dot{c}_t = \delta c_t + \sigma^{-1}\left(\phi_\pi - 1\right)\pi_t + \omega_{ck}k_t + \left(\sigma^{-1}\phi_\lambda + \chi_{c\lambda}\right)\hat{\lambda}_t + \sigma^{-1}u_t, \tag{D.126}$$

$$\dot{\pi}_t = -\kappa c_t + \rho\pi_t - \kappa\omega_q + \kappa\omega_k k_t, \tag{D.127}$$

$$\dot{\hat{q}}_t = \chi_{qc}c_t + \left(\phi_\pi - 1\right)\pi_t + \chi_{qq}\hat{q}_t + \chi_{qk}k_t + \left(\phi_\lambda + \chi_{q\lambda}\right)\hat{\lambda}_t + u_t, \tag{D.128}$$

$$\dot{k}_t = \chi_{kq}\hat{q}_t, \tag{D.129}$$

where $\chi_{c\lambda} = \frac{\lambda}{\sigma}\left[\left(\frac{C}{C^*}\right)^\sigma - 1\right]$, $\chi_{q\lambda} = \lambda\left(\frac{C}{C^*}\right)^\sigma \zeta_K$, and $\zeta_K = 1 - \frac{C^*}{C}$. It is immediate to see that if $\sigma = 1$ and $\phi_\lambda = -\chi_{q\lambda}$, the system of equations characterizing the equilibrium is independent of $\hat{\lambda}$.

Thus, there exists a monetary rule that makes the response of consumption, investment, output, and inflation to a monetary shock to be independent of movements in risk premia when the EIS = 1. However, the path of the nominal and real interest rates need to depend on the path of $\hat{\lambda}_t$. Hence, this "neutrality" result does not answer the question of what role do changes in asset prices play in the monetary transmission mechanism.

E Estimation of Fiscal Response to a Monetary Shock

We estimate the empirical IRFs using a VAR identified by a recursiveness assumption, as in Christiano, Eichenbaum and Evans (1999), extended to include fiscal variables. The variables included are: real GDP per capita, CPI inflation, real consumption per capita, real investment per capita, capacity utilization, hours worked per capita, real wages, tax revenues over GDP, government expenditures per capita, the federal funds rate, the 5-year constant maturity rate, and the real value of government debt per capita. We estimate a four-lag VAR using quarterly data for the period 1962:1-2007:3. The identification assumption of the monetary shock is as follows: the only variables that react contemporaneously to the monetary shock are the federal funds rate, the 5-year rate and the value of government debt. All other variables, including tax revenues and expenditures, react with a lag of one quarter. This assumption is the natural extension of Christiano et al. (1999) to a model with fiscal variables: while agents' decisions (in our case, households and the government) do not react to the shock contemporaneously, financial variables (in our case, the federal funds rate, the 5-year rate, and the value of government debt) immediately incorporate the information of the shock.

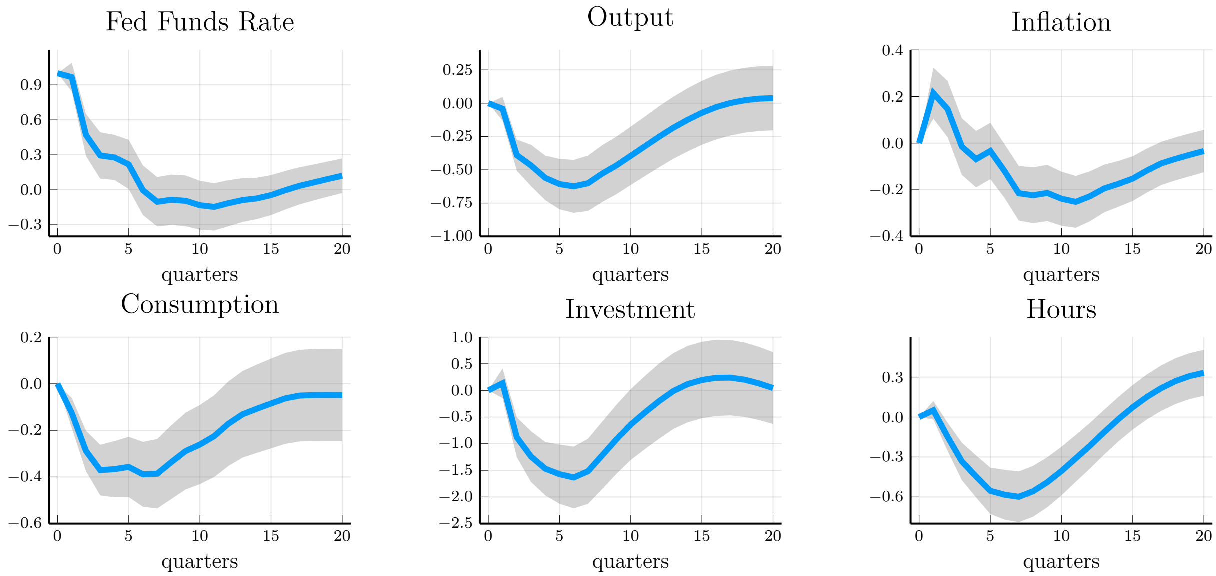


Figure E.1: Estimated IRFs.

Data sources. The data sources are: **Nominal GDP:** BEA Table 1.1.5 Line 1; **Real GDP:** BEA Table 1.1.3 Line 1, **Consumption Durable:** BEA Table 1.1.3 Line 4; **Consumption Non Durable:** BEA Table 1.1.3 Line 5; **Consumption Services:** BEA Table 1.1.3 Line 6; **Private Investment:** BEA Table 1.1.3 Line 7; **GDP Deflator:** BEA Table 1.1.9 Line 1; **Capacity Utilization:** FRED CUMFNS; **Hours Worked:** FRED HOANBS; **Nominal Hourly Compensation:** FRED COMPNFB; **Civilian Labor Force:** FRED CNP16OV; **Nominal Revenues:** BEA Table 3.1 Line 1; **Nominal Expenditures:** BEA Table 3.1 Line 21; **Nominal Transfers:** BEA Table 3.1 Line 22; **Nominal Gov't Investment:** BEA Table 3.1 Line 39; **Nominal Consumption of Net Capital:** BEA Table 3.1 Line 42; **Effective Federal Funds Rate (FF):** FRED FEDFUNDS; **5-Year Treasury Constant Maturity Rate:** FRED DGS5; **Market Value of Government Debt:** Hall, Payne and Sargent (2018).

All the variables are obtained from standard sources, except for the real value of debt, which we construct from the series provided by Hall et al. (2018). We transform the series into quarterly frequency by keeping the market value of debt in the first month of the

	(1) Revenues	(2) Interest Payments	(3) Transfers & Expenditures	(4) Debt in T	(5) Initial Debt	(1) - (2) - (3) + (4) - (5) Residual
Benchmark	10.54	36.2	2.68	1.42	-17.62	9.3
	[-14.11,35.18]	[20.07,52.33]	[-16.99,22.34]	[-14.77,17.61]	[-21.62,-13.63]	[-16.69,35.29]
Benchmark + EBP (shorter sample)	26.80	23.1	3.55	-7.4	-10.84	-3.6
	[10.6,42.82]	[8.18,38.02]	[-15.14,22.24]	[-30.1,15.3]	[-16.8,-4.88]	[-17.91,10.71]
Contemp. Output & Revenues	12.43	36.39	-0.02	4.03	-15.26	4.65
	[-12,36.85]	[19.73,53.05]	[-17.78,17.73]	[-11.11,19.18]	[-19.38,-11.14]	[-19.73,29.03]
Robustness 1	32.12	45.58	1.5	4.73	-16.26	-6.03
	[4.77,59.47]	[28.22,62.94]	[-18.37,21.36]	[-14.01,23.47]	[-20.09,-12.43]	[-32.81,20.75]
Contemp. Inflation	11.59	37.16	0.72	2.16	-18.37	5.77
	[-13.54,36.72]	[20.81,53.51]	[-18.33,19.77]	[-13.9,18.22]	[-22.68,-14.06]	[-20.24,31.77]
Robustness 2	11.59	37.16	0.72	2.16	-18.37	5.77
	[-13.54,36.72]	[20.81,53.51]	[-18.33,19.77]	[-13.9,18.22]	[-22.68,-14.06]	[-20.24,31.77]
Contemp. Output, Revenues & Infl.	15.32	38.67	-5	6.09	-16.66	-4.4
	[-8.62,39.27]	[22.67,54.67]	[-23.02,13.02]	[-9.39,21.57]	[-20.62,-12.7]	[-29.96,21.17]
Robustness 3	15.32	38.67	-5	6.09	-16.66	-4.4
	[-8.62,39.27]	[22.67,54.67]	[-23.02,13.02]	[-9.39,21.57]	[-20.62,-12.7]	[-29.96,21.17]

Table E.1: The impact on fiscal variables of a monetary policy shock

Note: Calculations correspond to a a 100 bps unanticipated interest rate increase. Confidence interval at 68% level.

quarter. This choice is meant to avoid capturing changes in the market value of debt arising from changes in the *quantity* of debt after a monetary shock instead of changes in *prices*.

VAR estimation. Figure E.1 shows the results. As is standard in the literature, we find that a contractionary monetary shock increases the federal funds rate and reduces output and inflation on impact. Moreover, the contractionary monetary shock reduces consumption, investment, and hours worked.

The Government's Intertemporal Budget Constraint. The fiscal response in the model corresponds to the present discounted value of transfers over an infinite horizon, that is, $\sum_{t=0}^{\infty} \tilde{\beta}^t T_t$, where $\tilde{\beta} = \frac{1-\lambda}{1+\rho_s}$. We next consider its empirical counterpart. First, we calculate a truncated intertemporal budget constraint from period zero to $\mathcal{T}$:

$$\underbrace{b_y b_0}_{\substack{\text{debt}\\\text{revaluation}}} = \sum_{t=0}^{\mathcal{T}} \tilde{\beta}^t \left[\underbrace{\tau y_t + \tau_t}_{\text{tax revenue}} - \underbrace{\tilde{\beta}^{-1} b_y (i^m_{t-1} - \pi_t - r^n)}_{\text{interest payments}} \right] - \underbrace{T_{0,\mathcal{T}} + \tilde{\beta}^{\mathcal{T}} b_y b_{\mathcal{T}}}_{\substack{\text{other transfers/expenditures}\\ \text{\& final debt}}} \tag{E.1}$$

The right-hand side of (E.1) is the present value of the impact of a monetary shock on fiscal accounts. The first term represents the change in revenues that results from the real effects of monetary shocks. The second term represents the change in interest payments on government debt that results from change in nominal rates. The last two terms are adjustments in transfers and other government expenditures, and the final debt position at period $\mathcal{T}$, respectively. In particular, $T_{0,\mathcal{T}}$ represents the present discounted value of transfers from period 0 through $\mathcal{T}$. Provided that $\mathcal{T}$ is large enough, such that (y_t, τ_t, i_t) have essentially converged to the steady state, then the value of debt at the terminal date, $b_{\mathcal{T}}$, equals (minus) the present discounted value of transfers and other expenditures from period $\mathcal{T}$ onward. Hence, the last two terms combined can be interpreted as the present discounted value of fiscal transfers from zero to infinity. Finally, the left-hand side represents the revaluation effect of the *initial* stock of government debt.

Table E.1 shows the impact on the fiscal accounts of a monetary policy shock, both in the data and in the estimated model. We first apply equation (E.1) to the data and check whether the difference between the left-hand side and the right-hand side is different from zero. The residual is calculated as

Residual = Revenues - Interest Payments - Transfers + Debt in $\mathcal{T}$ - Initial Debt

We truncate the calculations to quarter 60, that is, $\mathcal{T} = 60$ (15 years) in equation (E.1). The results reported in Table E.1 imply that we cannot reject the possibility that the resid-

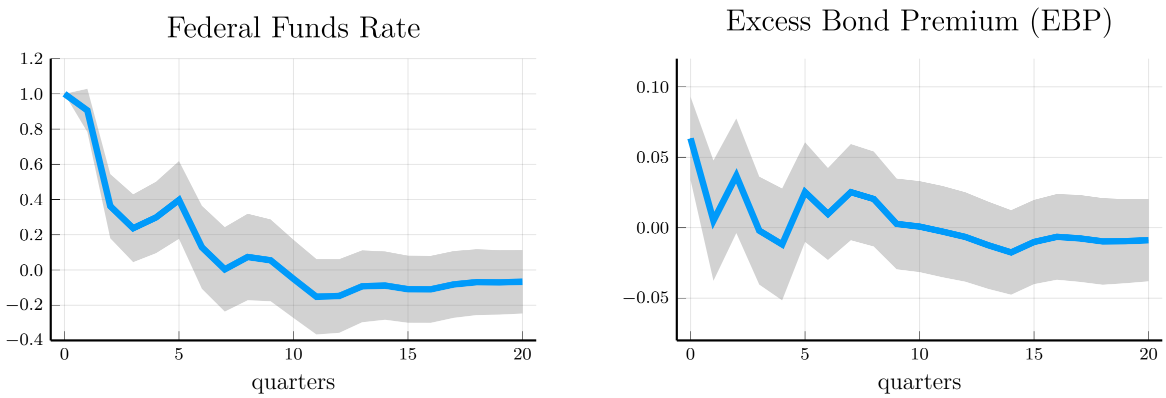


Figure E.2: IRFs for the federal funds rate and excess bond premium.

ual is zero and, therefore, we cannot reject the possibility that the intertemporal budget constraint of the government is satisfied in our estimation.

The adjustment of the fiscal accounts in the data corresponds to the patterns we observed in Figure 2. The response of initial debt is quantitatively important, and it accounts for the bulk of the adjustment in the fiscal accounts.

EBP. To estimate the response of the corporate spread in the data, we add the EBP measure of Gilchrist and Zakrajšek (2012) into our VAR (ordered after the fed funds rate). Since the EBP is only available starting in 1973, we reduce our sample period to 1973:1-2007:7. The estimated IRFs are in line with those obtained for the longer sample. We find a significant increase of the EBP on impact, of 6.5 bps, in line with the estimates in the literature. Moreover, Table E.1 shows that the estimated impact of the monetary shock on the fiscal accounts is in the ballpark of the benchmark case.

Robustness. To evaluate the sensitivity of our results to different identification assumptions, we consider alternative exercises that also impose the recursiveness assumption. We analyze three main specifications: i) output and revenues are allowed to respond contemporaneously to the monetary shock, ii) inflation is allowed to respond contemporaneously to the monetary shock, *iii*) output, revenues, and inflation are allowed to respond contemporaneously to the shock.

Table E.1 summarizes the results. The estimated effect of a monetary shock on fiscal variables is nearly identical across all cases. The implied response of the primary surplus ranges from 9 bps to 26 bps. While the upper bound is about three times larger than in our benchmark case, it remains orders of magnitude smaller than the fiscal backing implied by the MSV equilibrium.