Monetary Policy and Wealth Effects: The Role of Risk and Heterogeneity *

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Abstract
We study the role of wealth effects, i.e. the revaluation of stocks, bonds, and human wealth, in the monetary policy transmission mechanism. The analysis of wealth effects requires to incorporate realistic asset-pricing dynamics and heterogeneous households’ portfolios. Thus, we build an analytical heterogeneous-agents model with two main ingredients: i) rare disasters and ii) positive private debt. The model captures time-varying risk premia and precautionary savings in a linearized setting that nests the textbook New Keynesian model. Quantitatively, the model matches the empirical response of asset prices as well as the heterogeneous impact on borrowers and savers. We find that wealth effects induced by time-varying risk and private debt account for the bulk of the output response to monetary policy.

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1 Introduction

A long tradition in monetary economics emphasizes the role of wealth effects, i.e. the revaluation of real and financial assets, in the economy’s response to changes in monetary policy. Its importance can be traced back to both classical and Keynesian economists, such as Pigou, Patinkin, Metzler and Tobin.\(^1\) Keynes himself described the effects of interest rate changes as follows:

> There are not many people who will alter their way of living because the rate of interest has fallen from 5 to 4 per cent, if their aggregate income is the same as before. [...] Perhaps the most important influence, operating through changes in the rate of interest, on the readiness to spend out of a given income, depends on the effect of these changes on the appreciation or depreciation in the price of securities and other assets.

Recently, wealth effects have regained relevance. In an influential paper, Kaplan et al. (2018) build a quantitative heterogeneous-agents New Keynesian (HANK) model and find only a small role for the standard intertemporal-substitution channel, leading the way to a more important role for wealth effects.\(^2\) Any meaningful attempt to study wealth effects requires, however, bringing to the forefront the role of asset prices and the heterogeneity in households’ portfolios.\(^3\) This represents a challenge for standard New Keynesian models, as they fail to generate realistic asset-pricing movements, and for monetary models with richer asset pricing, as they require the use of complex global or higher-order perturbation methods. In this paper, we propose a new framework that generates rich asset-pricing dynamics and heterogeneous portfolios while preserving the simplicity of the New Keynesian model, allowing us to study the role of wealth effects in the response of the economy to monetary policy.

We build an analytical HANK model with two main ingredients: i) rare disasters and ii) positive private debt. Rare disasters allow us to capture both a precautionary savings motive and realistic risk premia. Private debt is an important component of households’ portfolios, representing 75% of GDP, and, as recently shown by Cloyne et al. (2020), borrowers account for the bulk of the response of aggregate consumption to changes in interest rates. Thus, by incorporating private debt, we are able to capture the role of revaluations in both gross and net asset positions.

The paper makes two main contributions. First, we provide a tractable unifying framework to study the role of risk and household heterogeneity in the monetary transmission mechanism. We obtain time-varying risk premia without the need for higher-order perturbation techniques, which allows us to provide a complete analytical characterization of the channels involved. Moreover, we capture key features of HANK models, such as precautionary savings and heterogeneous

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\(^1\)The revaluation of government liabilities was central to Pigou (1943) and Patinkin (1965), while Metzler (1951) considered stocks and money. Tobin (1969) focused on how monetary policy interacted with the value of real assets.

\(^2\) Cieslak and Vissing-Jorgensen (2020) show that policymakers track the behavior of stock markets because of its consumption wealth effect, while Chodorow-Reich et al. (2019) empirically establish the importance of this channel.

\(^3\)The effect of monetary policy on stock prices is considered by e.g. Bernanke and Kuttner (2005) and Kekre and Lenel (2020), while the effect on bonds is studied by e.g. Gertler and Karadi (2015) and Hanson and Stein (2015). The role of heterogeneous portfolios and the associated redistribution channel was originally considered by Auclert (2017).
marginal propensities to consume (MPCs), in a setting with positive private debt, a combination that has been elusive in the analytical HANK literature. Second, we quantify the importance of risk and private debt. We show that the model quantitatively captures key features of the monetary transmission mechanism, including important asset-pricing moments, such as the term premium, the equity premium, and corporate spreads, as well as the differential responses of borrowers and savers to monetary shocks observed in the data. When we decompose the total response of output to monetary policy, we find that time-varying risk explains roughly 50% of the output response, while the presence of private debt accounts for 30% of the response. Risk and household heterogeneity combined are, therefore, major drivers of the economy’s response to monetary policy.

We begin our analysis by considering an economy populated by two types of households, borrowers and savers, where borrowers are relatively impatient. Households are subject to borrowing constraints, and, in equilibrium, borrowers will be constrained at all times. By allowing households to borrow positive (but limited) amounts, we depart from most of the analytical HANK literature that focuses on the case of zero private liquidity. The zero liquidity assumption allowed the analytical literature to capture two key features of quantitative HANK models: a precautionary savings motive and heterogeneous MPCs. We capture these same two features in an economy with positive private debt by introducing an aggregate disaster risk, where the productivity of the economy is permanently reduced after a shock hits, as in the work of Barro (2006, 2009). Moreover, this formalization allows us to effectively discipline the magnitude of the precautionary savings motive with asset-pricing data.

We then study the impact of monetary shocks by perturbing the economy around a stationary equilibrium with positive aggregate risk instead of adopting the more common approach of approximating around a non-stochastic steady state. By perturbing around the stochastic stationary equilibrium, we are able to obtain time variation in precautionary motives and risk premia using a first-order approximation, while the standard approach would require a third-order approximation (see e.g. Andreasen (2012)). Moreover, by linearizing around an economy with zero monetary risk, we are able to solve for the stochastic stationary equilibrium in closed form, avoiding the need to compute the risky steady state numerically, as in Coeurdacier et al. (2011). This hybrid approach allows us to capture the effect of aggregate risk on asset prices in a linearized model.

Our first result states that output satisfies an aggregate Euler equation, where its sensitivity to interest rates depends on the disaster risk and on the level of private debt. With zero private liquidity and constant disaster probability, our economy features a discounted Euler equation, where output is less sensitive to future interest rate changes due to a precautionary motive, as in the incomplete-markets model of McKay et al. (2017). The presence of private debt acts in the op-

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4Rare disasters have been widely used to explain a range of asset-pricing facts; see e.g. Rietz (1988), Barro (2006), Gabaix (2008), Wachter (2013), Farhi and Gabaix (2016), and Barro and Liao (2020).
posite direction, as it pushes the economy towards *compounding* in the Euler equation, even with acyclical income inequality. We find that the second effect dominates in our calibration, so the aggregate Euler equation features compounding, even though, at the micro level, savers’ Euler equation always features discounting.

We turn next to the channels through which monetary policy affects the economy. We show that equilibrium output can be characterized as the sum of four terms: the *intertemporal-substitution effect* (ISE); the *inside wealth effect*, i.e. the change in valuation of assets in zero net supply; the *outside wealth effect*, i.e. changes in the valuation of assets in positive net supply; and a *time-varying risk effect*. For exposition purposes, we start by considering the case of a fiscally active regime, in the sense of Leeper (1991), and later show that all our analytical results extend to the more standard active monetary regime.

The ISE corresponds to the output response that operates through changes in the *timing* of output but not its overall (present value) level. While this channel is quantitatively important in the textbook New Keynesian model, we find that it has a marginal impact in the presence of heterogeneous agents and risk.

Most of the response in the economy can be explained by wealth effects and the associated time-varying risk effect. The inside wealth effect corresponds to a channel that is present only with heterogeneous MPCs and positive private debt. It captures the aggregate implications of the differential response of borrowers and savers to changes in interest payments. An increase in nominal interest rates creates a positive wealth effect on savers, as they receive a higher income from private lending, and a corresponding loss to borrowers. Given the higher MPC for borrowers, this generates a negative aggregate response of output on impact.

The outside wealth effect is the sum of the change in wealth for all households in the economy. This includes the change in the value of stocks, government bonds, and human wealth, *net* of the impact of discount rates on the present discounted value of consumption. We find that the outside wealth effect interacts with the presence of private debt in interesting ways. As we mentioned above, private debt introduces a force towards compounding in the Euler equation. This compounding *amplifies* the effect of changes in the value of households’ wealth on equilibrium output, analogously to the *debt-deflation effect*. Lower asset prices reduce aggregate demand, which lowers output and inflation. Lower inflation increases the real burden on borrowers, generating an extra effect on aggregate demand.

An important result of our analysis is that the outside wealth effect is tightly connected to the response of fiscal policy to monetary shocks. In particular, we show that the outside wealth effect is proportional to the revaluation of public debt and the fiscal backing, that is, the change in taxes

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5 The notion of inside/outside wealth is reminiscent of inside/outside money as used by Gurley and Shaw (1960), and, more recently, inside/outside liquidity by Holmstrom and Tirole (2011).

6 The fiscal regime allows us to isolate the impact of monetary policy on households’ balance sheets from the associated fiscal backing that typically operates in active monetary regimes. See Leeper and Leith (2016) for a discussion.

7 Note that previous analytical HANK models focused on either the case of heterogeneous MPC but no private debt, as in Bilbiie (2018), or positive private debt and no heterogeneity in MPC, as in Acharya and Dogra (2020).
and transfers in response to monetary shocks. Intuitively, in a closed economy, the government is the only trading counterpart to the household sector as a whole, so the outside wealth effect can be inferred from the impact of monetary policy on government finances. More importantly, this result implies that we can use standard VAR techniques to identify the fiscal response to a monetary shock and discipline the ability of the model to generate quantitatively meaningful wealth effects. These findings have important implications for the quantitative assessment of monetary models. We find that when constrained to match the estimated fiscal response, the standard RANK model generates a substantially weaker output response to monetary shocks than when fiscal backing is determined by a Taylor rule. Equivalently, these results imply that the standard Taylor equilibrium requires a (passive) fiscal response that is counterfactually large. It is in this context that the presence of heterogeneity and risk becomes particularly relevant, as these forces can compensate for the missing fiscal response.

Finally, we consider the role of time-varying risk, which has a significant impact on how output responds to monetary shocks. First, we study the case of a constant disaster probability. Even though the model is able to capture important unconditional asset-pricing moments, such as the level of the equity premium and an upward-sloping yield curve, it fails to generate the observed response of risk premia to monetary shocks. This failure has important real consequences, as aggregate risk has then only a minor impact on the response of output and inflation. When we introduce time-varying risk, allowing the model to simultaneously match how long-term bonds, corporate spreads, and equities respond to monetary shocks in the data, the impact on output increases almost threefold. This highlights the importance of matching the empirical response of asset prices to properly assess the role of risk in determining how monetary policy affects the economy.

To quantify the importance of the channels that are present in the model, we decompose the response of output by sequentially adding time-varying risk and private debt to the standard RANK model. We find that adding time-varying risk accounts for more than 50% of the overall output response, while private debt accounts for roughly 30%. Moreover, we find that time-varying risk has a larger impact on the economy in the presence of private debt and vice versa, showing the importance of considering risk and heterogeneity simultaneously.

**Literature review.** Wealth effects have a long tradition in monetary economics. Pigou (1943) relied on a wealth effect to argue that full employment could be reached even in a liquidity trap. Kalecki (1944) argued that these effects apply only to government liabilities, as inside assets cancel out in the aggregate, while Tobin highlighted the role of private assets and high-MPC borrowers.

Our work is closely related to two strands of literature. First, it relates to the analytical HANK

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8Tobin (1982) describes the role of inside assets: “The gross amount of these ‘inside’ assets was and is orders of magnitude larger than the net amount of the base. Aggregation would not matter if we could be sure that the marginal propensities to spend from wealth were the same for creditors and debtors. But if the spending propensity were systematically greater for debtors, even by a small amount, the Pigou effect would be swamped by this Fisher effect.”
literature, such as Werning (2015), Debotoli and Gali (2017), and Bilbiie (2018, 2019). While this literature focuses mostly on how the cyclicality of income interacts with differences in MPCs, we focus instead on how heterogeneous asset positions interact with differences in MPCs. We see these two channels as mostly complementary: even though Cloyne et al. (2020) do not find significant differences in income sensitivity across borrowers and savers, Patterson (2019) found a positive covariance between MPCs and the sensitivity of earnings to GDP across different demographic groups, suggesting that the income-sensitivity channel is operative for a different cut of the data. We share with Eggertsson and Krugman (2012) and Benigno et al. (2020) the emphasis on private debt, but they abstract from a precautionary motive and focus instead on the implications of deleveraging. Iacoviello (2005) also considers a monetary economy with private debt, but focuses instead on the role of housing as collateral.

Second, our paper is also closely related to work on how monetary policy affects the economy through changes in asset prices, including models with sticky prices, such as Caballero and Simsek (2020), and models with financial frictions, such as Brunnermeier and Sannikov (2016), Drechsler et al. (2018), and Di Tella (2019). In recent contributions, Kekre and Lenel (2020) consider the role of the marginal propensity to take risk in determining the risk premium and shaping the response of the economy to monetary policy, and Campbell et al. (2020) use a habit model to study the role of monetary policy in determining bond and equity premia. Our model highlights instead the role of heterogeneous MPCs, positive private liquidity, and disaster risk in an analytical framework that preserves the tractability of standard New Keynesian models.

A recent literature studies rare disasters and business cycles. Gabaix (2011) and Gourio (2012) consider a real business cycle model with rare disasters, while Andreasen (2012) and Isoré and Szczerbowicz (2017) allow for sticky prices. They focus on the effect of changes in disaster probability, while we study monetary shocks in an analytical HANK model with rare disasters.

Our result regarding how asset revaluations depend on fiscal variables is related to work on fiscal policy and asset prices. Croce et al. (2012) and Gomes et al. (2013) study how fiscal policy affects asset prices in neoclassical economies, while Jiang (2019) and Corhay et al. (2018) study exchange rates and bond returns, respectively, in a fiscally active regime.

Outline. The paper is organized as follows. Section 2 presents the model used in the analysis. It shows how heterogeneity, positive private liquidity, and risk feed into the the aggregate Euler equation. In Section 3 we study the equilibrium dynamics, focusing on the determination of inside and outside wealth effects. We turn to the role of risk in Section 4. We conclude in Section 5.

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9 A related literature focuses instead on the revaluation of housing, as in Berger et al. (2018) and Guren et al. (2018).
2  D-HANK: An Analytical Rare Disasters HANK Model

In this section, we consider an analytical HANK model with two main ingredients: the possibility of rare disasters and positive private liquidity. By introducing aggregate disaster risk, instead of the commonly adopted idiosyncratic income risk, we are able to capture a precautionary savings motive and an explicit role for liquidity in a setting with heterogeneous MPCs without having to keep track of a non-degenerate distribution of wealth.

2.1 The Model

Environment. Time is continuous and denoted by $t \in \mathbb{R}_+$. The economy is populated by households, firms, and a government. There are two types of households, borrowers and savers, who differ in their discount rates. A mass $0 \leq \mu_b < 1$ of households are borrowers and a mass $\mu_s = 1 - \mu_b$ are savers. Households can borrow or lend at a riskless rate, but they are subject to a borrowing constraint.

Firms can produce final or intermediate goods. Final-goods producers operate competitively and combine intermediate goods using a CES aggregator with elasticity $\epsilon > 1$. Intermediate-goods producers use labor as their only input and they face Rotemberg pricing adjustment costs.\(^{10}\) Intermediate-goods producers are subject to an aggregate productivity shock: with Poisson intensity $\lambda \geq 0$, they receive a shock that permanently reduces their productivity. This shock is meant to capture the possibility of rare disasters: low-probability, large drops in productivity and output, as in the work of Barro (2006, 2009). We say that periods that predate the realization of the shock are in the no-disaster state, and periods that follow the shock are in the disaster state. The disaster state is an absorbing state, and there are no further shocks after the disaster is realized.\(^{11}\)

The government sets fiscal policy, comprising a sales tax on intermediate-goods producers and transfers to borrowers and savers, and monetary policy, specified by a policy rule subject to a sequence of monetary shocks.

Households’ problem. A type-$j$ household, $j \in \{b, s\}$, chooses consumption $C_{j,t}$, and labor supply $N_{j,t}$, given an initial real value of riskless bonds $B_{j,t}$, to solve the following problem:

$$V_{j,t}(B_{j,t}) = \max_{[C_{j,t}, N_{j,t}] \geq 0} \mathbb{E}_t \left[ \int_t^{t^*} e^{-\rho(t-z)} \left( \frac{C_{j,t}^{1-\sigma}}{1-\sigma} - \frac{N_{j,t}^{1+\phi}}{1+\phi} \right) dz + e^{-\rho(t^*-t)} V_{j,t^*}(B_{j,t^*}) \right],$$

\(^{10}\)Rotemberg costs simplify the derivations in the disaster state, but they are not essential for our results.

\(^{11}\)Assuming an absorbing disaster state simplifies the presentation, but it can be easily relaxed, as shown in Appendix A.2. Allowing for partial recovery after a disaster, as in Barro et al. (2013) and Gourio (2012), introduces dynamics in the disaster state, but it does not change the main implications for the no-disaster state, which is our focus.
subject to the flow budget constraint

\[ \dot{B}_{j,t} = (i_t - \pi_t)B_{j,t} + \frac{W_t}{P_t} N_{j,t} + \Pi_{j,t} + \tilde{T}_{j,t} - C_{j,t} \]

and the borrowing constraint

\[ B_{j,t} \geq -\mathcal{D}_p, \]

where \( \rho_b > \rho_s > 0 \), \( W_t \) is the nominal wage, \( P_t \) is the price level, \( \Pi_{j,t} \) denotes real profits from corporate holdings, and \( \tilde{T}_{j,t} \) denotes government transfers. The random (stopping) time \( t^* \) represents the period in which the aggregate shock hits the economy. \( V_{j,t}^* (\cdot) \) and \( B_{j,t}^* \) denote, respectively, the value function and the value of bonds in the disaster state, where the star superscript is used throughout the paper to denote variables in the disaster state.

We assume that \( B_{b,0} > 0 \) and \( B_{b,0} = -\mathcal{D}_p \). For sufficiently large \( \rho_b \), borrowers are then constrained in all periods. We also assume that \( \Pi_{b,t} = 0 \), that is, firms are entirely owned by savers.\(^\text{12}\)

In Appendix A, we show that the labor supply is determined by the standard condition

\[ \frac{W_t}{P_t} = N_{j,t}^\phi C_{j,t}^r \]

and the Euler equation, if \( B_{j,t} > -\mathcal{D}_p \), is given by

\[ \frac{\dot{C}_{j,t}}{C_{j,t}^t} = \sigma^{-1} (i_t - \pi_t - \rho_j) + \frac{\lambda}{\sigma} \left[ \left( \frac{C_{j,t}^\sigma}{C_{j,t}^\rho} \right)^\sigma - 1 \right], \tag{1} \]

where \( C_{j,t}^t \) is the consumption of household \( j \) in the disaster state.\(^\text{13}\) The first term captures the usual intertemporal-substitution force present in RANK models. The second term captures the precautionary savings motive generated by disaster risk, and it is analogous to the precautionary motive that emerges in HANK models with idiosyncratic risk.

**Firms’ problem.** Intermediate-goods producers are indexed by \( i \in [0,1] \) and operate the linear technology \( Y_{i,t} = A_t N_{i,t} \). Productivity in the no-disaster state is given by \( A_t = A \) and productivity in the disaster state is given by \( A_t = A^* \), where \( 0 < A^* < A \). Intermediate-goods producers choose the rate-of-change of prices \( \pi_{i,t} = \dot{P}_{i,t} / P_{i,t} \) given the initial price \( P_{i,0} \), to maximize the expected discounted value of real (after-tax) profits subject to Rotemberg quadratic adjustment costs:

\[ Q_{i,t}(P_{i,t}) = \max_{|\pi_{i,t}| \leq \tau} \mathbb{E}_{t} \left[ \int_t^{t^*} \frac{\pi_{i,t}}{\eta_t} \left( \frac{1}{1 - \tau} \frac{P_{i,t}^z}{P_z} Y_{i,z} - \frac{W_z Y_{i,z}}{P_z} - \frac{\phi}{2} \frac{\pi_{i,z}^2}{\pi_{i,z}} \right) dz + \frac{\eta_{t^*}}{\eta_t} Q_{i,t^*}^+(P_{i,t^*}) \right], \tag{2} \]

\(^\text{12}\)Alternatively, we could have assumed that households can trade shares of the firms. In steady-state, borrowers would choose to sell their shares and firms would be entirely held by savers.

\(^\text{13}\)In discrete time, we obtain \( C_{j,t}^\sigma = (1 - \rho_j / (1 + \eta_t)) \left[ 1 - \lambda \Delta t \right] C_{j,t+\Delta t}^{-\sigma} + \lambda \Delta t (C_{j,t+\Delta t}^\sigma - C_{j,t+\Delta t}^{-\sigma}) \). After some rearrangement, we get \( \frac{C_{j,t+\Delta t}^\sigma - C_{j,t}^\sigma}{\Delta t} = -(\rho_j / (1 + \eta_t)) C_{j,t+\Delta t}^{-\sigma} - \lambda ((C_{j,t+\Delta t}^\sigma - C_{j,t+\Delta t}^{-\sigma}) + o(\Delta t) \), which gives equation (1) as \( \Delta t \to 0. \)
subject to the demand $Y_{i,t} = \left( \frac{P_{i,t}}{P_{r,t}} \right)^{-\varepsilon} Y_t$ and $\hat{\pi}_{i,t} = \pi_{i,t} P_{i,t}$, where $\eta_t$ denotes the stochastic discount factor (SDF) that is relevant to firms and $Q_{i,t}(P_t)$ denotes the firms’ value function in the disaster state. Note that the price level is a state variable in the firms’ problem and $\pi_{i,t}$ is a control variable. The parameter $\varphi$ controls the magnitude of the pricing adjustment costs. We assume that these costs are rebated to households, so they do not represent real resource costs. Moreover, as firms are owned by savers, we assume that firms discount profits using the SDF $\eta_t = e^{-\rho_t} C_{s,t}^{-\tau}$.

Combining the first-order condition and the envelope condition for problem (2), we obtain the non-linear New Keynesian Phillips curve

$$\pi_t = \left( i_t - \pi_t + \lambda \frac{\eta_t}{\eta_t} \right) \pi_t - \varphi^{-1}(\varepsilon - 1) \left( \frac{e^{-\rho_t} W_t}{\pi_t A} - (1 - \tau) \right) Y_t,$$  

assuming a symmetric initial condition $P_{i,0} = P_{0r}$ for all $i \in [0, 1]$.

**Government.** Government transfers to borrowers are determined by the policy rule $\bar{T}_{b,t} = \bar{T}_b(Y_t)$, where transfers depend on aggregate output and the elasticity of $\bar{T}_b(\cdot)$ determines the cyclicity of government transfers to borrowers and, ultimately, the cyclicity of borrowers’ consumption. The government’s flow budget constraint in the no-disaster state is given by

$$\bar{D}_{g,t} = (i_t - \pi_t) D_{g,t} + \sum_{j \in \{b,s\}} \mu_j \bar{T}_{j,t} - \tau Y_t,$$

and the No-Ponzi condition $\lim_{t \to \infty} \mathbb{E}_0[\eta_t D_{g,t}] \leq 0$, where $D_{g,t}$ denotes the real value of government debt and $D_{g,0} = D_g$ is given.

In the no-disaster state, monetary policy is determined by the policy rule

$$i_t = r_n + \varphi_{\pi} \pi_t + u_t,$$  

where $\varphi_{\pi} \geq 0$, $u_t$ represents monetary shocks, and $r_n$ denotes the real rate when $\pi_t = u_t = 0$ at all periods. In the disaster state, we assume that there are no monetary shocks, that is, $i_t^* = r_n^* + \varphi_{\pi} \pi_t^*$. By abstracting from the policy response after a disaster, we isolate the impact of changes in monetary policy during "normal times."

**Market clearing.** The market-clearing conditions for goods, labor, and bonds are given by

$$\sum_{j \in \{b,s\}} \mu_j C_{j,t} = Y_t, \quad \sum_{j \in \{b,s\}} \mu_j N_{j,t} = N_t, \quad \sum_{j \in \{b,s\}} \mu_j B_{j,t} = D_{g,t}.$$

2.2 Equilibrium dynamics

**Stationary equilibrium.** We define a stationary equilibrium as an equilibrium in which all variables are constant in each aggregate state. In particular, the economy will be in a stationary equi-
librium in the absence of monetary shocks, that is, \( u_t = 0 \) for all \( t \geq 0 \). Since variables are constant in each state, we drop time subscripts and write, for instance, \( C_{j,t} = C_j \) and \( C_{j,t}^* = C_j^* \). For ease of exposition, we follow Bilbiie (2019) and focus on a symmetric stationary equilibrium, where \( \tilde{T}_p \) implements the same consumption level for each household, and discuss the general case \( C_b \neq C_s \) in the appendix.

The natural interest rate, the real rate in the stationary equilibrium, is given by

\[
 r_n = \rho_s - \lambda \left[ \left( \frac{C_s}{C_s^*} \right)^\sigma - 1 \right],
\]

where \( 0 < C_s^* < C_s \). The presence of a precautionary motive depresses the natural interest rate relative to the one that would prevail in a non-stochastic economy. Moreover, we show in Appendix A.2 that the precautionary motive depends on the extent to which savers can self-insure. In particular, holding everything else constant, a higher level of private debt \( D_p \) implies a weaker precautionary motive and a higher natural interest rate. Public bonds do not serve the same role, as they are financed by taxes on savers.

**Log-linear dynamics.** Following the practice in the literature on monetary policy, we focus on a log-linear approximation of the equilibrium conditions. However, instead of linearizing around the non-stochastic steady state, we linearize the equilibrium conditions around the (stochastic) symmetric stationary equilibrium described above.\(^{14}\) Therefore, we are able to capture the effects of (time-varying) precautionary savings and risk premia in a linear setting.

Let lower-case variables denote log-deviations from the stationary equilibrium, e.g., \( c_{j,t} \equiv \log C_{j,t}/C_j \) and \( n_{j,t} \equiv \log N_{j,t}/N_j \). Borrowers’ consumption is given by

\[
c_{b,t} = (1 - \alpha)(w_t - p_t + n_{b,t}) + T_{b,t} - (i_t - \pi_t - r_n)d_p,
\]

where \( 1 - \alpha \equiv \frac{\text{WN}}{\text{PR}} \) is the labor share in the stationary equilibrium, \( T_{j,t} \equiv \frac{T_{j,t} - \tilde{T}_t}{T_t} \), and \( d_p \equiv \frac{D_p}{Y} \).

Using the fact that transfers satisfy \( T_{b,t} = T_{b}(Y)y_t \), and solving for the real wage, we obtain

\[
c_{b,t} = \chi_y y_t - \chi_i (i_t - \pi_t - r_n),
\]

where

\[
\chi_y \equiv \frac{T_{b}(Y) + (1 - \alpha)(1 + \phi)(1 + \phi^{-1}\sigma)}{1 + (1 - \alpha)\phi^{-1}\sigma}, \quad \chi_i \equiv \frac{d_p}{1 + (1 - \alpha)\phi^{-1}\sigma}.
\]

The coefficient \( \chi_y \) controls the cyclicality of income inequality and has been extensively studied by

\(^{14}\)Formally, we perturb the allocation around the economy where \( u_t = 0 \), while the standard approach would perturb around the economy where \( u_t = \lambda = 0 \). In particular, our method differs from the perturbation procedure considered by Fernández-Villaverde and Levintal (2018). It is also distinct from Coeurdacier et al. (2011), as we linearize around a stochastic steady state of an economy with no monetary shocks, instead of the stochastic steady state of the economy with both shocks. For a discussion of perturbation methods with jump processes, see Judd (1998).
the literature on analytical HANK models. We focus throughout the paper on the case in which
0 < μ_y < μ_b^{-1}, such that the consumption of both agents increases with y_t. The second term
is not present in the commonly studied case of zero private liquidity, d_p = 0, and it captures the
impact of monetary policy on the consumption of constrained agents that is not directly mediated
by aggregate output y_t. The coefficient χ_r plays an important role in the analysis that follows.

Savers’ Euler equation is given by

\[ c_{s,t} = \sigma^{-1}(i_t - \pi_t - r_n) + \lambda \left( \frac{C_s}{C^*_s} \right)^\sigma c_{s,t}. \]  

(6)

Combining condition (5) for borrowers’ consumption, equation (6) for the savers’ Euler equation,
and the market-clearing condition for goods, we can derive the evolution of aggregate output.
Proposition 1 characterizes the dynamics of aggregate output and inflation.

**Proposition 1** (Aggregate dynamics). The dynamics of output and inflation is described by the conditions

i. Aggregate Euler equation:

\[ \dot{y}_t = \bar{\sigma}^{-1}(i_t - \pi_t - r_n) + \delta y_t + v_t, \]  

(7)

where \( \bar{\sigma}^{-1}, \delta, \) and \( v_t \) are given by

\[ \bar{\sigma}^{-1} \equiv \frac{(1 - \mu_b)\sigma^{-1} - \mu_b \chi_y \rho}{1 - \mu_b \chi_y}, \quad \delta \equiv \lambda \left( \frac{C_s}{C^*_s} \right)^\sigma - \frac{\mu_b \chi_y}{1 - \mu_b \chi_y}, \quad v_t \equiv \frac{\mu_b \chi_y}{1 - \mu_b \chi_y} (\rho(i_t - r_n) - i_t). \]

ii. New Keynesian Phillips curve:

\[ \pi_t = \rho \pi_t - \kappa y_t, \]  

(8)

where \( \rho \equiv \rho_s + \lambda \) and \( \kappa \equiv \phi^{-1}(\varepsilon - 1)(1 - \tau)(\phi + \sigma). \)

**Proof.** See Appendix B.1.

Condition (7) represents the aggregate Euler equation for this economy. The aggregate Euler
equation has three terms. The first term, the product of the (aggregate) elasticity of intertemporal
substitution (EIS) and the real interest rate, corresponds to the one present in RANK models. The
dependence of the aggregate EIS on the cyclicality of inequality is well-known in the literature,
while the result that private liquidity may reduce \( \bar{\sigma}^{-1} \) is, to the best of our knowledge, new.\(^{16}\)

The second term, \( \delta y_t \), captures how the impact of real interest rate changes can be compounded
or discounted in equilibrium. This can be seen more clearly in the case of zero private liquidity,

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\(^{15}\) The role of \( \chi_y \), including the case where \( \chi_y > \mu_b^{-1} \), was originally considered by Bilbiie (2008). The cyclicality of income inequality also plays an important role in the aggregation results in Werning (2015) and Bilbiie (2018).

\(^{16}\) In the calibration for the numerical exercises, we obtain \( \bar{\sigma}^{-1} > 0 \). However, most of our results do not rely on this.
\( \chi_r = 0 \), such that \( v_t = 0 \) for all \( t \) and \( \delta > 0 \). Integrating the Euler equation forward, we obtain the expression

\[
y_t = -\delta^{-1} \int_t^\infty e^{-\delta(s-t)}(i_s - \pi_s - r_n) ds.
\]

This corresponds to the discounted Euler equation of McKay et al. (2017), where the effects of future changes in the real interest rate are dampened when \( \delta > 0 \). In contrast to McKay et al. (2016), we obtain a discounted Euler equation in an economy with aggregate disaster risk instead of idiosyncratic income risk.

More generally, the aggregate Euler equation (7) can feature compounding if we allow for positive private liquidity. Moreover, if \( \lambda \) is sufficiently small, we have that savers’ consumption satisfies a discounted Euler equation while the aggregate Euler equation features compounding. To better understand this result, consider again the case without private liquidity. If \( \chi_r = \lambda = 0 \) and \( \chi_y = 1 \), the response of the economy to a monetary shock coincides with the one in the RANK model. Even though only savers substitute intertemporally, the borrowers’ response to the reduction in output caused by the drop in demand by savers exactly compensates for their lack of an intertemporal substitution channel when \( \chi_y = 1 \). This is the main intuition behind the “as if” result of Bilbiie (2008) and Werning (2015). Introducing positive private liquidity adds a new effect. As a contractionary monetary shock depresses the economy and reduces inflation in all periods, it increases the real burden of debt for borrowers, amplifying the effect of the monetary shock. This amplification translates into a compounded response of output to future interest rate changes.

The third term in the aggregate Euler equation, \( v_t \), captures a direct effect of monetary policy on borrowers, one which is not mediated by the drop in demand by savers. To isolate this effect, suppose that \( \lambda = \kappa = 0 \), such that we can abstract from discounting or compounding. If we assume that \( i_T - r_n = y_T = 0 \) for some date \( T > 0 \), we obtain the following condition integrating the Euler equation forward:

\[
y_t = -\frac{1 - \mu_b}{1 - \mu_b \chi_y} \int_t^T (i_s - r_n) ds - \frac{\mu_b \chi_r}{1 - \mu_b} (i_t - r_n).
\]

The first term captures the response to a monetary shock in a HANK model with zero private liquidity. If \( \chi_y > 1 \), the response is amplified compared with what occurs in a RANK model, but this effect is still mediated by attempts by savers to substitute consumption intertemporally, as it is proportional to \( \sigma^{-1} \). In an economy with positive private liquidity, monetary policy directly affects borrowers, the high-MPC agents in this economy. Therefore, monetary policy has real effects even in the complete absence of intertemporal-substitution forces.

Finally, Proposition 1 defines the New Keynesian Phillips curve in this economy. The linearized Phillips curve coincides with the one obtained from models with Calvo pricing. As in a textbook New Keynesian model, inflation is given by the present discounted value of future
output gaps

$$\pi_t = \kappa \int_1^\infty e^{-\rho(s-t)} y_s ds.$$ 

One distinction relative to the standard formulation is that future output gaps are not discounted by the natural rate $r_n$, but by a higher rate $\rho > r_n$. This is a consequence of the riskiness of the firm’s value, so the appropriate discount rate incorporates an adjustment for risk.

**Asset prices.** Monetary policy affects the valuation of assets. For instance, the value of stocks in period 0 is given by

$$q_0 = \frac{Y}{Q} \int_0^\infty e^{-\rho t} [(1 - \tau) y_t - (1 - \alpha)(w_t - p_t + n_t)] dt - \int_0^\infty e^{-\rho t} \left[ i_t - \pi_t - r_n + \lambda \left( \frac{C_h}{\sigma C_s} \right) \frac{Q - Q^*}{Q} - \sigma c_{s,t} \right] dt. \quad (9)$$

Similarly, human wealth satisfies an analogous expression with wages and transfers as dividends. The expression shows that valuation of assets responds to changes in monetary policy through two channels: a *dividend channel*, e.g. changes in profits, wages, and transfers, and a *discount rate channel*, capturing changes in real interest rates and risk premia.\(^{17}\)

**Calibration.** In the numerical examples discussed in the next section, we adopt the following calibration, based mostly on the parameters adopted by Barro (2009). We choose $\lambda$ to match an annual disaster probability of 1.7%, and $A^*$ to match a drop in output of $1 - \frac{\gamma}{\gamma'} = 0.39$.\(^{18}\) The risk-aversion coefficient is set to $\sigma = 4$, a value within the range of reasonable values according to Mehra and Prescott (1985), but is substantially larger than $\sigma = 1$, a value often adopted in macroeconomic models. Note that the equity premium in this economy is

$$\frac{\Pi}{Q} + \frac{\mathbb{E}_t [dQ]}{Q dt} - r_n = \lambda \left[ \left( \frac{C_h}{C_s} \right)^\sigma - 1 \right] \frac{Q - Q^*}{Q},$$

where $Q$ is the value of intermediate-good firms in the stationary equilibrium. Our calibration implies an equity premium in the stationary equilibrium of 6.1%, in line with the observed equity premium of 6.5%. This suggests that the model is able to match movements in marginal utility caused by the rare disaster when $\sigma = 4$. Moreover, by setting $\sigma = 4$ we obtain a micro EIS of

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\(^{17}\)Compensation for holding risk is time-varying in the model. The price of disaster risk is given by $\lambda \left[ \left( \frac{C_h}{C_s} \right)^\gamma - 1 \right]$, as it represents the excess return on a unit exposure to disaster risk and is given up to first-order by $\lambda \left( \frac{C_h}{C_s} \right)^\sigma c_{s,t}$.

\(^{18}\)As discussed in Barro (2006), it is not appropriate to calibrate $A^*/A$ to the average magnitude of a disaster, given that empirically the size of a disaster is stochastic. We instead calibrate $A^*/A$ to match $\mathbb{E}_t[(C_h/C_s)^\gamma]$ using the empirical distribution of disasters reported in Barro (2009).
\( \sigma^{-1} = 0.25 \), in the ballpark of an EIS of 0.1 as recently estimated by Best et al. (2020).

The remaining parameters are chosen as follows. The discount rate of savers is chosen to match a natural interest rate of \( r_n = 1\% \). We assume a Frisch elasticity of one, \( \phi = 1 \), and set the elasticity of substitution between intermediate goods to \( \epsilon = 6 \), common values adopted in the literature. The fraction of borrowers is set to \( \mu_b = 30\% \), and the parameter \( \bar{d}_p \) is chosen to match a household debt-to-disposable income ratio of 1 (consistent with the U.S. Financial Accounts). The parameter \( \bar{d}_g \) is chosen to match a public debt-to-GDP ratio of 66\%, and, in the extension with long-term debt, we assume a duration of five years, consistent with the historical average for the United States. The tax rate is set to \( \tau = 0.27 \) and the parameter \( T^r_b(Y) \) is chosen such that \( \chi_y = 1 \), which requires countercyclical transfers to balance the procyclical wage income. A value of \( \chi_y = 1 \) is consistent with the evidence in Cloyne et al. (2020) that the net income of mortgagors and non-mortgagors reacts similarly to monetary shocks. The pricing cost parameter \( \varphi \) is chosen such that \( \kappa \) coincides with its corresponding value under Calvo pricing and an average period between price adjustments of three quarters. The half-life of the monetary shock is set to three and a half months to roughly match what we estimate in the data and, when considering an active monetary regime, we set \( \phi_r = 1.5 \).

3 Monetary Policy and Wealth Effects in D-HANK

In this section, we study how households’ balance sheets determine the aggregate impact of monetary policy on the economy. In particular, we focus on the role of wealth effects, i.e. the revaluation of households’ assets, including stocks, human wealth, and public and private bonds. As emphasized by the literature on fiscal-monetary interactions, though, wealth effects depend crucially on the fiscal response to monetary shocks.\(^{19}\) Thus, it is natural to start the study of wealth effects in the context of a fiscally active regime. We initially consider the case where the monetary authority chooses an exogenous path of nominal interest rates, \( \phi_m = 0 \), and the fiscal authority chooses an exogenous path of fiscal transfers to savers \( T_{s,t} \). We then consider the case of an active monetary regime and show that our results extend to that case as well.\(^{20}\)

3.1 The intertemporal budget constraint

Proposition 1 shows that output and inflation satisfy the following system of differential equations,

\[
\begin{bmatrix}
y_t \\
\pi_t
\end{bmatrix} =
\begin{bmatrix}
\delta & -\sigma^{-1} \\
-\kappa & \rho
\end{bmatrix}
\begin{bmatrix}
y_t \\
\pi_t
\end{bmatrix} +
\begin{bmatrix}
v_t \\
0
\end{bmatrix},
\]

\( (10) \)

\(^{19}\)See, for instance, the discussion in Leeper and Leith (2016) about the role of wealth effects in monetary analysis.

\(^{20}\)As argued by Cochrane (2018), the case of a fiscally active regime can also be of direct interest to economies which kept interest rates constant for a long period of time following the Great Recession.
where \( v_t \equiv \tilde{\sigma}^{-1}(i_t - r_n) + v_t \) is exogenous and depends only on the path of the nominal interest rate. The eigenvalues of the system are given by

\[
\bar{\omega} = \rho + \delta + \sqrt{(\rho + \delta)^2 + 4(\tilde{\sigma}^{-1} \kappa - \rho \delta)}, \quad \omega = \rho + \delta - \sqrt{(\rho + \delta)^2 + 4(\tilde{\sigma}^{-1} \kappa - \rho \delta)}.
\]

The following assumption, which we will assume holds for all subsequent analysis, guarantees that the eigenvalues are real-valued and that they have opposite signs, that is, \( \bar{\omega} > 0 \) and \( \omega < 0 \).

**Assumption 1.** The following condition holds: \( \rho \delta < \tilde{\sigma}^{-1} \kappa \).

Assumption 1 implies that the system lacks exactly one boundary condition under an interest rate peg. This is consistent with the results in Acharya and Dogra (2020), who find that indeterminacy under an interest rate peg requires a discounting parameter that is not overly large. Next, we show that the missing boundary condition can be provided by an *intertemporal budget constraint*.

An intertemporal budget constraint, computed with the SDF \( \eta_t \), holds with equality for both types of households,

\[
\mathbb{E}_0 \left[ \int_0^\infty \frac{\eta_t}{\eta_0} \mu_j C_{j,t} dt \right] = B_{b,0} + \mathbb{E}_0 \left[ \int_0^\infty \frac{\eta_t}{\eta_0} \left( \frac{W_t}{P_t} N_{j,t} + \Pi_{j,t} + \tilde{T}_{j,t} \right) dt \right],
\]

where the intertemporal budget constraint holds with equality for savers because of the transversality condition and holds with equality for borrowers as \( \lim_{t \to \infty} \mathbb{E}_t[\eta_t] = 0 \) and \( B_{b,0} \) is constant. Thus, the intertemporal budget constraint for each household is a necessary equilibrium condition. The next lemma establishes the sufficiency of an aggregate intertemporal budget constraint for pinning down the equilibrium. That is, it shows that if \( [y_t, \pi_t]_0^\infty \) satisfies system (10) and an aggregate intertemporal budget constraint (in its log-linear form), then we can determine the value of consumption and labor supply for each household, wages, and prices such that all equilibrium conditions are satisfied.

**Lemma 1.** Suppose \( [y_t, \pi_t]_0^\infty \) satisfy system (10) and the aggregate intertemporal budget constraint

\[
\int_0^\infty e^{-\rho t} \left( \mu_b c_{b,t} + (1 - \mu_b) c_{s,t} \right) dt = \Omega_0, \tag{11}
\]

where

\[
\Omega_0 = \int_0^\infty e^{-\rho t} \left[ (1 - \tau) y_t + T_t + \tilde{d}_g (i_t - \pi_t - r_n) \right] dt, \tag{12}
\]

\( \mu_b c_{b,t} + (1 - \mu_b) c_{s,t} = y_t \), and \( \tilde{d}_g \) is the public debt-to-GDP ratio. Then, \( y_t \) and \( \pi_t \) can be supported as part of a competitive equilibrium.

**Proof.** See Appendix B.2.

Condition (11) presents the (linearized) aggregate intertemporal budget constraint. Note that a simple rearrangement of (11) and (12) gives the government’s intertemporal budget constraint
(or valuation equation). By writing it this way, we make explicit the role of the outside wealth effect \( \Omega_0 \), which captures the revaluation of assets in positive net supply: stocks (the discounted value of profits), human wealth (the discounted value of labor income), and fiscal transfers. We follow Auclert (2019) and define the wealth effect net of the impact of changes in discount rates in the present discounted value of consumption, which can be thought of as a form of households’ liabilities. This explains the presence of the term involving public debt in the budget constraint, despite debt being short-term.\(^{21}\) When government debt is short-term, assets owned by the household sector as a whole have a shorter duration than their liabilities (planned consumption), so an increase in real interest rates generates a positive wealth effect, holding everything else constant. Similarly, an increase in output generates a positive wealth effect, as it increases the value of corporate holdings. In contrast, neither the wage nor the level of private debt enters the aggregate budget constraint directly, as they capture only a redistribution between borrowers and savers, holding everything else constant. Therefore, the outside wealth effect captures the response to monetary policy of the assets in positive net supply. Finally, note that the discount rate in the intertemporal budget constraint (11) is \( \rho \equiv \rho_s + \lambda \), instead of the natural rate \( r_n < \rho \), so the discount rate includes a risk-adjustment term.

### 3.2 Intertemporal substitution and wealth effects in D-RANK

We consider first the case of a representative-agent economy, that is, we assume \( \mu_b = 0 \). The next proposition characterizes the output response to a sequence of monetary policy shocks, for a given value of the outside wealth effect \( \Omega_0 \). We provide a full characterization of \( \Omega_0 \) in Section 3.4.

**Proposition 2** (Aggregate output in D-RANK). Suppose \( \mu_b = 0 \). Aggregate output is given by

\[
y_t = y_t^S + \frac{\frac{\omega}{\rho} - \delta}{\rho} \rho \Omega_0,
\]

where \( y_t^S \) is a function of the path of nominal interest rates and is given by

\[
y_t^S = -e^{\frac{F}{\sigma}}\int_t^\infty e^{\frac{-\omega(z-t)}{\rho}}(i_z - r_n)dz + e^{\frac{B}{\sigma}}\int_0^t (e^{\frac{-\omega z}{\rho}} - e^{\frac{-\omega}{\rho}}) (i_z - r_n)dz,
\]

where the coefficients \( e^{F}_{y,t} \) and \( e^{B}_{y,t} \) are positive, decreasing over time, and independent of \([i_t, T_{s,t}]_0^\infty\).

*Proof.* See Appendix B.3. \( \square \)

Proposition 2 shows that the response of output to a monetary policy shock can be decomposed into two terms. The first term corresponds to the response of output when the outside wealth

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\(^{21}\) Formally, the impact of changes in the interest rate on the present discounted value of consumption is \(-\frac{\omega}{\rho} \int_0^\infty e^{-\rho t}(i_t - \pi_t - r_n)dt\), and the corresponding impact on after-tax income is \(-\frac{1-\tau}{\rho} \int_0^\infty e^{-\rho t}(i_t - \pi_t - r_n)dt\), where \( T \) is the steady-state level of transfers. Combining the two and using \( C = (1-\tau)Y + T + r_n D_s \), we obtain \( \int_0^\infty e^{-\rho t} \delta (i_t - \pi_t - r_n)dt \) after dividing by \( Y \).
effect $\Omega_0$ is zero, and we refer to this term as the \textit{intertemporal-substitution effect} (ISE). The second term corresponds to the product of the partial equilibrium response of consumption to a change in wealth, $\rho \Omega_0$, and a \textit{general-equilibrium multiplier} (GE multiplier), $\frac{\sigma}{\rho} \Delta \omega$.

The ISE captures the equilibrium implications of the intertemporal-substitution channel. In the absence of outside wealth effects, monetary policy affects only the \textit{timing} of output, as the present value of economic activity is determined entirely by $\Omega_0$ (that is, we have that $\int_0^\infty e^{-\rho t} y^*_t dt = 0$ and $\int_0^\infty e^{-\rho t} y^*_t dt = \Omega_0$). Similar in logic to the substitution effect in introductory microeconomics, an increase in future nominal interest rates reduces consumption today, while past increases in interest rates tend to increase current consumption.\footnote{Our definition of ISE coincides with the textbook substitution effect in the limit case $\lambda = \kappa = 0$, where there is no distinction between changes in nominal and real rates, and there is no precautionary motive.} In this sense, the intertemporal-substitution channel of monetary policy operates simply by shifting demand over time, and it is ineffective in the absence of an intertemporal-substitution motive; that is, we obtain $y^*_t = 0$ in all periods if $\sigma^{-1} = 0$. Importantly, given a path of nominal interest rates, $y^*_t$ is uniquely determined.

The second term in expression (13) plays a crucial role, as the outside wealth effect determines the average level of output. Holding everything else constant, the impact on consumption of a wealth effect $\Omega_0$ would be simply $\rho \Omega_0$, as households attempt to smooth the impact of the change in wealth over time. However, the response of initial consumption is amplified in general equilibrium, as a positive wealth effect generates inflation, which reduces real interest rates and shifts consumption to the present. This effect can be quantitatively significant. Moreover, an important factor determining the size of the GE multiplier is the discounting parameter $\delta$. It can be shown that the GE multiplier in period 0 is decreasing in $\delta$. Therefore, the precautionary savings motive \textit{dampens} the effect of the outside wealth effect. In our calibrated example, the discounting parameter is $\delta = 3.1\%$ per quarter, roughly the same value as assumed by McKay et al. (2017) in

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{output_response}
\caption{Output dynamics in the D-RANK ($\mu_b = 0$) and the D-HANK ($\mu_b > 0$) economies.}
\end{figure}

Note: The path of the nominal interest rate is given by $i_t = r_n = e^{-\theta_n} (i_0 - r_n)$, where $i_0 - r_n$ equals 100 bps, and fiscal backing is given by $T_{s,t} = 0$ for all $t \geq 0$. 
their incomplete-markets economy.\textsuperscript{23}

Given our calibration, Figure 1 (left panel) shows the path of the ISE, the partial equilibrium impact of the wealth effect (PE-WE) $\rho \Omega_0$, the product of the wealth effect and GE multiplier, and the total response of output. The figure shows that, even though the partial equilibrium response to the wealth effect is quantitatively small, the GE multiplier is sufficiently large to make the initial output response 50% larger than the one in the ISE, indicating that wealth effects play a significant role even in this D-RANK model. Note, however, that the initial response of output is less than 0.1%, substantially smaller than in the textbook model, in part as a result of the lower value adopted for the EIS. We address these differences in Sections 3.5 and 4.

3.3 Intertemporal substitution and wealth effects in D-HANK

We consider next the case with heterogeneous agents by assuming again that $\mu_b > 0$. For ease of exposition, we focus on the case of exponentially decaying nominal interest rates; that is, we assume $i_t - r_n = e^{-\psi_m} (i_0 - r_n)$, where $\psi_m$ determines the persistence of monetary shocks. The next proposition extends the result of Proposition 2 to an economy with heterogeneous agents and positive private liquidity.

**Proposition 3** (Aggregate output in D-HANK). Suppose that $i_t - r_n = e^{-\psi_m} (i_0 - r_n)$. The path of aggregate output is then given by

$$y_t = \underbrace{\frac{1 - \mu_b}{1 - \mu_b \chi_Y^m}}_{\text{ISE}} y_t^S + \underbrace{\frac{H_b \chi_r}{1 - \mu_b \chi_Y^m} y_t^P}_{\text{inside wealth effect}} + \underbrace{\frac{\bar{\omega} - \delta e^{\omega t \rho}}{\rho} \rho \Omega_0,}_{\text{GE multiplier \times outside wealth effect}}$$

where $y_t^S$ is given by (14) and $y_t^P$ is given by

$$y_t^P = -\epsilon_{y,t}^F \psi_m \int_t^\infty e^{-\mu (z-t)} (i_z - r_n) dz + \epsilon_{y,t}^B \psi_m \int_0^t (e^{-\omega z} - e^{-\omega z}) (i_z - r_n) dz.$$  

**Proof.** See Appendix B.3.

Proposition 3 shows that output can now be decomposed into three terms, which capture the intertemporal-substitution channel, the revaluation of inside assets and the revaluation of outside assets. The first term corresponds to the same expression for the ISE obtained in the representative-agent economy, equation (14), multiplied by a factor that is increasing in the parameter that controls the (counter) cyclicity of inequality $\chi_Y$. This amplification lies at the heart of the mechanism in analytical HANK models with zero private liquidity.

\textsuperscript{23}The analysis of discounting in the Euler equation is related to an extensive body of literature that studies the forward-guidance puzzle, that is, the finding that, in New Keynesian models, the impact of monetary policy is larger for interventions further into the future. Several of the solutions to this puzzle imply an aggregate Euler equation with discounting, i.e. $\delta > 0$. Our analysis implies that the solutions to the puzzle usually operate by reducing the GE multiplier and the impact of wealth effects.
The second term corresponds to the inside wealth effect, and it is present only in economies with positive private debt and heterogeneous MPCs. The term \( y_1^p \) is analogous to the ISE in many respects. The coefficients \( e^{F}_{y,t} \) and \( e^{B}_{y,t} \) that determine the forward- and backward-looking interest-rate elasticities are the same as in the ISE, and the private wealth effect also operates by shifting demand over time, as it satisfies \( \int_0^\infty e^{-\rho t} y_t^p \, dt = 0 \). Moreover, \( y_1^p \) is also uniquely determined for a given path of the nominal interest rate. A key distinction between the two is that the persistence of monetary policy plays the role of the (micro) EIS \( \sigma^{-1} \) in equation (16). Therefore, while the ISE is equal to zero when we set \( \sigma^{-1} = 0 \), the inside wealth effect is zero when the monetary shock is permanent, \( \psi_m = 0 \).

An important implication of this result is that the effectiveness of monetary policy depends on the persistence of monetary shocks. For instance, by promising to keep interest rates low for a very long period of time, the monetary authority increases the persistence of the shock and, therefore, reduces the importance of inside wealth effects and the overall output response. To understand this result, note that an increase in interest rates has a negative impact on borrowers and a positive impact on savers. When the shock is temporary, the impact of the change in interest rates is initially larger on borrowers, as savers respond less strongly to the change in wealth to smooth consumption. If the shock is permanent, however, there is no reason to smooth the shock. In this case, the savers’ response coincides with the borrowers’ response, and the inside wealth effect is exactly zero. Thus, it is the variability of interest rates rather than the average level that matters for the inside wealth effect.

The third term in expression (15) corresponds to the product of the GE multiplier and the outside wealth effect, as in the case of the representative-agent economy described in Proposition 2. This does not mean, however, that heterogeneity and private liquidity have no role in shaping the impact of outside wealth effects. As discussed in the context of Proposition 1, positive private debt reduces the value of \( \delta \), a force towards compounding in the Euler equation. This increases the value of the initial GE multiplier, which ultimately amplifies the effect of changes in outside wealth on initial output.

We find that the presence of indebted hand-to-mouth agents substantially amplifies the impact of monetary policy in our calibrated example. Figure 1 compares the response to a given path of monetary shocks of a representative-agent economy (left panel) with the response of a heterogeneous-agents economy (right panel). Fiscal transfers to savers are set to \( T_{s,t} = 0 \) in both cases. Because \( \chi_y = 1 \) in our calibration, we abstract from the standard amplification mechanism previously explored in the analytical HANK literature. In particular, the allocation with \( \mu_b = 0 \) coincides with the allocation in the presence of heterogeneous agents, \( \mu_b > 0 \), but no private debt, \( \chi_r = 0 \). Despite abstracting from the usual amplification mechanism, we find that the model is able to strongly amplify the effects of monetary shocks. This is the result of both inside and outside wealth effects, as the ISE is roughly the same as in the case where \( \mu_b = 0 \). About 55% of the difference between the initial response of output in HANK and RANK models can be attributed
to the inside wealth effect, the direct impact of monetary policy on borrowers, while 40% of this
difference can be attributed to the outside wealth effect and its amplification. The presence of
indebted hand-to-mouth agents is enough to change the sign of the discounting parameter $\delta$, so
the economy now features compounding in the aggregate Euler equation, which increases the GE
multiplier and the importance of the outside wealth effect. The outside wealth effect $\Omega_0$ itself
also reacts more to a monetary shock in the case of $\mu_y > 0$, contributing to the amplification. The
determination of the outside wealth effect is discussed below in Section 3.4.

This pattern is roughly consistent with recent evidence in Cloyne et al. (2020), who study the
differential impact of monetary shocks on mortgagors and non-mortgagors (homeowners and
renters), a simple way of separating indebted households from households with large positions in
liquid and illiquid assets. First, they find a similar response of net income for both mortgagors
and non-mortgagors, consistent with the assumption that $\chi_y = 1$. Second, they find a significant
response of borrowing costs for mortgagors, consistent with a role for the inside wealth effect.
Importantly, in our model, the presence of private debt amplifies the effect of monetary policy
not only through the direct effect on interest payments (i.e. the inside wealth effect) but also by
magnifying general-equilibrium feedbacks, in line with their findings of strong GE mechanisms.

Inflation. The next proposition characterizes the response of inflation to monetary policy shocks
in the context of our heterogeneous-agent economy.

**Proposition 4 (Inflation in D-HANK).** Suppose $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$. The path of inflation is given by

$$\pi_t = \frac{1 - \mu_b}{1 - \mu_b \chi_y} \pi_t^S + \frac{\mu_b \chi_r}{1 - \mu_b \chi_y} \pi_t^P + \kappa e^{\omega^d} \Omega_0,$$

where $\pi_t^P = \psi_m \sigma \pi_t^S$, $\pi_t^S$ is a function of the path of nominal interest rates, and it is given by

$$\pi_t^S = e^{e_{\pi, t}} \sigma^{-1} \int_t^\infty e^{-\omega(z-t)} (i_z - r_n) dz + e^{B_{\pi, t}} \sigma^{-1} \int_0^t (e^{-\omega z} - e^{-\omega t}) (i_z - r_n) dz,$$

where $e_{\pi, t}$ is increasing over time and satisfies $e_{\pi, 0} = 0$, and $e^{B_{\pi, t}}$ is positive and decreasing over time.

**Proof.** See Appendix B.3. $\square$

Inflation can be analogously decomposed into three terms. The first two terms capture the
impact of the ISE and the inside wealth effect, while the last term captures the impact of the
outside wealth effect. Because $e^{B_{\pi, 0}} = 0$, the first two terms are initially zero $\pi_0^S = \pi_0^P = 0$. This
implies that initial inflation is determined entirely by the outside wealth effect, a consequence of
the forward-looking nature of the New Keynesian Phillips curve. Moreover, $\pi_t^S$ and $\pi_t^P$ are both
increasing in the nominal interest rate $i_z$, for any $z \geq 0$ and $t > 0$. That is, this economy features a

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24At the micro level, Di Maggio et al. (2017) shows that changes in interest payments have a significant effect on
consumption and that more heavily indebted households have higher MFC.
Neo-Fisherian behavior in the absence of the outside wealth effect, as an increase in interest rates leads to an increase in inflation. This sheds new light on how monetary policy controls inflation: monetary policy is able to reduce inflation by increasing interest rates only if it creates a negative net revaluation of households’ assets.

### 3.4 Outside Wealth Effects

So far, we have considered how output and inflation respond to the path of nominal interest rates given the value of \( \Omega_0 \). The outside wealth effect is, however, endogenous, as it depends on the path of output and inflation,

\[
\Omega_0 = \int_0^\infty e^{-\rho t} \left[ (1 - \tau) y_t + \mu_b T_b(Y) y_t + \mu_s T_{s,t} + \bar{d}_q(i_t - \pi_t - r_n) \right] dt, \tag{19}
\]

while output and inflation in turn depend on the outside wealth effect

\[
y_t = \hat{y}_t + (\bar{\omega} - \delta)e^{\omega t}\Omega_0, \quad \pi_t = \hat{\pi}_t + \kappa e^{\omega t}\Omega_0, \tag{20}
\]

where \((\hat{y}_t, \hat{\pi}_t)\) collect the terms that are a function only of \(i_t\).

This simultaneity reflects the fact that asset prices react to the level of aggregate demand, as shown by equation (19), and that spending decisions depend on the level of asset prices, as shown by (20). By combining these expressions, we can express \( \Omega_0 \) in terms of policy variables, that is, the path of nominal interest rates, \(i_t\), and the fiscal backing to the monetary shock, \(T_{s,t}\). In particular, we can express \( \Omega_0 \) as follows:

\[
\Omega_0 = \left[ 1 - \left( \tau - \mu_b T'_b(Y) + \frac{\kappa}{\bar{\omega} - \delta} \bar{d}_q \right) \right] \Omega_0 + \int_0^\infty e^{-\rho t} \left[ \mu_s T_{s,t} + \bar{d}_q(i_t - \hat{\pi}_t - r_n) \right] dt.
\]

The first term captures the impact of aggregate demand on the valuation of stocks, bonds, and human wealth, while the second term captures the impact of changes in monetary and fiscal variables that are not mediated by aggregate demand. Assumption 2 guarantees that outside wealth reacts less than one-to-one to aggregate demand.\(^{25}\)

**Assumption 2.** The following condition holds: \( \tau - \mu_b T'_b(Y) + \frac{\kappa}{\bar{\omega} - \delta} \bar{d}_q > 0. \)

The next proposition shows that the outside wealth effect can be expressed as the product of a multiplier and an autonomous term, that is, a term that does not depend directly on \( \Omega_0 \).

\(^{25}\)Assumption 2 implies that either the primary surplus or the cost of servicing the debt increases with economic activity, as captured by \( \Omega_0 \). It essentially implies that monetary policy has fiscal consequences.
Proposition 5. Suppose Assumption 2 holds. The outside wealth effect is then given by

\[
\Omega_0 = \frac{1}{\tau - \mu_s T_b(Y) + \frac{\kappa}{\alpha - \delta}} \int_0^\infty e^{-\rho t} \left[ \mu_s T_s(t) + \bar{d}_g (i_t - \bar{n}_t - r_n) \right].
\]

(21)

Proof. See Appendix B.4.

Proposition 5 introduces a novel relationship between the model-implied revaluation of assets in positive net supply, \(\Omega_0\), and the equilibrium path of policy variables.\textsuperscript{26} For example, expression (21) shows that, in the absence of any fiscal backing (\(T_s, t = 0\)) or government debt (\(\bar{d}_g = 0\)), the outside wealth effect is zero. Monetary policy still has an effect on the value of stocks and human wealth, as can be seen in (9), but the reduction in the value of households’ assets is exactly offset by the reduction in the value of households’ liabilities (in the form of consumption), as discussed in Section 3.1. Under Assumption 2, the aggregate demand effect cannot sustain a positive value of \(\Omega_0\) in the absence of a direct effect of policy variables.

By incorporating fiscal data into the analysis, this relationship provides a way to discipline the model’s economic forces. One can estimate the fiscal response to a monetary shock in the data and introduce the estimated values into expression (21) to obtain the model’s prediction for \(\Omega_0\). We do this next. However, our specification of the government’s budget constraint lacks a potentially important channel of fiscal adjustment: the revaluation of long-term bonds. Thus, we first extend our previous analysis to incorporate long-term bonds. As we will see, incorporating long-term bonds has the additional effect of introducing a time-varying term premium in the determination of the outside wealth effect.

**Long-term bonds and the term premium.** We introduce exponentially decaying coupons for government bonds, as in Woodford (2001), where government debt pays coupons \(e^{-\psi_d t}\) in period \(t\). The rate of decay \(\psi_d\) is inversely related to the bond’s maturity, where a perpetuity corresponds to \(\psi_d = 0\) and the limit \(\psi_d \to \infty\) corresponds to the case of short-term bonds considered until now.

We show in Appendix A.5 that the (linearized) price of the long bond, which is denoted by \(q_{L, t}\), is given by

\[
q_{L, 0} = -\int_0^\infty e^{-(\rho + \psi_d) t} \left[ i_t - r_n + r_L \sigma c_{st, t} \right] dt,
\]

(22)

where \(r_L\), the spread between long and short rates in the stationary equilibrium, is given by

\[
r_L = \lambda \left( \frac{C_s}{C_s^e} \right)^\sigma \frac{Q_L - Q_L^*}{Q_L}
\]

with \(Q_L\) and \(Q_L^*\) denoting the price of the long-term bond in the no-disaster and disaster stationary equilibrium, respectively. It can be shown that, in the stationary equilibrium, the spread \(r_L\) is

\textsuperscript{26}Recall that \(\bar{n}_t \equiv \frac{1 - \mu_s}{1 - \mu_s x_s} \pi_t^S + \frac{\mu_s x_s}{1 - \mu_s x_s} \pi_t^D\) is pinned down by the path of nominal rates, as given in Proposition 4.
strictly positive. Thus, our model generates an upward-sloping yield curve, where the yield on the long bond exceeds the natural (short) rate, consistent with the data.\footnote{The mechanism behind the upward-sloping yield curve is related to the lack of precautionary savings in the disaster state. We would obtain similar results by introducing expropriation and inflation in a disaster, as in Barro (2006).}

The yield on the long bond, expressed as deviations from the stationary equilibrium, is given by \(-Q_L^{-1} q_{L,0}\), which can be decomposed into two terms: a term depending on the path of nominal interest rates, as in the expectation hypothesis, and a term premium, \(Q_L^{-1} \int_0^\infty e^{-(\rho + \Psi_b)t} r_L \sigma_{c,s} dt\), capturing variations in the compensation for holding long-term bonds. Because the term premium can respond to monetary shocks, the expectation hypothesis does not hold in this economy. This is important since, as emphasized by Gertler and Karadi (2015) and Hanson and Stein (2015), the term premium accounts for the bulk of the response of long rates to changes in monetary policy.

With long-term bonds, the expression for \(\Omega_0\) becomes

\[\Omega_0 = \bar{d}_g q_{L,0} + \int_0^\infty e^{-\rho t} \left[ (1 - \tau) y \tau + T + \bar{d}_g (i_t - \pi_t - r_n + r_L \sigma_{c,s}) \right] dt.\] (23)

There are two important differences between the outside wealth effect with short-term bonds, given by equation (19), and the corresponding expression in the presence of long-term bonds, given by equation (23). First, the long-term bond introduces a revaluation effect, given by \(\bar{d}_g q_{L,0}\). Monetary policy affects the price of long-term bonds directly through changes in the short-term nominal interest rates and indirectly through its impact on the term premium. Second, the term premium now appears explicitly in the determination of outside wealth. Interestingly, while the value of stocks and human wealth also incorporate a premium for risk, only in the presence of long-term government bonds do variations in the price of risk directly affect households’ wealth. This is once again the manifestation that only the mismatch in exposure between the household sector and the government (in terms of both maturity and risk) matters for determining the change in the value of households’ wealth. Moreover, we show in Appendix B.4 that a version of Proposition 5 can be extended to the case with long-term bonds.

An implication of incorporating long-term bonds is that monetary shocks now have two opposing effects on \(\Omega_0\). First, an increase in interest rates allows households to reinvest their savings at a higher interest rate, which generates a positive wealth effect. Second, an increase in interest rates reduces the value of the long-term bonds held by households, creating a negative wealth effect. Under an extension of Assumption 2 for the case of long-term bonds, the next proposition shows that, if the maturity of government debt is sufficiently long, the second effect dominates, and an increase in nominal interest rates reduces the value of \(\Omega_0\), effectively extending the result in Cochrane (2018) to an economy with heterogeneous agents and disaster risk.

**Proposition 6 (Wealth effects and long-term bonds).** Under conditions given in the appendix, there exists a threshold \(\bar{q}_d > 0\) such that

\[\frac{\partial \Omega_0}{\partial q_{L,0}} < 0,\]
\textbf{Figure 2:} Estimated fiscal response to a monetary policy shock

Note: IRFs computed from a VAR identified by a recursiveness assumption, as in \textit{Christiano et al.} (1999). Variables included: real GDP per capita, CPI inflation, real consumption per capita, real investment per capita, capacity utilization, hours worked per capita, real wages, tax revenues over GDP, government expenditures per capita, federal funds rate, 5-year constant maturity rate and the real value of government debt per capita. We estimate a four-lag VAR using quarterly data for the period 1962:1-2007:3. The real value of government debt and the 5-year rate are ordered last, and the fed funds rate is ordered third to last. Gray areas are bootstrapped 95\% confidence bands. See \textit{Appendix C} for the details.

\textit{if the maturity of government debt is sufficiently long (ψₐ < \overline{τₐ}).}

\textbf{Proof.} See \textit{Appendix B.5}. \hfill \Box

Proposition 6 shows how the sign of the response of the outside wealth effect depends on the maturity of government debt. In the presence of short-term government debt, households’ assets have a shorter duration than their liabilities, so an increase in nominal interest rates generates a \textit{positive} wealth effect.\textsuperscript{28} As the duration of government debt increases, the sign of the mismatch is reversed, and increases in the nominal interest rates generate a \textit{negative} wealth effect. Note, however, that the overall \textit{magnitude} of the effect also depends on the response of stocks and human wealth, which end up amplifying the direct impact of monetary shocks through the multiplier effect.

\textbf{Fiscal response to monetary policy and the outside wealth effect.} Proposition 5 and its extension to long-term bonds show how the outside wealth effect, that is, the revaluation of assets in positive net supply, can be expressed as a function of the path of nominal interest rates and the fiscal response to monetary policy. Here, we estimate the counterpart of these objects in the data. We estimate a standard VAR augmented to incorporate fiscal variables, and compute empirical IRFs applying the recursiveness assumption of \textit{Christiano et al.} (1999). From the estimation, we obtain the three components necessary to calculate Ω₀ in the model: the path of the nominal interest rate,\textsuperscript{28}

\textsuperscript{28}This idea is analogous to the one behind the Pigou effect, as described in Patinkin (1948): “the private sector considered in isolation is, on balance, neither debtor nor creditor, when in its relationship to the government, it must be a net ‘creditor.’ (...) If we assume that government activity is not affected by the movements of the price level, then the net effect of a price decline must always be stimulatory.” We find similar effects by focusing on interest-rate changes instead of price-level changes.
the change in the initial value of government bonds, and the path of fiscal transfers. We provide the details of the estimation to Appendix C.

Figure 2 shows the dynamics of fiscal variables in the estimated VAR in response to a contractionary monetary shock. Government revenues fall in response to the contractionary shock, while government expenditures fall on impact and then turn positive, likely driven by the automatic stabilizer mechanisms embedded in the government accounts. The present value of interest payments increases by 69 bps and the initial value of government debt drops by 50 bps.\(^{29}\) In contrast, the present discounted value of expenditures, which we interpret as the empirical counterpart to the present discounted value of transfers \(T_t\), drops by 12 bps.\(^{30}\) Moreover, we cannot, at the 95% confidence level, reject the possibility that the present discounted value of the primary surplus does not change in response to monetary shocks and that the increase in interest payments is entirely compensated by the initial reduction in the value of government bonds.

Given the value of \(T_t\) estimated from the data, we can now solve for \(\Omega_0\). Figure 3 shows the path of output, inflation, and the sum of stocks and human wealth in response to an increase in nominal interest rates for different values of the level and maturity of government debt, and consequently, different values of \(\Omega_0\). In the absence of outside wealth effects, output falls on impact, but inflation is initially zero and eventually positive. The presence of long-term bonds is enough to avoid the Neo-Fisherian result, while short-term bonds make the response of inflation even more positive, as there is now a positive wealth effect. The value of stocks plus human wealth drops on impact for all specifications, with larger drops associated with a stronger outside wealth effect. The figure also shows the dynamics for the case of active monetary policy, which we discuss next.

\(^{29}\)The present discounted value of interest payments is calculated as \(\sum_{t=0}^{T} \left( \frac{1}{1+\rho} \right)^t \left[ d \left( \hat{r}_{t+1} - \hat{\rho}_t \right) \right]\), where \(T\) is the truncation period, \(\hat{r}_{t+1}\) is the IRF of the 5-year rate estimated in the data, and \(\hat{\rho}_t\) is the IRF of inflation. We choose \(T = 60\) quarters, when the main macroeconomic variables, including government debt, are back to their pre-shock values. Other present value calculations follow a similar logic.

\(^{30}\)In the data, expenditures also include the response of government consumption and investment. When run separately, however, we cannot reject the possibility that the sum of these two components is equal to zero in response to monetary shocks.
3.5 The case of active monetary policy

So far, we have focused on the case of a fiscally active regime, where \( \phi_\pi = 0 \) and the sequence of nominal interest rates and fiscal transfers is given exogenously. Next, we consider the case where monetary policy is active and the coefficient \( \phi_\pi \) satisfies a (generalized) Taylor principle. First, we provide the conditions under which there is a unique bounded solution to the three-equation dynamic system for \([i_t, y_t, \pi_t]_0^\infty\), given by the Taylor rule (4), the aggregate Euler equation (7), and the New Keynesian Phillips curve (8).

**Lemma 2** (Generalized Taylor Principle). There is a unique bounded solution to the dynamic system for \([i_t, y_t, \pi_t]_0^\infty\) if and only if the (generalized) Taylor principle is satisfied, that is, \( \phi_\pi > \max \{ \phi_1^1, \phi_2^2 \} = \phi_\pi^* \), where

\[
\phi_1^1 = \frac{\bar{\sigma}^{-1}_\iota - \rho \delta}{\bar{\sigma}^{-1}_\iota + \frac{\mu_b \chi r \rho}{1 - \mu_b \chi r}}, \quad \phi_2^2 = 1 - \left( \rho + \lambda \left( \frac{C_s}{C_s^*} \right)^{\iota} \right) \frac{1 - \mu_b \chi_y}{\mu_b \chi_y}. \tag{24}
\]

**Proof.** See Appendix B.6. \(\square\)

The Taylor coefficient \( \phi_\pi \) must satisfy the conditions \( \phi_\pi > \phi_1^1 \) and \( \phi_\pi > \phi_2^2 \). The second threshold goes to \(-\infty\) as \( \chi r = 0 \), so it is only relevant for economies with positive private debt. In the case of zero private liquidity \( \chi r = 0 \), we recover the result in Acharya and Dogra (2020) that discounting \( (\delta > 0) \) makes the Taylor principle more likely to be satisfied, while compounding \( (\delta < 0) \) makes the Taylor principle less likely to be satisfied, as \( \phi_\pi^* = 1 - \frac{\rho \delta}{\bar{\sigma}^{-1}_\iota} \). The standard result in RANK models, \( \phi_\pi^* = 1 \), is obtained when \( \delta = 0 \). The presence of indebted hand-to-mouth agents breaks the link between discounting and determinacy. In particular, it can be shown that we can simultaneously have \( \phi_\pi^* < 1 \) and compounding in the Euler equation \( \delta < 0 \).

The next lemma states the equilibrium path of nominal interest rates and fiscal transfers in the active monetary policy regime.

**Lemma 3** (Active monetary policy). Suppose that the Taylor principle is satisfied and the monetary shock is given by \( u_t = e^{-\psi_m t} u_0 \), and \( \psi_m > |\omega| \).\(^3\) The equilibrium path of nominal interest rates is then given by

\[ i_t - r_t = e^{-\psi_m t} \varepsilon_{i,u} u_0. \tag{25} \]

The equilibrium path of fiscal transfers to savers satisfies the condition

\[ \int_0^\infty e^{-\rho t} \mu_s T_s ds = (\tau - \mu_b T_{b'}(Y)) \Omega_0 - \frac{\varepsilon_{r,u} u_0}{\rho + \psi_m}, \tag{26} \]

where \( i_t - \pi_t - r_t = \varepsilon_{r,u} u_t \) and the outside wealth effect \( \Omega_0 \) is given by

\[ \Omega_0 = -\varepsilon_{\Omega,u} \left( \frac{1 - \mu_b}{1 - \mu_b \chi_y} \sigma^{-1} + \frac{\mu_b \chi_r}{1 - \mu_b \chi_y} \phi_m \right) u_0, \]

\(^3\)The assumption that monetary shocks are not too persistent, \( \psi_m > |\omega| \), guarantees that an increase in \( u_0 \) raises \( i_t \).
where \( \epsilon_{t,u}, \epsilon_{r,u}, \) and \( \epsilon_{\Omega,u} \) are positive constants.

Proof. See Appendix B.7.

Lemma 3 shows that the equilibrium nominal interest rate inherits the persistence and sign of the monetary shock. The outside wealth effect responds negatively to an increase in \( u_0 \), and the effect is amplified in the economy with positive private liquidity. The fiscal authority responds to a contractionary monetary shock, \( u_0 > 0 \), by raising taxes on (or reducing transfers to) savers, and this response is stronger the larger is the outside wealth effect generated by the active monetary regime. Lemma 3 also provides a way to construct the fiscal response to a monetary shock in an active monetary regime.

The next proposition shows that the value of output and inflation obtained using the formulas derived under the fiscally active regime coincide with the solution under the active monetary regime.

**Proposition 7.** Suppose Assumption 2 holds, the Taylor principle is satisfied, and \( u_t = e^{-\psi_{nt}} u_0 \). Given \( i_t \) and \( T_{s,t} \) satisfying (25) and (26), let \( y_t \) and \( \pi_t \) be given by expressions (15) and (17), for \( \Omega_0 \) given by (21). Then, \( y_t \) and \( \pi_t \) coincide with the equilibrium output and inflation in the active monetary regime.

Proof. See Appendix B.8.

An important implication of Proposition 7 is that, not only there is no loss of generality in taking the path of monetary and fiscal variables as given, but the active monetary regime with the Taylor rule (4) is, in a sense, a special case where the present value of \( T_{s,t} \) takes on a specific value (given by (26)). Standard formulations of the Fiscal Theory of the Price Level correspond to another special case, as it is typically assumed that \( T_{s,t} = 0 \). Even though active and passive monetary regimes describe the off-equilibrium behavior of fiscal and monetary authorities very differently, the equilibrium implications of these regimes can be summarized by the equilibrium path of the nominal interest rate and the fiscal response to a monetary shock \( T_{s,t} \).

**The role of fiscal policy in active monetary regimes.** It may sound surprising that our expressions derived under a fiscally active regime also apply to the case of active monetary policy. After all, it is well-known that, in active monetary regimes, the exact specification of the government’s budget constraint is irrelevant to the determination of the response of the economy to monetary policy. The previous analysis, however, does not contradict these results.

Our findings state that, when monetary policy has fiscal consequences (recall Assumption 2), there is a tight connection between the determination of outside wealth effects and the response of fiscal policy to monetary shocks.\(^{32}\) In particular, the model predicts that the outside wealth effect

\(^{32}\)When monetary policy has no fiscal consequences, i.e., \( \tau = T^*_p(Y) = \mathcal{J}_p = 0 \), the equilibrium is indeterminate under a fiscally active regime. This indeterminacy is resolved in a monetary-active regime by imposing a monetary rule that guarantees that there is a unique bounded equilibrium (see e.g., Cochrane (2011)). In this case, it is not possible to express \( \Omega_0 \) in terms of policy variables, as in (21), so the model loses a potential testable implication.
is proportional to policy variables *irrespective of* the policy regime that generated that response. While this result has no impact on the theoretical determination of equilibrium in a monetary-active regime, it has important implications for how we confront the model with the data. First, it allows us to bypass the (often controversial) determination of the monetary-fiscal interactions that determine government policy. If one has data on the response of interest rates and fiscal variables to a monetary shock, then it is not necessary to impose a policy rule to obtain a (unique) prediction of the model. Second, this approach allows us to *discipline* one of the key channels of the policy transmission mechanism: the outside wealth effect.

To see why this is important, consider Figure 3 again. The solid line shows the behavior of output and inflation under an active monetary regime. The effect on output and inflation is substantially larger in the active monetary regime than in the active fiscal regimes. This implies a stronger (negative) outside wealth effect in the active monetary regime and, as indicated by condition (26), a stronger response of fiscal policy. Importantly, the fiscal response implied by the Taylor rule adopted here is 104 bp, almost nine times that estimated in the data. That is, the active monetary regime implies a *counterfactual* fiscal response to sustain the equilibrium. Thus, by incorporating fiscal data into the analysis, we are able to show that the active monetary regime generates a strong response of the outside wealth effect by imposing a counterfactual large response of fiscal policy. In the next section, we show that heterogeneous agents and time-varying risk premia can compensate quantitatively for the absent fiscal response.

4 Time-Varying Risk and Monetary Policy

In this section, we study the role of risk in the transmission of monetary shocks. We start by showing that the model with constant disaster probability is unable to match the response of asset prices to monetary shocks. As a consequence, we find that risk plays only a minor role in the response of output to changes in monetary policy. We then introduce time-varying disaster risk, consistent with *Gourio* (2012) and *Gabaix* (2012), and show that, not only is the model able to capture the response of stocks and bonds to monetary shocks, but risk now plays an important role in shaping how the economy responds to changes in monetary policy.

4.1 The limitations of the constant disaster risk model

We have seen in Section 3 that heterogeneity in an economy with private debt substantially amplifies the economic response to monetary shocks. We next consider the role of aggregate risk. First, consider the response of asset prices to monetary shocks. Figure 4 shows that the yield on the long bond increases by 6.5 bps, which implies a decline of the value of the bond of 32 bps (given a 5-year duration), less than half of the response estimated by the VAR in Section 3.4. Moreover, movements in the yield of the long bond are almost entirely explained by the path of nominal interest rates, while the term premium is nearly indistinguishable from zero. This stands in sharp
contrast to the evidence reported in Gertler and Karadi (2015) and Hanson and Stein (2015), who
find that long rates respond substantially to monetary shocks and that the change in yields is
explained for the most part by the term premium. Similarly, it can be shown that most of the
response of stocks in the model is explained by movements in interest rates instead of changes
in risk premia, a finding that is inconsistent with the evidence documented in e.g. Bernanke and
Kuttner (2005).

Figure 4 enables us to compare the response of output to monetary shocks for the heterogeneous-
agent economy with aggregate risk (D-HANK) and without aggregate risk (HANK), and similarly
for the representative-agent economy. We find that risk has only a minor impact on the response of
output. Aggregate risk increases the value of the discounting parameter δ, which reduces the GE
dultiplier and dampens the initial impact of the monetary shock. Given that the term premium
barely moves, disaster risk plays only a small role in determining the outside wealth effect. In
contrast, the role of heterogeneity can be clearly seen by comparing the response of the D-HANK
and D-RANK economies.

Therefore, while introducing a constant disaster probability allows the model to capture impor-
tant unconditional asset-pricing moments, such as the (average) risk premium or the upward-
sloping yield curve, the model is unable to match key conditional moments, in particular, the re-
sponse of asset prices to monetary policy. The limitations of the model with constant disaster
probability in matching conditional asset-pricing moments were recognized early on in the litera-
ture, leading to an assessment of the implications of time-varying disaster risk, as in Gabai
(2012) and Wachter (2013). Next, we consider how time-varying disaster risk affects the asset-pricing
response to monetary shocks and, ultimately, its impact on real economic variables.

4.2 Asset-pricing implications of time-varying risk

We now consider a simple extension of our baseline model that allows us to capture the response
of asset prices to monetary shocks in a parsimonious way. Motivated by the literature on time-
varying risk premia and rare disasters, we allow for a time-varying disaster probability.\(^3\) Given our interest in how asset prices respond to monetary shocks, we assume the following form for the disaster probability: \(\lambda_t = \lambda(i_t - r_n)\), for a given function \(\lambda(\cdot)\). In our linearized setting, the only relevant parameter is the semi-elasticity of the disaster probability with respect to monetary shocks in the stationary equilibrium, that is, \(\varepsilon_\lambda \equiv \lambda'(0)/\lambda(0)\). We focus on the case in which \(\varepsilon_\lambda \geq 0\), and the economy under study until now corresponds to the special case in which \(\varepsilon_\lambda = 0\).

Time-varying risk plays a role, for instance, when considering the price of a long-term bond in the economy with non-zero \(\varepsilon_\lambda\):

\[
q_{L,0} = -\int_0^\infty e^{-(\rho + \psi_t)t}(i_t - r_n)dt - \int_0^\infty e^{-(\rho + \psi_t)t}r_L(\sigma c_{S,t} + \varepsilon_\lambda (i_t - r_n))dt.
\]

As before, the yield on long-term bonds depends on the path of future nominal rates and on a term premium. The term premium is a function of savers’ consumption, a channel that is quantitatively small, and of movements in the disaster probability. We calibrate \(\varepsilon_\lambda\) to match the initial response of the 5-year yield on government bonds. Consistent with Gertler and Karadi (2015) and our own estimates reported in Appendix C, we find that a 100 bps increase in the nominal interest rate leads to an increase in the 5-year yield of roughly 20 bps. Figure 5 shows the response of the yield on the long bond and the contributions of the path of future interest rates and the term premium. We find that the bulk of the reaction of the 5-year yield reflects movements in the term premium, a finding that is consistent with the evidence.

The model is also able to capture the responses of other asset prices, which are not directly targeted in the calibration. We consider first the response of the corporate spread, the difference between the yield on a corporate bond and the yield on a government bond (without risk of default) with the same promised cash flow. This corresponds to the way in which the GZ spread is computed in the data by Gilchrist and Zakrajšek (2012). Let \(e^{-\psi_t t}\) denote the coupon paid by the corporate bond. We assume that the monetary shock is too small to trigger a corporate default,

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\(^3\)See Tsai and Wachter (2015) for a review of this literature.
but the corporate bond defaults if a disaster occurs, where lenders recover the amount $1 - \zeta_c$ in case of default. We calibrated $\psi_c$ and $\zeta_c$ to match a duration of 6.5 years and a credit spread of 200 bps in the stationary equilibrium, which is consistent with the estimates reported by Gilchrist and Zakrajšek (2012). Note that the calibration targets the unconditional level of the credit spread. We evaluate the model on its ability to generate an empirically plausible conditional response to monetary shocks.

The price of the corporate bond can be computed analogously to the computation of the long-term government bond

$$q_{C,0} = -\int_0^\infty e^{-(\rho+\psi_t)t}(i_t - r_n)dt - \int_0^\infty e^{-(\rho+\psi_t)t} \left[ \lambda \left( \frac{C_s}{C_e} \right)^\sigma \frac{Q_C - Q^*_C}{Q_C} (\sigma \zeta_t + \epsilon_{\lambda, t}(i_t - r_n)) \right] dt,$$

where $Q_C$ and $Q^*_C$ denote the price of the corporate bond in the stationary equilibrium in the no-disaster and disaster states, respectively. Given the price of the corporate bond, we can compute the corporate spread. Figure 5 shows that the corporate spread responds to monetary shocks by 8.9 bps. We introduce the excess bond premium (EBP) in our VAR and find an increase in the EBP of 6.5 bps and an upper bound of the confidence interval of 10.9 bps, consistent with the prediction of the model. Thus, even though this was not a targeted moment, time-varying risk is able to produce quantitatively plausible movements in the corporate spread.

Another moment that is not targeted by the calibration is the response of stocks to monetary shocks. We find a substantial response of stocks to changes in interest rates, which is explained mostly by movements in the risk premium. In contrast to the empirical evidence, we find a positive response of dividends to a contractionary monetary shock. This is the result of the well-known feature of sticky-prices models that profits are strongly countercyclical. This counterfactual prediction could be easily solved by introducing some form of wage stickiness. Despite the positive response of dividends, the model generates a decline in stocks of 2.15% in response to a 100 bps increase in interest rates, which is smaller than the point estimate of Bernanke and Kuttner (2005), but is still within their confidence interval. Fixing the degree of countercyclicality of profits would likely bring the response of stocks closer to their point estimate.

### 4.3 Time-varying risk and the monetary policy transmission mechanism

Having established that the model with time-varying risk is able to capture the response of asset prices to monetary shocks, we turn now to consider how risk affects the monetary policy transmission mechanism. Consider first the savers’ Euler equation,

$$\dot{c}_{s,t} = \sigma^{-1}(i_t - \pi_t - r_n) + \lambda \left( \frac{C_s}{C_e} \right)^\sigma \epsilon_{s,t} + \sigma^{-1} p_d \epsilon_{\lambda}(i_t - r_n),$$

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34We follow common practice in the asset-pricing literature and report the response of a levered claim on firms’ profits, using a debt-to-equity ratio of 0.5, as in Barro (2006).
where \( p_d \equiv \lambda \left[ \left( \frac{C_t}{\overline{C}} \right) \delta - 1 \right] \) is the price of disaster risk, i.e. the required compensation for a unit exposure to the disaster risk. Time-varying risk introduces a new precautionary savings term for savers, as risk now reacts directly to monetary policy, which will ultimately amplify the impact of changes in nominal rates. By matching the empirical response of asset prices to monetary policy, we are effectively disciplining the magnitude of this precautionary motive with asset-pricing data.

Given the savers’ optimality condition above, we obtain an aggregate Euler equation analogous to the one derived in Proposition 1,

\[
y_t = \bar{\sigma}^{-1}(i_t - \pi_t - r_n) + \delta y_t + v_t,
\]

where \( \bar{\sigma} \) and \( \delta \) are the same as in Proposition 1, and \( v_t \) is now given by

\[
v_t = \frac{\mu_b \chi_r}{1 - \mu_b \chi_y} (\psi_m + \rho)(i_t - r_n) + \frac{1 - \mu_b}{1 - \mu_b \chi_y} \sigma^{-1} p_d \varepsilon \lambda (i_t - r_n).
\]

The first term corresponds to the private wealth effect discussed in Section 3. The second term is new and captures the impact of time-varying risk. The next proposition shows how this new term affects the determination of output in this economy.

**Proposition 8** (Aggregate output with time-varying risk). Suppose that \( i_t - r_n = e^{-\psi_u} (i_0 - r_n) \). The path of aggregate output is then given by

\[
y_t = \frac{1 - \mu_b}{1 - \mu_b \chi_y} y_t^S + \frac{\mu_b \chi_r}{1 - \mu_b \chi_y} y_t^P + \frac{1 - \mu_b}{1 - \mu_b \chi_y} y_t^R + \frac{\overline{\omega} - \delta}{\rho} e^{\omega t} \rho \Omega_0,
\]

where \( y_t^S \) and \( y_t^P \) are given by (14) and (16), respectively, and \( y_t^R = p_d \varepsilon \lambda y_t^S \).

**Proof.** See Appendix B.3. \( \square \)

Proposition 8 shows that, given \( \Omega_0 \), time-varying risk amplifies the response of output in a way that is similar to that of the ISE and inside wealth effect, and the magnitude of the amplification depends crucially on the price of disaster risk \( p_d \) and the degree of time-varying risk captured by \( \varepsilon \lambda \). Figure 6 shows the response of output to a monetary shock for the D-HANK (\( \mu_b > 0 \)) and D-RANK (\( \mu_b = 0 \)) economies. First, we see that introducing time-varying risk substantially amplifies the response of output to monetary policy. Output reacts by 1.05% to a 100 bp increase in the nominal interest rate, which is consistent with the empirical estimates of e.g. Miranda-Agrippino and Ricco (2020). This stands in sharp contrast to the impact on output in the absence of heterogeneity and time-varying risk. If we were to shut down these two channels, the initial response of output would be only –0.14%, a more than sevenfold reduction in the impact of monetary policy. Time-varying risk (TVR) and the outside wealth effect are the two main components determining the output dynamics, representing 39% and 47% of the output response, respectively. In contrast,
the ISE accounts for only 6.5% of the output response, indicating that intertemporal substitution plays only a minor role in the monetary transmission mechanism. Interestingly, heterogeneity interacts with time-varying risk. The left panel in Figure 6 shows that the contribution of the outside wealth effect declines substantially in the D-RANK economy, indicating that the impact of risk is amplified in an economy with heterogeneous agents and private debt. Analogously, one can show that the effect of private debt is amplified in the presence of time-varying risk.

Alternatively, we can quantify the role of risk and private debt as follows: let $y_t^{RANK}$ denote the response of output in the representative-agent economy without risk, $y_t^{D-RANK}$ denote the response of output in a model with time-varying risk, but no heterogeneity, and $y_t^{D-HANK}$ denote the response of output in the full model with both ingredients. We can then apply the following identity,

$$y_t^{D-HANK} = y_t^{RANK} + \Delta_{TVR} + \Delta_{PD},$$

where $\Delta_{TVR} = y_t^{D-RANK} - y_t^{RANK}$ is the contribution of time-varying risk and $\Delta_{PD} = y_t^{D-HANK} - y_t^{D-RANK}$ is the contribution of private debt. Consistent with the results reported above, we find that time-varying risk and private debt explain the bulk of the output response. The contribution of time-varying risk is 50.3% and the contribution of heterogeneity is 30.3%, with the remaining contribution representing the channels in the RANK model.\footnote{Results are similar if we first introduce private debt and then time-varying risk.}

As we have seen, heterogeneity amplifies the effect of time-varying risk on the economy. Moreover, time-varying risk is important for properly capturing the heterogeneous response to monetary shocks. Figure 7 shows that borrowers are disproportionately affected by monetary shocks. The magnitude of the relative response of borrowers and savers, however, is too large in the economy without time-varying risk. The drop in borrowers’ consumption is 7 times greater than the decline in savers’ consumption with a constant disaster probability, while it is 3 times greater in the economy with time-varying risk. Cloyne et al. (2020) estimate a relative peak response of
mortgagors and home-owners of roughly 3.6. Therefore, allowing for time-varying risk is also important if we want to capture the heterogeneous impact of monetary policy.

4.4 The role of fiscal backing

We have found that time-varying risk and heterogeneity substantially amplify the impact of monetary policy on the economy. To properly assess the importance of these two channels, however, it was crucial to control for the implicit fiscal backing, as discussed in Section 3.4. Figure 8 illustrates this point. In the three panels, we show the impact of a monetary shock that leads to an increase in nominal interest rates on impact of 100 bps. In the left panel, we consider a RANK economy ($\mu_b = \lambda = 0$) with the standard value for the EIS ($\sigma^{-1} = 1$) and fiscal backing implicitly determined by a standard Taylor rule, corresponding to the textbook New Keynesian model. In the middle panel, we consider the same economy but the fiscal backing corresponds to the value estimated in the data. The right panel shows our D-HANK model with time-varying risk and the calibrated value of the EIS, $\sigma^{-1} = 0.25$. The response of the textbook economy is only slightly smaller than that of our D-HANK economy despite the lack of time-varying risk or heterogeneous agents. The main difference is the value of the implicit fiscal backing, which is almost ten times larger in the textbook economy compared with the one we estimated in the data. The response of output drops by almost half when the fiscal backing is the same as in the data.

Note that the value of the EIS also plays an important role. Even with fiscal backing directly from the data, the response of output is still significant, only slightly less than that in our D-RANK with time-varying risk. But this same response comes from very different channels. In the RANK economy, the ISE accounts for roughly 40% of the output response, while in our D-RANK the ISE accounts for less than 7% of that response. Therefore, the strong response of the ISE in RANK relies to a great extent on having a value of the EIS that is much larger than the empirical estimates, as in Best et al. (2020).
Figure 8: Output in RANK vs D-HANK with time-varying risk.

Note: The first two panels show output in RANK ($\mu_b = \lambda = 0$) with unit EIS ($\sigma^{-1} = 1$). In the left panel, fiscal backing is determined by a Taylor rule, while in the middle panel fiscal backing corresponds to the value estimated in the data. The right panel corresponds to the D-HANK economy with time-varying risk and the estimated fiscal backing.

Thus, one can attribute the quantitative success of the RANK model (see, for example, Christiano et al. (2005)) to implausibly large fiscal backing in response to monetary shocks and a strong intertemporal-substitution channel, which compensate for missing heterogeneous agents and risk channels. Once we discipline the fiscal backing with data and calibrate the EIS to the estimates obtained from microdata, the limitations of the standard RANK model become apparent. Our model shows that it is possible to incorporate indebted hand-to-mouth agents and time-varying aggregate risk without sacrificing the tractability of standard macro models.

5 Conclusion

In this paper we provide a novel unified framework in which we analyze the role of heterogeneity and risk in a tractable linearized New Keynesian model. We are able to study the implications of positive private liquidity and heterogeneous MPCs, a combination that has been proven elusive to the analytical HANK literature. Moreover, we capture both important unconditional asset-pricing moments, such as the equity premium and an upward-sloping yield curve, and conditional moments, such as the response of government bonds, corporate bonds, and equities, in response to monetary shocks. Despite its richness, the model can be fully characterized in closed-form.

We show how monetary policy affects the economy through the intertemporal-substitution channel as well as inside and outside wealth effects, and time-varying risk premia. We find that wealth effects play an important role in the transmission of monetary shocks. Quantitatively, we find that time-varying risk explains roughly 50% of the output response, while the presence of private debt accounts for 30% of the response.

The methods introduced in this paper can be applied beyond the current model. For instance, it can be applied to a full quantitative HANK model with idiosyncratic risk, extending the results of Ahn et al. (2018) to allow for time-varying risk premia. Alternatively, one could introduce risky household debt, or a richer capital structure for firms, and study the pass-through of monetary
policy to households and firms. These methods may enable us to bridge the gap between the extensive existing work on heterogeneous agents and monetary policy and the emerging literature on the role of asset prices in the transmission of monetary shocks.
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Appendix: For Online Publication

A Derivations for Sections 2 and 3

In this appendix, we provide the derivations of the expressions provided in Section 2 and 3. We consider first the optimality conditions for the non-linear model, then the derivation of the stationary equilibrium, and, finally, the log-linear equilibrium conditions.

A.1 The non-linear model

**Derivation of labor supply and Euler equation.** The household problem is given by

\[ V_{j,t}(B_j) = \max_{[C_{j,t}, N_{j,t}] \geq 0} \mathbb{E}_t \left[ \int_{t}^{t^*} e^{-\rho(z-t)} \left( \frac{C_{j,t}^{1-\sigma}}{1-\sigma} - \frac{N_{j,t}^{1+\phi}}{1+\phi} \right) dz + e^{-\rho(t^*-t)} V_{j,t^*}(B_{j,t^*}) \right], \]

subject to the conditions

\[ B_{j,t} = (i_t - \pi_t) B_{j,t} + \frac{W_{j,t}}{P_t} N_{j,t} + \frac{\Pi_{j,t}}{P_t} + \tilde{T}_{j,t} - C_{j,t}, \quad B_{j,t} \geq -\overline{D}_p, \]

given the initial condition \( B_{j,0} = B_j \geq -\overline{D}_p. \)

The HJB equation is given by

\[ \rho_j V_{j,t} = \max_{C,N} \left\{ \frac{C_{j,t}^{1-\sigma}}{1-\sigma} - \frac{N_{j,t}^{1+\phi}}{1+\phi} + \frac{\partial V_{j,t}}{\partial B} \left[ (i_t - \pi_t) B_{j,t} + \frac{W_{j,t}}{P_t} N_{j,t} + \frac{\Pi_{j,t}}{P_t} + \tilde{T}_{j,t} - C_{j,t} \right] + \lambda_t (V_{j,t} - V_{j,t}), \right\}, \]

where we allow for the arrival rate \( \lambda_t \) to be potentially time-varying.

The first-order conditions are given by

\[ C_{j,t}^{1-\sigma} = \frac{\partial V_{j,t}}{\partial B}, \quad N_{j,t}^{1+\phi} = \frac{\partial V_{j,t}}{\partial B} \frac{W_{j,t}}{P_t}, \]

The solution is also subject to the state-constraint boundary condition\(^{36}\)

\[ \frac{\partial V_{j,t}(-\overline{D}_p)}{\partial B} \geq \left( -(i_t - \pi_t) \overline{D}_p + \frac{W_{j,t}}{P_t} N_{j,t} + \frac{\Pi_{j,t}}{P_t} + \tilde{T}_{j,t} \right)^{-\sigma}, \]

which implies that \( \dot{B}_{j,t} \geq 0 \) at \( \dot{B}_{j,t} = -\overline{D}_p \), guaranteeing that the borrowing constraint is not violated.

Combining the first-order conditions, we obtain the labor-supply condition

\[ \frac{W_t}{P_t} = N^{\phi}_t C^\tau_{j,t}, \]

which coincides with the expression provided in the main text.

\(^{36}\text{See Achdou et al. (2017) for a discussion of state-constraint boundary conditions in the context of continuous-time savings problems with borrowing constraints.}\)
The envelope condition with respect to $B_{j,t}$ for an unconstrained household is given by

$$\frac{\partial V_{i,t}}{\partial B} = (i_t - \pi_t) \frac{\partial V_{i,t}}{\partial B} + \mathbb{E}_t \left[ \frac{\partial V_{i,t}}{\partial B} \right] \frac{dt}{dt}.$$  

Combining the expression above with the first-order condition for consumption, we obtain

$$\mathbb{E}_t \left[ dC_{j,t}^{-\sigma} \right] = -(i_t - \pi_t - \rho_j) C_{j,t}^{-\sigma}.$$  

Expanding the expectation of the marginal utility, we obtain the Euler equation for savers

$$\frac{\partial C_{s,t}}{\partial C_{s,t}} = \sigma^{-1} (i_t - \pi_t - \rho_j) + \frac{\lambda_t}{\sigma} \left( \frac{C_{s,t}}{C_{s,t}} \right)^{\sigma} - 1,$$

which coincides with expression (6) provided in the main text for $\lambda_t = \lambda$.

**Derivation of the non-linear NK Phillips curve.** Final goods are produced according to the production function

$$Y_t = \left( \int_0^1 Y_{i,t}^{\frac{1}{1-\epsilon}} \right)^{1/(1-\epsilon)}.$$

The problem of the final-goods producer is given by

$$\max_{[Y_{i,t}]_{i=0}} P_t \left( \int_0^1 Y_{i,t}^{\frac{1}{1-\epsilon}} dt \right)^{1/(1-\epsilon)} - \int_0^1 P_{i,t} Y_{i,t} di.$$

The demand for variety $i$ and the price level are given by

$$Y_{i,t} = \left( \frac{P_{i,t}}{P_t} \right)^{-\epsilon} Y_t, \quad P_t = \left( \int_0^1 P_{i,t}^{1-\epsilon} di \right)^{1/(1-\epsilon)}.$$

The intermediate-goods producers’ problem is given by

$$Q_{i,t}(P_t) = \max_{[\pi_{i,t}]_{i=1}} \mathbb{E}_t \left[ \int_t^{t'} \frac{\eta_i}{\eta_t} \left( 1 - \tau \right) \frac{P_{i,t}}{P_t} Y_{i,t} - \frac{W_{i,t}}{Y_t} - \frac{\varphi_i^2}{2} (j) \right] ds + \frac{\eta_t^{\ast}}{\eta_t} Q_{i,t}^{\ast},$$

subject to $Y_{i,t} = \left( \frac{P_{i,t}}{P_t} \right)^{-\epsilon} Y_t$ and $P_{i,t} = \pi_{i,t} P_{i,t}$, given $P_{i,t} = P_t$.

The HJB equation for this problem is

$$0 = \max_{\pi_{i,t}} \eta_t \left( (1 - \tau) \frac{P_{i,t}}{P_t} Y_{i,t} - \frac{W_{i,t}}{Y_t} - \frac{\varphi_i^2}{2} (j) \right) dt + \mathbb{E}_t \left[ d(\eta_t Q_{i,t}) \right].$$

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Applying Ito’s lemma for jump processes, we obtain
\[
\frac{E_t[d(\eta_t Q_{i,t})]}{\eta_t dt} = \frac{\partial Q_{i,t}}{\partial t} + \frac{\partial Q_{i,t}}{\partial P_t} \pi_{i,t} P_t + \lambda_t (Q_{i,t}^* - Q_{i,t}) - (\pi_t - \pi_i) Q_{i,t} + \lambda_t \left( \frac{\eta_t - \eta_i}{\eta_t} \right) (Q_{i,t}^* - Q_{i,t}) \\
= \frac{\partial Q_{i,t}}{\partial t} + \frac{\partial Q_{i,t}}{\partial P_t} \pi_{i,t} P_t - (\pi_t - \pi_i) Q_{i,t} + \lambda_t \left( \frac{C_{i,t}}{C_t} \right)^\sigma (Q_{i,t}^* - Q_{i,t}),
\]
using the fact that \( \frac{E_t[d\eta_t]}{\eta_t} = -(\pi_t - \pi_i) dt \).

Firms’ HJB equation can be written as
\[
0 = \max_{\pi_{i,t}} \left( (1-\tau) \frac{P_{i,t}}{P_t} - \frac{W_t}{P_t} \right) \left( \frac{P_{i,t}}{P_t} \right)^{-\epsilon} \pi_t\frac{\pi_t}{\pi_t} + \frac{\partial Q_{i,t}}{\partial t} + \frac{\partial Q_{i,t}}{\partial P_t} \pi_{i,t} P_t - (\pi_t - \pi_i) Q_{i,t} + \lambda_t \left( \frac{\eta_t^*}{\eta_t} \right) (Q_{i,t}^* - Q_{i,t}),
\]
where \( \eta_t^* \equiv e^{-\rho t} \left( C_{i,t} \right)^{-\sigma} \).

The first-order condition is given by
\[
\frac{\partial Q_{i,t}}{\partial P_t} P_{i,t} = \varphi \pi_{i,t}.
\]

The envelope condition with respect to \( P_{i,t} \) is given by
\[
0 = \left( (1-\epsilon)(1-\tau) \frac{P_{i,t}}{P_t} + \epsilon \frac{W_t}{P_t} \right) \left( \frac{P_{i,t}}{P_t} \right)^{-\epsilon} \pi_t\frac{\pi_t}{\pi_t} + \frac{\partial^2 Q_{i,t}}{\partial t^2} + \frac{\partial^2 Q_{i,t}}{\partial P_t^2} \pi_{i,t} P_t P_t + \frac{\partial Q_{i,t}}{\partial P_t} \pi_{i,t} P_t P_t - (\pi_t - \pi_i) \frac{\partial Q_{i,t}}{\partial P_t}.
\]

Multiplying the first-order condition by \( \eta_t \) and computing the expected change over time, we obtain
\[
\left( \frac{\partial^2 Q_{i,t}}{\partial t^2} + \frac{\partial^2 Q_{i,t}}{\partial P_t^2} \pi_{i,t} P_t P_t \right) P_{i,t} + \frac{\partial Q_{i,t}}{\partial P_t} \pi_{i,t} P_t P_t + \lambda_t \left( \frac{\eta_t^*}{\eta_t} \right) \frac{\partial Q_{i,t}}{\partial P_t} P_{i,t} = \varphi \pi_{i,t}.
\]

Combining the previous two equations, we obtain
\[
0 = \left( (1-\epsilon)(1-\tau) \frac{P_{i,t}}{P_t} + \epsilon \frac{W_t}{P_t} \right) \left( \frac{P_{i,t}}{P_t} \right)^{-\epsilon} \pi_t + \frac{\partial Q_{i,t}}{\partial t} + (\pi_t - \pi_i) \varphi \pi_{i,t} - (\pi_t - \pi_i) \frac{\partial Q_{i,t}}{\partial P_t}.
\]

Rearranging the expression above, we obtain the non-linear New Keynesian Phillips curve
\[
\pi_t = (\pi_t - \pi_i + \lambda_t \left( \frac{\eta_t^*}{\eta_t} \right) \pi_t - (\epsilon - 1) \varphi^{-1} \left( \frac{e}{e-1} \frac{W_t}{P_t} - (1-\tau) \right) \pi_t,
\]
where we assumed that \( P_{i,t} = P_t \) for all \( i \in [0,1] \).

**A.2 Stationary equilibrium**

**Introducing recurrent shocks.** Suppose aggregate productivity follows the following process:\footnote{The process can be easily generalized to allow for trend growth, \( dA_t = \delta A_t dt - \xi A_t dN_t \). The expressions in the text apply to this case as well after the variables are properly detrended.}

\[
\frac{dA_t}{A_t} = -\zeta dN_t
\]
given \( A_0 = A \) and \( 0 < \zeta < 1 \), where \( \mathcal{N}_t \) is a Poisson process with arrival rate \( \lambda_t = \lambda(A_t) \).

Note that the above process implies that aggregate productivity after a Poisson event is given by \( A(1 - \zeta) \equiv A^\bullet \). The setting discussed in the main text corresponds to the case where \( \lambda(A) > 0 \) and \( \lambda(A_t) = 0 \) for \( A_t < A \). This formulation also captures the case of recurrent shocks assumed by Barro (2009), where \( \lambda_t \) is constant and positive for all \( t \).

**Consumption determination in a stationary equilibrium.** In a stationary equilibrium, all variables are independent of calendar time and depend only on aggregate productivity \( A_t \), so they are constant between the realization of disasters. We can then write, for instance, consumption and output as \( C_{j,t} = C_j(A_t) \) and \( Y_t = Y(A_t) \), where \( C_j(\cdot) \) and \( Y(\cdot) \) are functions we need to solve for.

First, note that, imposing \( \dot{S}_{s,t} = 0 \) in the Euler equation (1), we obtain the natural rate \( r_n(A) \)

\[
r_n(A) = \rho_s - \lambda(A) \left[ \left( \frac{C_s(A)}{C_s(A(1 - \zeta))} \right)^\sigma - 1 \right].
\]

The consumption of borrowers and savers satisfy the conditions

\[
C_b(A) = \left[ A(1 - \tau)(1 - e^{-1}) \right]^{1+\phi} \frac{C_b^\phi}{C_s^\phi}(A) + T_b(A) - r_n(A)\bar{D}_p \tag{28}
\]

\[
C_s(A) = \left[ A(1 - \tau)(1 - e^{-1}) \right]^{1+\phi} \frac{C_s^\phi}{C_s^\phi}(A) + \frac{1 - (1 - \tau)(1 - e^{-1})}{1 - \mu_b} Y(A) - \frac{\mu_b T_b(A)}{1 - \mu_b} - \frac{r_n(A)(1 - \mu_b)}{1 - \mu_b} \bar{D}_p, \tag{29}
\]

where \( T_b(A) \) represents the level of transfers as a function of productivity, and we used the labor supply condition to solve for \( N_b \) and \( N_s \) and the fact that the real wage is given by

\[
\frac{W_t}{P_t} = A_t(1 - \tau)(1 - e^{-1}),
\]

obtained by imposing \( \pi_t = 0 \) in the non-linear New Keynesian Phillips curve (3).

Given that \( Y(A) = \sum_{j \in \{b,s\}} \mu_j C_j(A) \), the above conditions provide a pair of functional equations that determine \( C_j(A) \). To ease notation, we write \( C_j \equiv C_j(A) \) and \( C_j^\bullet \equiv C_j(A(1 - \zeta)) \) to denote variables in the no-disaster and disaster states, respectively.

**Symmetric stationary equilibrium.** Note that, using \( \mu_b = 0 \) and \( C_s = Y \) in the expression for \( C_s \), we can solve for output in a representative-agent economy:

\[
\bar{Y}(A) = A^{1+\phi} \left[ (1 - \tau)(1 - e^{-1}) \right]^{1+\phi}.
\]

We obtain the output level \( \bar{Y} \) in the economy with \( \mu_b > 0 \) if \( T_b(A) \) satisfies

\[
T_b(A) = \left[ 1 - (1 - \tau)(1 - e^{-1}) \right] \bar{Y}(A) + \left( \rho_s - \lambda(A) \left[ \left( \frac{\bar{Y}(A)}{\bar{Y}(A(1 - \zeta))} \right)^\sigma - 1 \right] \right) \bar{D}_p. \tag{30}
\]
Plugging the value of \( T_b(A) \) into the expression for \( C_j(A) \), we obtain

\[
C_b(A) = \left[ A(1 - \tau)(1 - e^{-1}) \right]^{1 + \phi} \left[ \frac{C_b^*}{C_b(A(1 - \zeta))} \right]^\sigma + \left[ 1 - (1 - \tau)(1 - e^{-1}) \right] \sum_{j \in \{k, s\}} \mu_j C_j(A)
\]

\[
+ \lambda(A) \frac{\mu_k D_p}{1 - \mu_b} \left[ \left( \frac{C_s(A)}{C_s(A(1 - \zeta))} \right) - \left( \frac{\bar{Y}(A)}{\bar{Y}(A(1 - \zeta))} \right) \right]^\sigma
\]

\[
C_s(A) = \left[ A(1 - \tau)(1 - e^{-1}) \right]^{1 + \phi} \left[ \frac{C_s^*}{C_s(A(1 - \zeta))} \right]^\sigma + \left[ 1 - (1 - \tau)(1 - e^{-1}) \right] \sum_{j \in \{k, s\}} \mu_j C_j(A)
\]

\[
+ \lambda(A) \frac{\mu_k D_p}{1 - \mu_b} \left[ \left( \frac{C_s(A)}{C_s(A(1 - \zeta))} \right) - \left( \frac{\bar{Y}(A)}{\bar{Y}(A(1 - \zeta))} \right) \right]^\sigma
\]

A solution to the above system is \( C_j(A) = \bar{Y}(A) \). In this case, we obtain a symmetric stationary equilibrium, where consumption in both households is the same. Moreover, consumption is a power function of productivity, which implies that \( C_s^* \) is given by

\[
C_s^* = (1 - \zeta)^{\frac{1 + \phi}{\sigma}} C_s.
\]

Therefore, in a symmetric stationary equilibrium, the real interest rate is given by

\[
r_n(A) = \rho_s - \lambda(A) \left[ (1 - \zeta)^{-\phi \frac{1 + \phi}{\sigma \xi}} - 1 \right].
\]

**Role of private debt.** In a symmetric stationary equilibrium, the government taxes all of the income savers receive by lending to borrowers, so the natural interest rate is independent of the level of private debt. To allow a role for private debt in determining the natural rate, we assume that transfers to borrowers satisfy the condition

\[
T_b(A^*) = \left[ 1 - (1 - \tau)(1 - e^{-1}) \right] \bar{Y}(A^*),
\]

and transfers to borrowers are given by (30) if \( A_t \neq A^* \). This provides the minimal deviation from the symmetric stationary equilibrium that allows a role for private debt in determining the natural rate.

**Proposition 9.** Suppose \( T_b(A^*) \) is given by (31) and \( r_n > 0 \). Let \( C_s = C_s(A) \) and \( C_s^* = C_s(A^*) \), where \( A^* = A(1 - \zeta) \). Then, the natural interest rate \( r_n(A) \) is given by

\[
r_n(A) = \rho_s - \lambda(A) \left[ \left( \frac{C_s}{C_s^*} \right)^\sigma - 1 \right],
\]

and it is strictly increasing in \( D_p \).

**Proof.** Consider the determination of \( (C_s^*, C_b^*) \). The condition for \( C_b^* \) is given by

\[
0 = \left[ A^*(1 - \tau)(1 - e^{-1}) \right]^{\frac{1 + \phi}{\xi}} (C_b^*)^{-\frac{\xi}{\sigma}} + \left[ 1 - (1 - \tau)(1 - e^{-1}) \right] \bar{Y}(A^*) - r_n^*(A^*; C_b^*) D_p - C_b^*,
\]

where the notation \( r_n^*(A^*; C_b^*) \) makes explicit that the natural interest rate depends on \( C_b^* \).

The right-hand side of the above expression is positive for sufficiently small \( C_b^* \), it is negative for sufficiently large \( C_b^* \), and it is strictly decreasing in \( C_b^* \). Therefore, there is a unique solution to the above
non-linear equation, given the values of $C_s^*$ and $\overline{D}_p$, which we denote by $C_s^*(C_s^*, \overline{D}_p)$. Applying the implicit function theorem to the above expression, we can show that $C_s^*(C_s^*, \overline{D}_p)$ is strictly increasing in $C_s^*$. Moreover, $C_s^*(C_s^*, \overline{D}_p)$ approaches zero as $C_s^*$ approaches zero and it is bounded as $C_s^*$ approaches infinity.

Savers’ consumption is determined by $g(C_s^*) = 0$, where $g(C_s^*)$ is given by

$$g(C_s^*) = \left[ A^*(1 - \tau)(1 - e^{-1}) \right]^{1 + \phi} (C_s^*)^{-\phi} - (1 - \tau)(1 - e^{-1}) C_s^*$$

$$+ \mu_b \frac{(1 - (1 - \tau)(1 - e^{-1}))(C_s^*(C_s^*, \overline{D}_p) - \overline{Y}(A^*)) + r_n^*(A^*; C_s^*) \overline{D}_p}{(1 - \mu_b)}.$$ 

Using the properties of $C_s^*(\cdot)$, we infer that $g(\cdot)$ approaches infinity as $C_s^* \to 0$ and it approaches $-\infty$ as $C_s^* \to \infty$. The continuity of $g(\cdot)$ then implies that a solution $C_s^*(\overline{D}_p)$ exists. It remains to show that the solution is unique. Applying the implicit function theorem, it can be shown that the term in the second line of the above expression is strictly decreasing in $C_s^*$, so $g(C_s^*)$ is also strictly decreasing and there is a unique solution to $g(C_s^*) = 0$.

Applying the implicit function theorem, we obtain

$$\frac{\partial C_s^*}{\partial \overline{D}_p} = \frac{\mu_b r_n^*}{1 - \mu_b} \left[ 1 - \frac{1 - (1 - \tau)(1 - e^{-1})}{1 + \frac{e}{\phi} [A^*(1 - \tau)(1 - e^{-1})]^{1 + \phi} (C_s^*)^{-\phi - \frac{\phi}{\mu_b}} \right] > 0,$$

Given that the solution coincides with $\overline{Y}(A^*)$ if $\overline{D}_p = 0$, we conclude that $C_s^*(\overline{D}_p) > \overline{Y}(A^*)$ for $\overline{D}_p > 0$. Since $C_s = \overline{Y}(A)$, $r_n$ is increasing in $\overline{D}_p$ and it is larger than in the symmetric stationary equilibrium. □

### A.3 Risk premium

The value of the intermediate-goods firms satisfies the standard pricing condition

$$\frac{\Pi_t}{Q_t} dt + \frac{\mathbb{E}_t[d(\eta_t Q_t)]}{\eta_t Q_t} = 0,$$

where $\Pi_t$ represents the sum of the profit of the intermediate-goods producer and the rebate to households of the cost of adjusting prices.

Rearranging the pricing condition, we obtain

$$\frac{\Pi_t}{Q_t} dt + \frac{\mathbb{E}_t[dQ_t]}{Q_t} = \left[ \frac{(i_t - \pi_t) + \lambda_t (C_{s,t}^*)^{-\sigma} - C_{s,t}^{-\sigma} \frac{Q_t - Q_t^*}{C_{s,t}^{-\sigma}}}{\text{interest rate}} \right] dt.$$

In a stationary equilibrium, profits are given by $\Pi_t = e^{-1}(1 - \tau)Y_t$ and the value of the firm is given by $Q_t = Q(A_t)$. We can write the above expression as follows:

$$\frac{\Pi_t}{Q_t} = r_{n,t} + \lambda_t C_{s,t}^{\sigma} \left( 1 - \frac{\Pi_t}{Q_t} \frac{Q_t^*}{\Pi_t} \frac{\Pi_t^*}{\Pi_t} \right).$$

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Rearranging the above expression, we obtain

\[ \frac{\Pi_t}{Q_t} = \frac{\rho_s + \lambda_t}{1 + \lambda_t \frac{C_i}{C_{i,j}} \frac{Q}{Q_t} \Pi_t^f}. \]

Consider first the case where \( \lambda_t > 0 \) in the no-disaster state and \( \lambda_t = 0 \) in the disaster state. Therefore, \( \Pi_t^f = \rho_s Q_t^f \) and \( \Pi_t / Q_t \) can be written as

\[ \frac{\Pi_t}{Q_t} = \frac{\rho_s (\rho_s + \lambda_t)}{\rho_s + \lambda_t \frac{C_i}{C_{i,j}} (1 - \xi_Y)}, \]

where \( 1 - \xi_Y \equiv \frac{Y^*}{Y} \).

The risk premium can then be written as

\[ \frac{\Pi}{Q} dt + \frac{\lambda Q^* - Q}{Q} - r_n = \frac{\lambda}{\alpha} \left( (1 - \xi_Y)^{-\alpha} - 1 \right) \left( 1 - \frac{(\rho_s + \lambda_t)(1 - \xi_Y)}{\rho_s + \lambda_t \frac{C_i}{C_{i,j}}} \right), \]

using the fact that \( \frac{\Pi_t^f}{\Pi_t} = 1 - \xi_Y \).

Suppose now that \( \lambda_t = \lambda \) for all \( t \geq 0 \). The dividend-yield is then given by

\[ \frac{\Pi_t}{Q_t} = r_n + \lambda (1 - \xi_y)^{-\sigma} b_y, \]

using the fact that \( \frac{\Pi}{Q} = \frac{\Pi^r}{Q^r} \).

The risk premium is given by

\[ \frac{\Pi}{Q} dt + \frac{\lambda Q^* - Q}{Q} - r_n = \frac{\lambda}{\alpha} \left( (1 - \xi_Y)^{-\alpha} - 1 \right) \xi_Y, \]

which coincides with the expression for the risk premium in Barro (2009).

### A.4 Log-linear dynamics

We use lower-case variables to denote log-deviations from the stationary equilibrium, e.g. \( c_{i,t} \equiv \log C_{i,t} / C_j \) and \( n_{i,t} = \log N_{i,t} / N_j \). We derive the equilibrium conditions for the general case where \( C_b \) may differ from \( C_s \), and then specialize to the \( C_b = C_s \) case considered in Section 2.

**Labor supply and market clearing.** The labor supply condition can be written as

\[ w_t - p_t = \phi n_{i,t} + \sigma c_{j,t}. \]

Log-linearizing the market-clearing conditions for consumption and labor, we obtain

\[ \mu^c_b c_{b,t} + (1 - \mu^c_b) c_{s,t} = y_t, \quad \mu^b_b n_{b,t} + (1 - \mu^b_b) n_{s,t} = n_t, \]

where \( \mu^c_b \equiv \frac{\mu^c_b}{Y} \) and \( \mu^b_b \equiv \frac{\mu^b_b}{N} \).

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Given $c_{b,t}$ and $y_t$, we can use the above equations to solve for the real wage, savers’ consumption, and labor supply for both agents. From the labor-supply condition, we obtain

$$n_{b,t} = n_{b,t} + \phi^{-1}\sigma(c_{b,t} - c_{s,t})$$

$$= n_{b,t} + \phi^{-1}\sigma(1 - \mu_b^c)(c_{b,t} - y_t),$$

using the market-clearing condition for goods to eliminate $c_{s,t}$.

Plugging the above expression into the market-clearing condition for labor, we obtain

$$n_{b,t} = (1 + \phi^{-1}\sigma)y_t - \phi^{-1}\sigma c_{b,t} + \phi^{-1}\sigma \frac{\mu_b^c - \mu_b^s}{1 - \mu_b^c}(y_t - c_{b,t}).$$

The real wage is given by

$$w_t - p_t = (\phi + \sigma)y_t + \sigma \frac{\mu_b^c - \mu_b^s}{1 - \mu_b^c}(y_t - c_{b,t}).$$

**Borrowers’ consumption.** Linearizing borrowers’ budget constraint, we obtain

$$c_{b,t} = \frac{WN_b}{PC_b}(w_t - p_t + n_{b,t}) + T_{b,t} - \alpha\pi_t - r_n\bar{a}_p.$$

where $T_{b,t} = \frac{T_{b,t} - T_b}{c_b}$, and $\bar{a}_p = \frac{T_{b,t}}{c_b}$.

Plugging the expressions for the real wage and labor supply into the above expression, we obtain

$$c_{b,t} = \frac{WN_b}{PC_b} \left[ (1 + \phi^{-1})(\phi + \sigma)y_t - \phi^{-1}\sigma c_{b,t} + (1 + \phi^{-1})\sigma \frac{\mu_b^c - \mu_b^s}{1 - \mu_b^c}(y_t - c_{b,t}) \right]$$

$$+ T_{b,t} - \alpha\pi_t - r_n\bar{a}_p.$$

Transfers to borrowers are given by

$$T_{b,t} = \psi_b y_t,$$

where $\psi_b = T_b'(\bar{\gamma}).$

Combining the previous two conditions, we obtain

$$c_{b,t} = \chi_y y_t - \chi_f (\alpha\pi_t - r_n),$$

where

$$\chi_y = \psi_b + \frac{WN_b}{PC_b} \left[ (1 + \phi^{-1})(\phi + \sigma) + (1 + \phi^{-1})\sigma \frac{\mu_b^c - \mu_b^s}{1 - \mu_b^c} \right]$$

$$\frac{1}{1 + \frac{WN_b}{PC_b} \left[ \phi^{-1}\sigma + (1 + \phi^{-1})\sigma \frac{\mu_b^c - \mu_b^s}{1 - \mu_b^c} \right]}$$

$$\chi_f = \frac{WN_b}{PC_b} \left[ \frac{\mu_b^c - \mu_b^s}{1 - \mu_b^c} \right].$$

The expression in the text is obtained by imposing $C_b = C_s = Y$, so $\mu_b^c = \mu_b^s = \mu_b$ and $1 - \alpha = \frac{WN}{\bar{P}}$. 

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Savers’ consumption. From the borrowers’ consumption and market clearing, we obtain

$$c_{s,t} = \frac{1 - \mu_b X_y}{1 - \mu_b} y_t + \frac{\mu_b X_x}{1 - \mu_b} (i_t - \pi_t - r_n).$$

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Flow budget constraints. The flow budget constraint for savers can be written as

$$\bar{b}_s \dot{b}_{s,t} = (i_t - \pi_t - r_n)\bar{b}_s + r_n \bar{b}_s \dot{b}_{s,t} + \frac{WN_s}{PC_s} (w_t - p_t + n_{s,t}) + \frac{(1 - \tau) y_t - (1 - \alpha)(w_t - p_t + y_t)}{1 - \mu_b}$$

$$+ T_{s,t} - c_{s,t},$$

where $1 - \alpha \equiv \frac{WN}{\bar{r}}$ is the labor share, $T_{s,t} \equiv \frac{T_{s,t} - \bar{T}_s}{\bar{r}_s}$, and $\bar{b}_s \equiv \frac{\bar{b}_s}{\bar{r}_s}$.

The government’s budget constraint is given by

$$\bar{a}_g \dot{a}_{g,t} = (i_t - \pi_t - r_n)\bar{a}_g + r_n \bar{a}_g \dot{a}_{g,t} + \sum_{j \in \{b_s\}} \mu_j^s T_{j,t} - \tau y_t.$$

Stocks and human wealth. The value of the firm satisfies the condition

$$\frac{\Pi_t}{Q_t} + \frac{\dot{Q}_t}{Q_t} = i_t - \pi_t + \lambda_t \left( \frac{C_{s,t}}{C_{s,t}} \right)^\sigma \left( 1 - \frac{Q^*}{Q_t} \right).$$

After some rearrangement, the above expression can be written as

$$\dot{Q}_t = \left[ i_t - \pi_t + \lambda_t \left( \frac{C_{s,t}}{C_{s,t}} \right)^\sigma \right] Q_t - \lambda_t \left( \frac{C_{s,t}}{C_{s,t}} \right)^\sigma Q^* - \Pi_t.$$

Log-linearizing the above expression, we obtain

$$\dot{q}_t = \rho q_t + (i_t - \pi_t - r_n) + \lambda \left( \frac{C_s}{C_s} \right)^\sigma \frac{Q - Q^*}{Q} (\sigma c_{s,t} + e\lambda_j (i_t - r_n)) -$$

$$\frac{Y}{Q} ((1 - \tau) y_t - (1 - \alpha)(w_t - p_t + n_t)),$$

where we defined $q_t \equiv (Q_t - Q) / Q$.

Solving the above equation forward, we obtain

$$q_0 = \frac{Y}{Q} \int_0^\infty e^{-\rho t} \left[ (1 - \tau) y_t - (1 - \alpha)(w_t - p_t + n_t) \right] dt$$

$$- \int_0^\infty e^{-\rho t} \left[ i_t - \pi_t - r_n + \lambda \left( \frac{C_s}{C_s} \right)^\sigma \frac{Q - Q^*}{Q} (\sigma c_{s,t} + e\lambda_j (i_t - r_n)) \right] dt.$$

The change in the value of stocks depends on the change in dividends, firms’ profits, and on changes in the discount rate, captured by changes in the real interest rate and the risk premium. Note that the present discount value of savers’ consumption is given by

$$\int_0^\infty e^{-\rho t} c_{s,t} dt = \frac{\mu_b X_y}{1 - \mu_b} \int_0^\infty e^{-\rho t} (i_t - \pi_t - r_n) dt + \frac{1 - \mu_b X_y}{1 - \mu_b} \Omega_0,$$
using \( c_{s,t} = \frac{1-\mu_0 X_t}{\mu_0} y_t + \frac{\mu_0 X_t}{\mu_0} (i_t - \pi_t - r_n) \).

For a given value of \( \Omega_0 \), an increase in real rates will raise the risk premium on average. Therefore, the response of the risk premium will amplify the response of real rates when \( \Omega_0 = 0 \).

Similarly, if we define human wealth as the present discount value of (after-tax) labor income \( H_t = E_t \left[ \int_t^\infty \frac{w_t}{\eta t} \left( \frac{W_T}{T} N_T + T_z \right) dt \right] \), we can then write the linearized value of human wealth as follows:

\[
h_0 = \frac{Y}{H} \int_0^\infty e^{-\rho t} \left( (1 - \alpha) (w_t - p_t + n_t) + T_t \right) dt - \int_0^\infty e^{-\rho t} \left[ i_t - \pi_t - r_n + \lambda \left( \frac{C_s}{C^*} \right) \frac{H - H^*}{H} (\sigma c_{s,t} + \epsilon_{\lambda,t} (i_t - r_n)) \right] dt.
\]

### A.5 Long-term bonds

**Portfolio problem and pricing.** Suppose savers now face a portfolio problem where they choose how much to invest in short-term and long-term bonds. For simplicity, we assume that borrowers issue only short-term bonds and the government issues only long-term (nominal) bonds. Let \( \tilde{Q}_{L,t} = \frac{Q_{L,t} e^{-\psi_L t}}{P_t} \) denote the real value of a long-term bond that pays the coupon \( e^{-\psi_L t} / P_t \) in real terms, where \( P_t \) is the price level at period \( t \). The return on the long-term bond is given by

\[
dR^*_t = \left[ \frac{1}{\tilde{Q}_{L,t}} + \frac{\tilde{Q}_{L,t}}{\tilde{Q}_{L,t} - \psi_t - \pi_t} \right] dt + \frac{\tilde{Q}_{L,t}^* - \tilde{Q}_{L,t}}{\tilde{Q}_{L,t}} dN_t,
\]

where \( \tilde{Q}_{L,t} \) denotes the time-derivative of the price of the bond conditional on there being no disasters.

Let \( B_{s,t} = B_{s,t}^S + \tilde{Q}_{L,t} B_{s,t}^L \) denote the total value of bonds held by savers, that is, the sum of short-term \( (B_{s,t}^S) \) and long-term \( (B_{s,t}^L) \) bonds. Let \( \omega_{j,t} = \frac{B_{j,t}}{B_{s,t}} \) denote the share invested in the long-term bond. The law of motion of savers’ total bond position is given by

\[
dB_{j,t} = \left[ (i_t - \pi_t + \omega_{j,t} r_{L,t}) B_{j,t} + \frac{W_{j,t}}{P_t} N_{j,t} + \frac{\Pi_{j,t}}{P_t} + T_{j,t} - C_{j,t} \right] dt + \omega_{s,t} B_{j,t} \frac{Q_{L,t}^* - Q_{L,t}}{Q_{L,t}} dN_t,
\]

where \( r_{L,t} = \frac{1}{\tilde{Q}_{L,t}} + \frac{\tilde{Q}_{L,t}}{\tilde{Q}_{L,t} - \psi_t - i_t} \) is the excess return on long-term bonds conditional on there being no disasters.

The HJB equation for the households’ problem is given by

\[
\rho_j V_{j,t} = \max_{c_{s,t},o,N} \left\{ \frac{C_{j,t}^{1-\sigma}}{1-\sigma} - \frac{N_{j,t}^{1+\phi}}{1+\phi} + \frac{\partial V_{j,t}}{\partial B} \left[ (i_t - \pi_t + \omega_{j,t} r_{L,t}) B_{j,t} + \frac{W_{j,t}}{P_t} N_{j,t} + \frac{\Pi_{j,t}}{P_t} + T_{j,t} - C_{j,t} \right] + \psi_{\lambda,j} (V^*_{j,t} - V_{j,t}) \right\},
\]

where \( V^*_{j,t} \) is evaluated at \( B_{j,t} + \omega_{j,t} B_{j,t} \frac{Q_{L,t}^* - Q_{L,t}}{Q_{L,t}} \).

The first-order condition for the portfolio problem is given by

\[
\frac{\partial V_{j,t}}{\partial B} r_{L,t} + \lambda_{\lambda,j} \frac{\partial V^*_{j,t}}{\partial B} \frac{Q_{L,t}^* - Q_{L,t}}{Q_{L,t}} = 0.
\]

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Rearranging the above expression, we obtain the expression for the term premium,

\[ r^*_L,t = \lambda \left[ \left( \frac{C_{s,t}}{C_{s,t}} \right)^\sigma - 1 \right] \frac{Q_{L,t} - Q_{L,t}^*}{Q_{L,t}}, \]

where \( r^*_L,t \equiv r_{L,t} + \lambda t \frac{Q_{L,t}^* - Q_{L,t}}{Q_{L,t}}. \)

Note that the envelope condition for \( B \) is given by

\[ \rho_j \frac{\partial V_{j,t}}{\partial B} = \frac{\partial V_{j,t}}{\partial B} (i_t - \pi_t + \omega_{j,t} r_{L,t}) + \mathbb{E} \left[ \frac{dV_{j,t}}{dB} \right] + \lambda t \frac{\partial V_{j,t}^*}{\partial B} \omega_{j,t} \frac{Q_{L,t}^* - Q_{L,t}}{Q_{L,t}}. \]

Combining the above expression with the first-order condition for \( \omega_{k,t} \), we obtain the same Euler equation as in the case of short-term bonds.

**Stationary equilibrium.** The price of the long-term bond in the disaster state is given by

\[ Q_t^L = \frac{1}{i_t^* + \psi_d}, \]

where \( i_t^* = r_n^* \) is the yield on the long-term bond.

The price of the long-term bond in the no-disaster state satisfies

\[ \frac{1}{Q_L} - \psi_d - r_n = \lambda \left( \frac{C_s}{C_s^*} \right)^\sigma \frac{Q_L - Q_t^L}{Q_L}. \]

Rearranging the above expression, we obtain

\[ Q_L = \frac{1 + \lambda \left( \frac{C_s}{C_s^*} \right)^\sigma Q_t^L}{\psi_d + \rho}. \]

The yield on the long-term bond is given by

\[ i_L = \frac{\psi_d + \rho}{1 + \lambda \left( \frac{C_s}{C_s^*} \right)^\sigma Q_t^L} - \psi_d \]

\[ = r_n + \lambda \left( \frac{C_s}{C_s^*} \right)^\sigma \frac{r_n^* - r_n}{\psi_d + \psi_d + \lambda \left( \frac{C_s}{C_s^*} \right)^\sigma}. \]

Note that \( i_L > r_n \) and the difference is decreasing in \( \psi_d \), so the yield increases with the bond duration.

**Linearized price of the long-term bond.** The pricing condition can be written as

\[ \frac{1}{Q_{L,t}} + \frac{Q_{L,t}}{Q_{L,t}^*} - \psi_d - i_t + \lambda t \left( \frac{C_{s,t}}{C_{s,t}^*} \right)^\sigma \frac{Q_{L,t}^* - Q_{L,t}}{Q_{L,t}} = 0. \]
Rearranging the above expression, we obtain

\[ Q_{L,t} = (i_t + \psi_d)Q_{L,t} - 1 + \lambda_t \left( \frac{C_{s,t}}{C_{L,t}} \right)^\sigma (Q_{L,t} - Q_{L,t}^*). \]

Linearizing the above expression, we obtain

\[ \dot{q}_{L,t} = (\rho + \psi_d)q_{L,t} + i_t - r_n + \lambda \left( \frac{C_s}{C_L} \right)^\sigma \frac{Q_L - Q_L^*}{Q_L} (\sigma c_{s,t} + \epsilon_{\lambda,t}(i_t - r_n)). \]

Solving the above equation forward, we obtain

\[ q_{L,0} = - \int_0^\infty e^{-\rho t} e^{-\psi_d t} \left[ i_t - r_n + \lambda \left( \frac{C_s}{C_L} \right)^\sigma \frac{Q_L - Q_L^*}{Q_L} (\sigma c_{s,t} + \epsilon_{\lambda,t}(i_t - r_n)) \right] dt. \]

The yield on the long-term bond can then be written as

\[ i_{L,0} - i_L = (i_L + \psi_d) \int_0^\infty e^{-\rho t} e^{-\psi_d t} \left[ i_t - r_n + \lambda \left( \frac{C_s}{C_L} \right)^\sigma \frac{Q_L - Q_L^*}{Q_L} (\sigma c_{s,t} + \epsilon_{\lambda,t}(i_t - r_n)) \right] dt, \]

using the fact that \( Q_L^{-1} = i_L + \psi_d. \)

**Yield curve.** Let \( Q_{L,t}(\tau) \) denote the price of a (real) zero-coupon bond maturing \( \tau \) periods ahead. The value of the bond is given by

\[ Q_{L,t}(\tau) = E_t \left[ e^{-\rho_t \tau} \left( \frac{C_{s,t+\tau}}{C_{s,t}} \right)^{-\sigma} \right] = e^{-\rho_t \tau} \left( \frac{C_{s,t+\tau}}{C_{s,t}} \right)^{-\sigma} + (1 - e^{-\rho_t \tau}) e^{-\rho_t \tau} \left( \frac{C_{s,t+\tau}}{C_{s,t}} \right)^{-\sigma}. \]

In a stationary equilibrium, we obtain

\[ Q_L(\tau) = e^{-\rho_s \tau} + (1 - e^{-\rho_s \tau}) e^{-\rho_s \tau} \left( \frac{C_s}{C_L} \right)^{-\sigma}. \]

The yield on the bond is given by \( r_L(\tau) \equiv - \log Q_L(\tau)/\tau \) and can be written as

\[ r_L(\tau) = \rho_s + \lambda - \frac{1}{\tau} \log \left[ 1 + (e^{\lambda \tau} - 1) \left( \frac{C_s}{C_L} \right)^\sigma \right]. \]

Approximating the above expression, we obtain

\[ r_L(\tau) = r_n + \lambda \left( \left( \frac{C_s}{C_L} \right)^\sigma - 1 \right) \tau + O(\tau^2), \]

which is increasing in \( \tau. \)

**A.6 Corporate bonds**

Let \( Q_{C,t} \) denote the value of a *corporate bond*, an asset that pays off coupons \( e^{-\psi_c t} \) in nominal terms in the absence of default. We assume that monetary shocks are too small to trigger default, so there is no default in the no-disaster state, and that the bond suffers a loss \( 1 - \zeta_c. \)
The value of the corporate bond in the disaster state is given by

\[ Q_C^* = \frac{1 - \xi_c}{r_n + \psi_c}. \]

In the stationary equilibrium, the value of the corporate bond in the no-disaster state is given by

\[ Q_C = \frac{1 + \lambda \left( \frac{C_i}{C^*_i} \right)^{\sigma} Q_C^*}{\psi_c + \rho}. \]

The yield on the corporate bond is given by \( i_{C,t} = Q_{C,t}^{-1} - \psi_C \). The corporate spread, the difference between the yield on the corporate bond and a government bond with the same coupons, is given by in the stationary equilibrium

\[ r_C = \frac{\psi_c + \rho}{1 + \lambda \left( \frac{C_i}{C^*_i} \right)^{\sigma}} - \frac{\psi_c + \rho}{1 + \lambda \left( \frac{C_i}{C^*_i} \right)^{\sigma} Q_C^* / (1 - \xi_c)}. \]

The linearized price of the corporate bond is given by

\[ q_{C,0} = -\int_0^\infty e^{-(\rho + \psi_C)t} \left[ i_t - r_n + \lambda \left( \frac{C_i}{C^*_i} \right)^{\sigma} \frac{Q_C - Q_C^*}{Q_C} (\sigma c_{s,t} + \epsilon_{\lambda,t}(i_t - r_n)) \right] dt. \]

The yield on the corporate bond is \( i_{C,0} - i_c = -Q_C^{-1} q_{C,0} \), which can be written as

\[ i_{C,0} - i_c = (i_c + \psi_c) \int_0^\infty e^{-(\rho + \psi_C)t} \left[ i_t - r_n + \lambda \left( \frac{C_i}{C^*_i} \right)^{\sigma} \frac{Q_C - Q_C^*}{Q_C} (\sigma c_{s,t} + \epsilon_{\lambda,t}(i_t - r_n)) \right] dt, \]

using the fact that \( Q_C^{-1} = i_c + \psi_c \).

The corporate spread is \( r_{C,0} = i_{C,0} - i_{C,0} \), where \( i_{C,0} \) is the yield of a government bond with the same coupons as the corporate bond.

\[ r_{C,0} = r_C \int_0^\infty e^{-(\rho + \psi_C)t} (i_t - r_n)dt + \left[ (i_c + \psi_c)(i_c - i) - (i_c + \psi_c)(i_c - i) \right] \int_0^\infty e^{-(\rho + \psi_C)t} (\sigma c_{s,t} + \epsilon_{\lambda,t}(i_t - r_n))dt, \]

where \( i_C - i = \lambda \left( \frac{C_i}{C^*_i} \right)^{\sigma} \frac{Q_C - Q_C^*}{Q_C} \) is the difference between the corporate bond yield and the short-term nominal rate in the stationary equilibrium and \( i_C - i \) is the corresponding object for a bond without default risk.

**B Proofs**

**B.1 Proof of Proposition 1**

*Proof.* Consider first the New Keynesian Phillips curve

\[ \pi_t = \left( i_t - \pi_t + \lambda_t \frac{\eta_t}{\eta_t} \right) \pi_t - (e - 1) \varphi^{-1} \left( \frac{e}{e - 1} \frac{W}{P} e^{\psi_t} - \frac{1}{A} (1 - \tau) \right) Y e^{\psi_t}. \]
Linearizing the above expression, and using \( \frac{e_t W 1}{\varepsilon \pi} = (1 - \tau) \), we obtain

\[
\pi_t = \left( r_n + \lambda \left( \frac{C_s}{C_s^*} \right)^\sigma \right) \pi_t - \varphi^{-1}(\varepsilon - 1)(1 - \tau)(w_t - p_t).
\]

Using the expression for \( w_t - p_t \), we obtain

\[
\pi_t = (\rho_s + \lambda) \pi_t - \kappa y_t,
\]

where \( \kappa \equiv \varphi^{-1}(\varepsilon - 1)(1 - \tau)(\phi + \sigma) \) and we used the fact that \( r_n + \lambda \left( \frac{C_s}{C_s^*} \right)^\sigma = \rho_s + \lambda \).

Consider next the generalized Euler equation. From the market-clearing condition for goods and borrowers’ consumption, we obtain

\[
c_{s,t} = \frac{1 - \mu_b\chi_y}{1 - \mu_b} y_t + \frac{\mu_b\chi_r}{1 - \mu_b} (i_t - \pi_t - r_n).
\]

The Euler equation for savers is given by

\[
\dot{c}_{s,t} = \sigma^{-1}(i_t - \pi_t - r_n) + \lambda \left( \frac{C_s}{C_s^*} \right)^\sigma c_{s,t} + \sigma^{-1}p_d\varepsilon_{\lambda,i}(i_t - r_n),
\]

where \( p_d \equiv \lambda \left[ \left( \frac{C_s}{C_s^*} \right)^\sigma - 1 \right] \) is the price of disaster risk and \( \varepsilon_{\lambda,i} \equiv \lambda'(0)/\lambda(0) \) is the semi-elasticity of \( \lambda_t = \lambda(i_t - r_n) \) with respect to \( i_t \).

Combining the previous two conditions, we obtain

\[
\dot{y}_t = \frac{1 - \mu_b}{1 - \mu_b\chi_y} \sigma^{-1}(i_t - \pi_t - r_n) + \frac{1 - \mu_b}{1 - \mu_b\chi_y} \left[ \lambda \left( \frac{C_s}{C_s^*} \right)^\sigma c_{s,t} + \sigma^{-1}p_d\varepsilon_{\lambda,i}(i_t - r_n) \right] - \frac{\mu_b\chi_r}{1 - \mu_b\chi_y} (i_t - \pi_t)
\]

\[
= \left[ \frac{1 - \mu_b}{1 - \mu_b\chi_y} \sigma^{-1} - \frac{\mu_b\chi_r}{1 - \mu_b\chi_y} \right] (i_t - \pi_t - r_n) + \left[ \lambda \left( \frac{C_s}{C_s^*} \right)^\sigma - \frac{\mu_b\chi_r}{1 - \mu_b\chi_y} \right] y_t - \frac{\mu_b\chi_r}{1 - \mu_b\chi_y} (i_t - \rho(i_t - r_n))
\]

\[
+ \frac{1 - \mu_b}{1 - \mu_b\chi_y} \sigma^{-1}p_d\varepsilon_{\lambda,i}(i_t - r_n),
\]

using the fact that \( r_n = \rho - \lambda \left( \frac{C_s}{C_s^*} \right)^\sigma \).

Allowing for the disaster risk to depend on output, \( \lambda_t = \lambda(Y_t) \), the linearized Euler equation for savers is

\[
\dot{c}_{s,t} = \sigma^{-1}(i_t - \pi_t - r_n) + \lambda(Y) \left( \frac{C_s}{C_s^*} \right)^\sigma c_{s,t} + \frac{\lambda(Y)}{\sigma} \left[ \left( \frac{C_s}{C_s^*} \right)^\sigma - 1 \right] \varepsilon_{\lambda,Y} t.
\]

The generalized Euler equation assumes the same form as before, with the coefficient \( \delta \) given by

\[
\delta \equiv \lambda(Y) \left( \frac{C_s}{C_s^*} \right)^\sigma - \frac{\mu_b\chi_r}{1 - \mu_b\chi_y} \kappa + \frac{(1 - \mu_b)}{1 - \mu_b\chi_y} \sigma^{-1}p_d\varepsilon_{\lambda,Y}.
\]
B.2 Proof of Lemma 1

Proof. We first derive the (non-linear) intertemporal budget constraint, then derive its log-linearized version and show the sufficiency of system (10) and the intertemporal budget constraint.

Non-linear intertemporal budget constraint. The dynamics of the SDF can be written as

\[
\frac{d\eta_t}{\eta_t} = -(i_t - \pi_t) dt + \frac{(C_{s,t})^{-\sigma} - C_{s,t}^{-\sigma}}{C_{s,t}^{-\sigma}} (dN_t - \lambda_t dt).
\]

Applying Ito’s lemma to \(d(\eta_t B_{j,t})\), we obtain

\[
\mathbb{E}[d(\eta_t B_{j,t})] = \eta_t B_{j,t} dt - (i_t - \pi_t) \eta_t B_{j,t} dt.
\]

Integrating \(d(\eta_t B_{j,t})\) and taking expectations, we obtain

\[
\mathbb{E}_0[\eta_t B_{j,t}] - \eta_0 B_{j,0} = \mathbb{E}_0 \left[ \int_0^t \eta_z \left( \frac{W_z}{P_z} N_{j,z} + \Pi_{j,z} + \tilde{T}_{j,z} - C_{j,z} \right) dz \right].
\]

Note that \(\lim_{t \to \infty} \mathbb{E}_0[\eta_t B_{j,t}] = 0\), as \(B_{j,t}\) is constant in equilibrium and \(\lim_{t \to \infty} \mathbb{E}_0[\eta_t] = 0\). The borrowing constraint implies that \(\lim_{t \to \infty} \mathbb{E}_0[\eta_t B_{j,t}] \geq 0\), which, combined with the No-Ponzi condition for the government and the previous result for borrowers, implies that \(\lim_{t \to \infty} \mathbb{E}_0[\eta_t B_{j,t}] = 0\). Therefore, we conclude that \(\lim_{t \to \infty} \mathbb{E}_0[\eta_t B_{j,t}] = 0\) for both types of households.

Taking the limit as \(t \to \infty\) of the previous expression, using the fact that \(\lim_{t \to \infty} \mathbb{E}_0[\eta_t B_{j,t}] = 0\), and aggregating across households, we obtain

\[
\mathbb{E}_0 \left[ \int_0^\infty \frac{\eta_t}{\eta_0} (\mu_b C_{b,t} + (1 - \mu_b) C_{s,t}) dt \right] = D_{g,0} + \mathbb{E}_0 \left[ \int_0^\infty \frac{\eta_t}{\eta_0} (1 - \tau) Y_t + \tilde{T}_t dz \right],
\]

using the fact that \(\frac{W_t}{P_t} N_t + (1 - \mu_b) \Pi_{s,t} = (1 - \tau) Y_t\).

The above expression can then be written as

\[
Q_{C,0} = D_{g,0} + Q_{Y,0}
\]

where \(Q_{C,0}\) is the initial value of the consumption claim and \(Q_{Y,0}\) is the initial value of a claim on after-tax wages and profits.

Log-linearized intertemporal budget constraint. The linearized budget constraint can be written as

\[
Q_{C,0} = D_{g,0} + Q_{Y,0}.
\]
where $q_{C,0} \equiv \log \frac{Q_{C,0}}{Q_C}$, $q_{Y,0} \equiv \log \frac{Q_{Y,0}}{Q_Y}$, and $Q_C$ and $Q_Y$ denote the value of the consumption claim and the (after-tax) income claim, respectively, in the no-disaster state of the stationary equilibrium.

Following a derivation analogous to the one for the value of stocks and human wealth in Section A.4, we obtain

$$q_{C,0} = \frac{Y}{Q_C} \int_0^\infty e^{-pt} \left[ \mu_b c_{b,t} + (1 - \mu_b) c_{s,t} \right] dt - \int_0^\infty e^{-pt} \left[ i_t - \pi_t - r_n + \lambda \left( \frac{C_S}{C^*_S} \right)^\sigma \frac{Q_C - Q_C^*}{Q_C} p_{d,t} \right] dt$$

$$q_{Y,0} = \frac{Y}{Q_Y} \int_0^\infty e^{-pt} \left[ (1 - \tau) y_t + T_t \right] dt - \int_0^\infty e^{-pt} \left[ i_t - \pi_t - r_n + \lambda \left( \frac{C_S}{C^*_S} \right)^\sigma \frac{Q_Y - Q_Y^*}{Q_Y} p_{d,t} \right] dt,$$

where $T_t = \mu_b T_{b,t} + (1 - \mu_b) T_{s,t}$, $p_{d,t} \equiv \sigma c_{s,t} + \epsilon_{\lambda,t}(i_t - r_n)$ is the (log-linear) price of disaster risk.

The intertemporal budget constraint can then be written as

$$\int_0^\infty e^{-pt} \left[ \mu_b c_{b,t} + (1 - \mu_b) c_{s,t} \right] dt = \bar{d}_g q_{L,0} + \int_0^\infty e^{-pt} \left[ (1 - \tau) y_t + T_t \right] dt +$$

$$\int_0^\infty e^{-pt} \frac{Q_C - Q_Y}{Y} (i_t - \pi_t - r_n) + \int_0^\infty e^{-pt} \left[ \lambda \left( \frac{C_S}{C^*_S} \right)^\sigma \left( \frac{Q_C - Q_Y}{Q_Y} - \frac{Q_C^* - Q_Y^*}{Q_Y} \right) p_{d,t} \right] dt.$$

The first term on the right-hand side of the above expression represents the dividend effect on households’ assets, the second term captures the net interest-rate effect, and the third term captures the net risk-premium effect.

In a stationary equilibrium, we infer that $Q_C = \bar{D}_S + Q_Y$ and $Q_C^* = \bar{D}_S + \bar{D}_g \frac{Q_Y - Q_s}{Q_L} + Q_Y$, which allows us to write

$$\Omega_0 = \int_0^\infty e^{-pt} \left[ (1 - \tau) y_t + T_t + \bar{d}_g (i_t - \pi_t - r_n + r_L p_{d,t}) \right] dt - \bar{d}_g \int_0^\infty e^{-(\rho + \psi) t} \left[ i_t - r_n + r_L p_{d,t} \right] dt,$$

where $r_L \equiv \lambda \left( \frac{C_S}{C^*_S} \right)^\sigma \frac{Q_Y - Q_s}{Q_L}$ and we used the expression for $q_{L,0}$ derived in Section A.5.

** Sufficiency of the intertemporal budget constraint.** Suppose $[y_t, \pi_t]_0^\infty$ satisfy system (10) and the intertemporal budget constraint (11) in the no-disaster state. We will show that $[y_t, \pi_t]_0^\infty$ can be supported as an equilibrium. Consider first the disaster state. We set $T_{s,t} = \rho b_s b_{s,t}$, where $t^*$ denotes the time the economy switches to the disaster state. All the remaining variables are equal to zero in the disaster state. The equation for the real wage, $w_t^* - p_t^* = (\phi + \sigma)y_t^* = 0$, and the labor-supply condition for each household, $w_t^* - p_t^* = \sigma c_{i,t}^* + \phi n_{i,t}^*$, are also satisfied. The same applies to the savers’ Euler equation and the market clearing condition for goods, labor, and bonds. Borrowers’ and savers’ budget constraint are satisfied,

$$c_{b,t}^* = (1 - \alpha)(w_t^* - p_t^* + n_{b,t}^*) + T_{b,t}^* - (i^* - \pi^* - \rho_s) \bar{d}_p$$

$$\bar{b}_b b_{s,t} = \bar{b}_s \rho_b b_{s,t} + (1 - \alpha)(w_t^* - p_t^* + n_{s,t}^*) + \frac{(1 - \tau) y_t^* - (1 - \alpha)(w_t^* - p_t^* + n_t^*)}{1 - \mu_b} + T_{s,t}^*$$

$$+ (i^* - \pi^* - \rho_s) \bar{b}_s - c_{s,t}^*,$$

which implies that $b_{s,t}$ is constant in the disaster state at the $b_{s,t}^*$ level.

Consider now the no-disaster state. The real wage is given by $w_t - p_t = (\phi + \sigma)y_t$. Borrowers’
and savers’ consumption is given by
\[ c_{b,t} = X_y y_t - X_t (i_t - \pi_t - r - n), \quad c_{s,t} = \frac{1 - \mu_b X_y}{1 - \mu_b} y_t + \frac{\mu_b X_t}{1 - \mu_b} (i_t - \pi_t - r_n), \]
and the labor supply is given by \[ n_{j,t} = \phi^{-1}(w_t - p_t) - \phi^{-1} \sigma c_{j,t}. \]

By construction, the market-clearing condition for goods and labor, the labor supply for each household, and the budget constraint for borrowers are all satisfied. Because \( y_t \) satisfies the aggregate Euler equation, the savers’ Euler equation is also satisfied. Because \( \pi_t \) satisfies the New Keynesian Phillips curve, the optimality condition for firms is satisfied. Bond holdings by savers and government debt evolve according to
\[
\bar{b}_s \hat{b}_{s,t} = \bar{b}_s r_n b_{s,t} + (1 - \alpha)(w_t - p_t + n_{s,t}) + \frac{(1 - \tau)y_t - (1 - \alpha)(w_t - p_t + n_t)}{1 - \mu_b} + \tau_{s,t}
\]
\[
+ (i - \pi_t - r_n) \bar{b}_s - c_{s,t},
\]
\[
\bar{g}_g \hat{g}_{g,t} = \bar{g}_g r_n d_{g,t} + T_t - \tau y_t + (i - \pi_t - r_n) \bar{g}_g,
\]
where \( b_{s,0} = d_{g,0} = 0. \)

The market-clearing condition for bonds \( (1 - \mu_b) \bar{b}_s b_{s,t} = \bar{g}_g d_{g,t} \) is satisfied in all periods. The only condition that remains to be checked is the No-Ponzi condition for the government or, equivalently, the aggregate intertemporal budget constraint. Because condition (11) is satisfied, the No-Ponzi condition for the government is also satisfied.

\[ \Box \]

### B.3 Proof of Propositions 2, 3, and 4

**Proof.** We consider here the equilibrium dynamics for the general case \( \mu_b \geq 0. \) The results in Proposition 2 are a special case, with \( \mu_b = 0. \) We can write dynamic system (10) in matrix form as follows:
\[ \hat{Z}_t = AZ_t + BV_t, \]
where \( B = [1, 0]' \).

Applying the spectral decomposition to matrix \( A \), we obtain
\[ A = V \Omega V^{-1}, \]
where
\[
V = \begin{bmatrix} \frac{\rho - \omega}{\kappa} & \frac{\rho - \omega}{\kappa} \\ 1 & 1 \end{bmatrix} ; \quad V^{-1} = \frac{\kappa}{\omega - \omega} \begin{bmatrix} -1 & \frac{\rho - \omega}{\kappa} \\ 1 & -\frac{\rho - \omega}{\kappa} \end{bmatrix} ; \quad \Omega = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} .
\]

Decoupling the system, we obtain
\[ \hat{z}_t = \Omega z_t + b v_t, \]

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where \( z_t = V^{-1}Z_t \) and \( b = V^{-1}B \).

Solving the equation with a positive eigenvalue forward and the one with a negative eigenvalue backward, we obtain
\[
z_{1,t} = -b_1 \int_t^\infty e^{-\bar{\omega}(z-t)} v_z dz
\]
\[
z_{2,t} = e^{\omega t} z_{2,0} + b_2 \int_0^t e^{\omega(t-z)} v_z dz.
\]

Rotating the system back to the original coordinates, we obtain output and inflation
\[
y_t = V_{12} \left( V^{21} y_0 + V^{22} \pi_0 \right) e^{\omega t} - V_{11} V^{11} \int_t^\infty e^{-\bar{\omega}(z-t)} v_z dz + V_{12} V^{21} \int_0^t e^{\omega(t-z)} v_z dz
\]
\[
\pi_t = V_{22} \left( V^{21} y_0 + V^{22} \pi_0 \right) e^{\omega t} - V_{21} V^{21} \int_t^\infty e^{-\bar{\omega}(z-t)} v_z dz + V_{22} V^{21} \int_0^t e^{\omega(t-z)} v_z dz,
\]
where \( V^{ij} \) is the \((i,j)\) entry of matrix \( V^{-1} \).

Integrating \( e^{-\rho t} y_t \) and using the intertemporal budget constraint,
\[
\Omega_0 = V_{12} \left( V^{21} y_0 + V^{22} \pi_0 \right) \frac{1}{\rho - \omega} - \frac{1}{\rho - \omega} V_{11} V^{11} \int_0^\infty \left( e^{-\bar{\omega} t} - e^{-\rho t} \right) v_t dt + \frac{1}{\rho - \omega} V_{12} V^{21} \int_0^\infty e^{-\rho t} v_t dt.
\]

Rearranging the above expression, we obtain
\[
V_{12} \left( V^{21} y_0 + V^{22} \pi_0 \right) = (\rho - \omega) \Omega_0 + \frac{\rho - \omega}{\rho - \omega} V_{11} V^{11} \int_0^\infty \left( e^{-\bar{\omega} t} - e^{-\rho t} \right) v_t dt - V_{12} V^{21} \int_0^\infty e^{-\rho t} v_t dt.
\]

Consumption is then given by
\[
y_t = \bar{y}_t + (\rho - \omega) e^{\omega t} \Omega_0,
\]
where
\[
\bar{y}_t = -\frac{\bar{\omega} - \rho}{\bar{\omega} - \omega} \int_t^\infty e^{-\bar{\omega}(z-t)} v_z dz + \frac{\bar{\omega} - \delta}{\bar{\omega} - \omega} \int_0^t e^{\omega(t-z)} v_z dz - \frac{\rho - \omega}{\rho - \omega} e^{\omega t} \int_0^\infty e^{-\bar{\omega} z} v_z dz.
\]

Inflation is given by
\[
\pi_t = \bar{\pi}_t + \kappa e^{\omega t} \Omega_0,
\]
where
\[
\bar{\pi}_t = \frac{\kappa}{\omega - \omega} \int_t^\infty e^{-\bar{\omega}(z-t)} v_z dz + \frac{\kappa}{\omega - \omega} \int_0^t e^{\omega(t-z)} v_z dz - \frac{\kappa}{\omega - \omega} e^{\omega t} \int_0^\infty e^{-\bar{\omega} z} v_z dz.
\]
We can write \( y^S_t \) and \( \pi^S_t \) as follows:

\[
\dot{y}_t = -e^{F}_{y,t} \int_0^\infty e^{-\omega(z-t)} v_z dz + e^{B}_{y,t} \int_0^t (e^{-\omega z} - e^{-\omega z}) v_z dz
\]

\[
\dot{\pi}_t = e^{F}_{\pi,t} \int_0^\infty e^{-\omega(z-t)} v_z dz + e^{B}_{\pi,t} \int_0^t (e^{-\omega z} - e^{-\omega z}) v_z dz,
\]

where

\[
e^{F}_{y,t} = \frac{\omega - \rho}{\omega - \omega} + (\omega - \delta)e^{-(\omega - \omega)t}, \quad e^{B}_{y,t} = \frac{\rho - \omega}{\omega - \omega} e^{\omega t},
\]

\[
e^{F}_{\pi,t} = \frac{\kappa}{\omega - \omega} (1 - e^{-(\omega - \omega)t}), \quad e^{B}_{\pi,t} = \frac{\kappa}{\omega - \omega} e^{\omega t},
\]

and

\[
v_t = \frac{1 - \mu_b}{1 - \mu_b \chi_y} \sigma^{-1}(i_t - r_n) - \frac{\mu_b \chi_y}{1 - \mu_b \chi_y} i_t.
\]

If \( \mu_b = 0 \), then \( \dot{y}_t = y^S_t \), as shown in Proposition 2. If \( i_t - r_n = e^{-\psi_m t}(i_0 - r_n) \), then \( i_t = -\psi_m(i_t - r_n) \). This allows us to write

\[
\dot{y}_t = \frac{1 - \mu_b}{1 - \mu_b \chi_y} y^S_t + \frac{\mu_b \chi_y}{1 - \mu_b \chi_y} \dot{y}_t^p, \quad \dot{\pi}_t = \frac{1 - \mu_b}{1 - \mu_b \chi_y} \pi^S_t + \frac{\mu_b \chi_y}{1 - \mu_b \chi_y} \dot{\pi}_t^p,
\]

consistent with the results in Propositions 3 and 4.

For future reference, we solve for the value of \( \dot{y}_t \) and \( \dot{\pi}_t \) in the case where \( v_t = e^{-\psi_m t} v_0 \). Output is given by

\[
\dot{y}_t = -\frac{(\omega - \rho) + (\omega - \delta)e^{-(\omega - \omega)t}}{\omega - \omega} e^{-\psi_m t} v_0 + \frac{\rho - \omega}{\omega - \omega} e^{\omega t} \left[ \frac{1 - e^{-(\omega + \psi_m)t}}{\omega + \psi_m} - \frac{1 - e^{-(\omega - \omega)t}}{\omega - \omega} \right] v_0
\]

\[
= -\frac{\rho + \psi_m}{(\omega + \psi_m)(\omega + \psi_m)} e^{-\psi_m t} v_0 + \frac{\omega - \delta}{(\omega + \psi_m)(\omega + \psi_m)} e^{\omega t} v_0,
\]

where we assumed \( \psi_m \neq -\omega \).

Inflation is given by

\[
\dot{\pi}_t = \frac{\kappa}{\omega - \omega} (1 - e^{-(\omega - \omega)t}) e^{-\psi_m t} v_0 + \frac{\kappa}{\omega - \omega} e^{\omega t} \left[ \frac{1 - e^{-(\omega + \psi_m)t}}{\omega + \psi_m} - \frac{1 - e^{-(\omega - \omega)t}}{\omega - \omega} \right] v_0
\]

\[
= -\frac{\kappa}{(\omega + \psi_m)(\omega + \psi_m)} e^{-\psi_m t} v_0 + \frac{\kappa}{(\omega + \psi_m)(\omega + \psi_m)} e^{\omega t} v_0.
\]
B.4 Proof of Proposition 5

Proof. We derive the Fiscal Keynesian Cross in the case with long-term bonds, that is, we use expression (23) for $\Omega_0$, which can be written as follows,

$$
\Omega_0 = \int_0^\infty e^{-\rho t} \left[ (1 - \tau) y_t + \mu_b T_b(Y) y_t + (1 - \mu_b) T_s,t + \bar{d}_g (i_t - \pi_t - r_n + r_L p_{d,t}) \right] dt
$$

$$
- \bar{d}_g \int_0^\infty e^{-(\rho+\psi_d)t} (i_t - r_n + r_L p_{d,t}) dt
$$

where $P_{d,t} \equiv \lambda_t \left( \frac{C_{t}^d}{C_{t-1}^d} \right)^\sigma$ denotes the price of disaster risk and $p_{d,t} \equiv \log P_{d,t}/P_d$ denotes the log-deviation from the stationary equilibrium.

We can write the price of disaster risk as follows

$$
p_{d,t} \equiv \sigma c_{s,t} + \epsilon_{\lambda,i}(i_t - r_n)
$$

$$
= \sigma \left[ 1 - \frac{\mu_b \chi_y}{1 - \mu_b} \right] y_t + \sigma \frac{\mu_b \chi_r}{1 - \mu_b} (i_t - \pi_t - r_n) + \epsilon_{\lambda,i}(i_t - r_n)
$$

$$
= \sigma \left[ 1 - \frac{\mu_b \chi_y}{1 - \mu_b} \right] \hat{y}_t + \sigma \frac{\mu_b \chi_r}{1 - \mu_b} (i_t - \hat{\pi}_t - r_n) + \epsilon_{\lambda,i}(i_t - r_n) + \epsilon_{p_{d,t}, \Omega} \omega^\Omega_0
$$

$$
\hat{p}_{d,t} + \epsilon_{p_{d,t}, \Omega} \omega^\Omega_0,
$$

where $\epsilon_{p_{d,t}, \Omega} \equiv \sigma \left( \frac{1 - \mu_b \chi_y}{1 - \mu_b} (\omega - \delta) - \frac{\mu_b \chi_r}{1 - \mu_b} \right)$.

Using the fact that $y_t = \hat{y}_t + (\omega - \delta) \omega^\Omega_0, \int_0^\infty e^{-\rho t} y_t dt = \Omega_0$, and $\pi_t = \hat{\pi}_t + \kappa \omega^\Omega_0$, we obtain

$$
\Omega_0 = \epsilon_{\omega, \Omega} \Omega_0 + \int_0^\infty e^{-\rho t} \left[ (1 - \mu_b) T_s,t + \bar{d}_g (i_t - \hat{\pi}_t - r_n + r_L \hat{p}_{d,t}) \right] dt
$$

$$
- \bar{d}_g \int_0^\infty e^{-(\rho+\psi_d)t} (i_t - r_n + r_L \hat{p}_{d,t}) dt
$$

where

$$
\epsilon_{\omega, \Omega} \equiv 1 - \left[ \tau - \mu_b T_b(Y) + \frac{\kappa}{\omega - \delta} \bar{d}_g - \frac{\psi_d r_t \bar{d}_g}{\rho + \psi_d - \omega} \epsilon_{p_{d,t}, \Omega} \right].
$$

Rearranging the expression for $\Omega_0$, we obtain

$$
\Omega_0 = \frac{1}{1 - \epsilon_{\omega, \Omega}} \left[ \int_0^\infty e^{-\rho t} \left[ \mu_s T_{s,t} + \bar{d}_g (i_t - \hat{\pi}_t - r_n + r_L \hat{p}_{d,t}) \right] dt
$$

$$
- \bar{d}_g \int_0^\infty e^{-(\rho+\psi_d)t} (i_t - r_n + r_L \hat{p}_{d,t}) dt \right].
$$

(35)

In the limit $\psi_d \rightarrow \infty$, we obtain the result in Proposition 5, as $r_L = 0$ and the second integral above is equal to zero.
Assumption 3. The following condition holds: \( \tau - \mu_b T'_b(Y) + \frac{\kappa}{\sigma - \delta} \bar{d} \hat{g} - \frac{\psi_d \bar{d}}{\rho + \psi_d - \bar{\omega}} \epsilon_{p,t} \Omega > 0. \)

Assumption 3 is the extension of Assumption 2 to an economy with long-term government bonds. This implies that \( \epsilon_{\Omega, \Omega} < 1. \)

B.5 Proof of Proposition 6

Proof. Consider first the case in which \( \psi_d \to \infty. \) Using expression (35) and setting \( T_{s,t} = 0, \) we obtain

\[
\Omega_0 = \frac{1}{1 - \epsilon_{\Omega, \Omega}} \int_0^\infty e^{-\rho t} \bar{d} \hat{g}(i_t - \hat{\pi}_t - r_n) dt,
\]

where \( \epsilon_{\Omega, \Omega} < 1 \) by Assumption 3.

To solve for the response of \( \Omega_0 \) to \( i_0, \) we first compute the impact of the nominal interest rate on the present discounted value of \( \hat{\pi}_t. \) In the absence of outside wealth effects, inflation is given by expression (34). The present discounted value is given by

\[
\int_0^\infty e^{-\rho t} \hat{\pi}_t dt = \frac{\bar{\sigma}(\bar{\omega} - \rho)}{(\bar{\omega} + \psi_m)(\rho + \psi_m)} \nu_0,
\]

using the fact that \( \kappa \bar{\sigma}^{-1} = (\bar{\omega} - \rho)(\rho - \omega). \)

Note that \( \nu_2 \) can be written as

\[
\nu_0 = \left[ \bar{\sigma}^{-1} + \frac{\mu_b \chi_r}{1 - \mu_b \chi_y} (\psi_m + \rho) \right] (i_0 - r_n).
\]

We can then write the present discounted value of \( i_t - \hat{\pi}_t - r_n \) as follows,

\[
\int_0^\infty e^{-\rho t} (i_t - \hat{\pi}_t - r_n) dt = \frac{i_0 - r_n}{\rho + \psi_m} - \frac{(\bar{\omega} - \rho)}{(\bar{\omega} + \psi_m)(\rho + \psi_m)} \left[ 1 + \frac{\mu_b \chi_r}{(1 - \mu_b) \bar{\sigma}^{-1} - \mu_b \chi_y \rho} (\psi_m + \rho) \right] (i_0 - r_n)
\]

\[
= \frac{1}{\bar{\omega} + \psi_m} \left[ 1 - \frac{(1 - \mu_b) \bar{\sigma}^{-1} - \mu_b \chi_y \rho}{1 - \mu_b \chi_y} \right] (i_0 - r_n),
\]

using the fact that \( \bar{\sigma}^{-1} = \frac{(1 - \mu_b) \sigma^{-1} - \mu_b \chi_y \rho}{1 - \mu_b \chi_y}. \)

If \( \psi_d \to \infty, \) then \( \Omega_0 > 0 \) if and only if the following condition is satisfied,

\[
\bar{\omega} < \frac{1 - \mu_b}{\mu_b \chi_r} \bar{\sigma}^{-1},
\]

which is always the case if \( \mu_b = 0 \) or \( \chi_r = 0. \)

Therefore, in a representative-agent economy, \( \Omega_0 \) responds positively to an increase in interest rates with short-term bonds. This is not always the case in the economy with heterogeneous
agents. Note that if we take the limit as both \(\sigma^{-1}\) and \(\lambda\) go to zero, in a way such that Assumption 1 is always satisfied, then \(\bar{\omega}\) converges to

\[
\bar{\omega} = \max \left\{ \rho - \frac{\mu_b \chi_r \kappa}{1 - \mu_b \chi_y}, 0 \right\}.
\]

If the above term is positive, then for \(\sigma^{-1}\) and sufficiently small \(\lambda\), an increase in nominal interest rates will reduce \(\Omega_0\) even with short-term bonds. If \(\mu_b\) is sufficiently close to zero or one, then nominal interest rates raise \(\Omega_0\) even in the economy with heterogeneous agents if the government issues only short-term bonds.

Consider next the case with long-term bonds. In the case where \(\psi_d = 0\), the derivative of \(\Omega_0\) with respect to \(i_0\) can be written as

\[
\frac{\partial \Omega_0}{\partial i_0} = -\frac{\tilde{d}_g}{1 - \epsilon \Omega} \frac{\bar{\omega} - \rho}{(\bar{\omega} + \psi_m)(\rho + \psi_m)} \left[ 1 + \frac{\mu_b \chi_r}{(1 - \mu_b)\sigma^{-1} - \mu_b \chi_r \rho} (\psi_m + \rho) \right] (i_0 - r_n).
\]

Note that \(\bar{\omega} - \rho\) satisfies

\[
\bar{\omega} - \rho = \frac{\delta - \rho + \sqrt{(\delta - \rho)^2 + 4\sigma^{-1} \kappa}}{2},
\]

so \(\sigma > 0\) implies \(\bar{\omega} - \rho > 0\).

Therefore, \(\frac{\partial \Omega_0}{\partial i_0} < 0\) for \(\psi_d = 0\). By continuity, there exists a positive \(\bar{\psi}_d\) such that \(\frac{\partial \Omega_0}{\partial i_0} < 0\) if \(\psi_d < \bar{\psi}_d\). \(\square\)

### B.6 Proof of Lemma 2

**Proof.** First, note that we can write \(v_t\) as follows,

\[
v_t = \frac{1 - \mu_b}{1 - \mu_b \chi_y} \sigma^{-1} (i_t - r_n) - \frac{\mu_b \chi_r}{1 - \mu_b \chi_y} i_t + \frac{1 - \mu_b}{1 - \mu_b \chi_y} \sigma^{-1} p_d e_\lambda (i_t - r_n)
\]

\[
= \left[ \frac{1 - \mu_b}{1 - \mu_b \chi_y} \sigma^{-1} + \frac{1 - \mu_b}{1 - \mu_b \chi_y} \sigma^{-1} p_d e_\lambda \right] (\phi_{\pi t} + \phi_{\pi t} + \dot{u}_t)
\]

\[
= \sigma^{-1} \left[ 1 + \frac{1 - \mu_b}{1 - \mu_b \chi_y} \sigma^{-1} p_d e_\lambda \right] \phi_{\pi t} + \frac{\mu_b \chi_r}{1 - \mu_b \chi_y} \phi_{\pi t} + \dot{v}_t,
\]

where \(\dot{v}_t = \left[ \frac{1 - \mu_b}{1 - \mu_b \chi_y} \sigma^{-1} + \frac{1 - \mu_b}{1 - \mu_b \chi_y} \sigma^{-1} p_d e_\lambda \right] u_t - \frac{\mu_b \chi_r}{1 - \mu_b \chi_y} \dot{u}_t\).

\[
v_t = \frac{\mu_b \chi_r}{1 - \mu_b \chi_y} (\psi_m + \rho) (i_t - r_n) + \frac{1 - \mu_b}{1 - \mu_b \chi_y} \sigma^{-1} p_d e_\lambda (i_t - r_n).
\]
The dynamic system for $y_t$ and $\pi_t$ can now be written as

$$\begin{bmatrix} \dot{y}_t \\ \dot{\pi}_t \end{bmatrix} = \begin{bmatrix} \delta & -\delta^{-1}(1 - \phi_\pi) \\ -\kappa & \rho \end{bmatrix} \begin{bmatrix} y_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \bar{v}_t,$$

(37)

where $\delta \equiv \delta + \frac{\mu_b X_r}{1 - \mu_b X_y} \phi_\pi \kappa$ is exogenous and depends only on the path of the nominal interest rate.

The eigenvalues of the system above are given by

$$\bar{\omega}_T = \frac{\rho + \delta + \sqrt{(\rho + \delta)^2 + 4(\delta^{-1}(1 - \phi_\pi)\kappa - \rho \delta)}}{2},$$

$$\omega_T = \frac{\rho + \delta - \sqrt{(\rho + \delta)^2 + 4(\delta^{-1}(1 - \phi_\pi)\kappa - \rho \delta)}}{2}.$$

The system has a unique bounded solution if both eigenvalues have positive real parts. A necessary condition for the eigenvalues to have positive real parts is

$$\rho + \delta + \frac{\mu_b X_r}{1 - \mu_b X_y} \phi_\pi \kappa > 0 \iff \phi_\pi > -(\rho + \delta) \left( \frac{\mu_b X_r \kappa}{1 - \mu_b X_y} \right)^{-1} = 1 - \left( \rho + \lambda \left( \frac{C_s}{C_s^*} \right)^\sigma \right) \frac{1 - \mu_b X_y}{\mu_b X_r \kappa}.$$

If the condition above is violated, then the real part of $\omega_T$ is negative. Another necessary condition for the eigenvalues to have positive real parts is

$$\delta^{-1}(1 - \phi_\pi) \kappa < \rho \left[ \delta + \frac{\mu_b X_r}{1 - \mu_b X_y} \phi_\pi \kappa \right],$$

which after some rearrangement gives us

$$\phi_\pi > \frac{\delta^{-1} \kappa - \rho \delta}{\delta^{-1} \kappa + \frac{\mu_b X_r \phi_\pi}{1 - \mu_b X_y}}.$$

If the above condition is violated, then the eigenvalues are real-valued and $\omega_T < 0$. This establishes the necessity of the condition

$$\phi_\pi > \max \left\{ 1 - \rho \lambda \left( \frac{C_s}{C_s^*} \right)^\sigma, 1 - \left( \rho + \lambda \left( \frac{C_s}{C_s^*} \right)^\sigma \right)^\sigma \frac{1 - \mu_b X_y}{\mu_b X_r \kappa} \right\}.$$

Suppose that the above condition is satisfied. If the eigenvalues are complex-valued, then the above condition guarantees that $\rho + \delta > 0$, so the eigenvalues’ real part is positive. If the eigenvalues are real-valued, the above condition guarantees that $\sqrt{(\rho + \delta)^2 + 4(\delta^{-1}(1 - \phi_\pi)\kappa - \rho \delta)} < \rho + \delta$, so $\omega_T > 0$. Therefore, the condition is necessary and sufficient for both eigenvalues to have a positive real part.

$\square$
B.7 Proof of Lemma 3

Proof. First, we derive the solution to $[y_t, \pi_t]_{t=0}^\infty$. We then solve for the path of the nominal interest rate using the Taylor rule (4), and we solve for the value of fiscal transfers to savers using the intertemporal budget constraint (11).

Solution to the dynamic system. The dynamic system for $[y_t, \pi_t]_{t=0}^\infty$ is given by

$$
\begin{bmatrix}
\dot{y}_t \\
\dot{\pi}_t
\end{bmatrix} =
\begin{bmatrix}
\delta & -\theta^{-1}(1 - \Phi) \\
-\kappa & \rho
\end{bmatrix}
\begin{bmatrix}
y_t \\
\pi_t
\end{bmatrix} +
\begin{bmatrix}
1 \\
0
\end{bmatrix} \tilde{\nu}_t.
$$

(38)

In matrix form, the system is given by

$$
\dot{Z}_t = \tilde{A} Z_t + B \tilde{\nu}_t,
$$

where $B = [1, 0]'$.

Applying the spectral decomposition to matrix $\tilde{A}$, we obtain

$$
\tilde{A} = \tilde{V} \Omega_T \tilde{V}^{-1},
$$

where

$$
\tilde{V} = \begin{bmatrix}
\frac{\rho - \pi_T}{\kappa} & \frac{\rho - \omega_T}{\kappa} \\
1 & 1
\end{bmatrix} ; \\
\tilde{V}^{-1} = \frac{\kappa}{\omega_T - \pi_T}
\begin{bmatrix}
-1 & \frac{\rho - \omega_T}{\kappa} \\
1 & -\frac{\rho - \pi_T}{\kappa}
\end{bmatrix} ; \\
\Omega_T = \begin{bmatrix}
\frac{\omega_T}{\pi_T} & 0 \\
0 & \omega_T
\end{bmatrix}.
$$

Decoupling the system, we obtain

$$
\dot{z}_t = \Omega_T \tilde{z}_t + \tilde{b} \tilde{\nu}_t,
$$

where $\tilde{z}_t = \tilde{V}^{-1} Z_t$ and $\tilde{b} = \tilde{V}^{-1} B$.

Solving the system forward, we obtain

$$
z_{1,t} = -\tilde{V}^{11} \int_t^\infty e^{-\pi_T(z-t)} \tilde{V}_z dZ$$

$$
z_{2,t} = -\tilde{V}^{21} \int_t^\infty e^{-\omega_T(z-t)} \tilde{V}_z dZ,$$

where $\tilde{V}^{ij}$ is the $(i, j)$ entry of the matrix $\tilde{V}^{-1}$.
Rotating the system back to the original coordinates, we obtain output and inflation

\[ y_t = -\tilde{V}_{11}V^{11} \int_t^\infty e^{-\bar{\gamma}_T(z-t)} \tilde{v}_z dz - \tilde{V}_{12}V^{21} \int_t^\infty e^{-\bar{\gamma}_R(z-t)} \tilde{v}_z dz \]

\[ \pi_t = -\tilde{V}_{21}V^{11} \int_t^\infty e^{-\bar{\gamma}_T(z-t)} \tilde{v}_z dz - \tilde{V}_{22}V^{21} \int_t^\infty e^{-\bar{\gamma}_R(z-t)} \tilde{v}_z dz. \]

We rewrite the above expression as follows,

\[ y_t = -\frac{\bar{\omega}_T - \rho}{\bar{\omega}_T - \bar{\omega}_T} \int_t^\infty e^{-\bar{\gamma}_T(z-t)} \tilde{v}_z dz + \frac{\bar{\omega}_T - \rho}{\bar{\omega}_T - \bar{\omega}_T} \int_t^\infty e^{-\bar{\gamma}_T(z-t)} \tilde{v}_z dz \]

\[ \pi_t = -\frac{\kappa}{\bar{\omega}_T - \bar{\omega}_T} \int_t^\infty \left( e^{-\bar{\gamma}_T(z-t)} - e^{-\bar{\gamma}_T(z-t)} \right) \tilde{v}_z dz, \]

where \( \tilde{v}_t = \frac{1 - \mu_b}{1 - \mu_b \gamma} \sigma^{-1} u_t - \frac{\mu_b \gamma}{1 - \mu_b \gamma} \tilde{u}_t \).

Using the fact that \( u_t = e^{-\psi_m t} u_0 \), we obtain

\[ y_t = -\frac{\rho + \psi_m}{\bar{\omega}_T + \psi_m} \left( \frac{1 - \mu_b}{1 - \mu_b \gamma} \sigma^{-1} + \frac{\mu_b \gamma}{1 - \mu_b \gamma} \psi_m \right) u_t \]

\[ \pi_t = -\frac{\kappa}{\bar{\omega}_T + \psi_m} \left( \frac{1 - \mu_b}{1 - \mu_b \gamma} \sigma^{-1} + \frac{\mu_b \gamma}{1 - \mu_b \gamma} \psi_m \right) u_t, \]

where \( (\bar{\omega}_T + \psi_m)(\bar{\omega}_T + \psi_m) = \bar{\sigma}^{-1} \kappa (\phi_r - 1) + (\bar{\sigma} + \psi_m)(\rho + \psi_m) > 0. \)

**Nominal interest rate.** The nominal interest rate is given by

\[ i_t = r_n + \left[ 1 - \frac{\kappa \phi_r}{(\bar{\omega}_T + \psi_m)(\bar{\omega}_T + \psi_m)} \left( \frac{1 - \mu_b}{1 - \mu_b \gamma} \sigma^{-1} + \frac{\mu_b \gamma}{1 - \mu_b \gamma} \psi_m \right) \right] u_t \]

\[ = r_n + \frac{(\bar{\sigma} + \psi_m)(\rho + \psi_m) - \bar{\sigma}^{-1} \kappa (\bar{\sigma} + \psi_m)}{(\bar{\sigma} + \psi_m)(\rho + \psi_m) - \bar{\sigma}^{-1} \kappa (1 - \phi_r)} u_t. \]

Note that the term in brackets in the above expression is positive as \( \psi_m \to \infty \). If \( \psi_m = 0 \), then the expression is given by

\[ i_t - r_n = \left[ \frac{\bar{\omega}_T \omega_T - \kappa \phi_r}{\bar{\omega}_T \omega_T - \kappa \phi_r} \left( \bar{\sigma}^{-1} + \frac{\mu_b \gamma}{1 - \mu_b \gamma} (\rho + \psi_m) \right) \right] u_t \]

\[ = -\left[ \bar{\sigma}^{-1} \kappa - \rho \bar{\sigma} + \kappa \phi_r \frac{\mu_b \gamma}{1 - \mu_b \gamma} (\rho + \psi_m) \right] \frac{u_t}{\omega_T \omega_T} \]

\[ = -\left[ \bar{\sigma}^{-1} \kappa - \rho \bar{\sigma} + \kappa \phi_r \frac{\mu_b \gamma}{1 - \mu_b \gamma} \psi_m \right] \frac{u_t}{\omega_T \omega_T}, \]

which is negative under Assumption 1.

Therefore, nominal interest rates may increase or decrease in response to an increase in \( u_t \), depending on the persistence of the monetary shock.
**Fiscal transfers.** To determine the value of fiscal transfers to savers, we need first to solve for the real interest rate and for the wealth effect. The real interest rate is given by

$$i_t - \pi_t - r_n = \left[ 1 - \frac{\kappa(\phi_\pi - 1)}{(\omega_T + \psi_m)(\omega_T + \psi_m)} \left( 1 - \frac{\mu_b}{1 - \mu_b \chi_y} \sigma^{-1} + \frac{\mu_b \chi_r}{1 - \mu_b \chi_y} \psi_m \right) \right] u_t$$

$$= \frac{(\rho + \psi_m) \left( \delta + \frac{\mu_b \chi_r}{1 - \mu_b \chi_y} + \psi_m \right)}{\bar{\sigma}^{-1} \kappa(\phi_\pi - 1) + (\delta + \psi_m)(\rho + \psi_m)} u_t.$$

The wealth effect is given by

$$\Omega_0 = -\frac{1}{(\omega_T + \psi_m)(\omega_T + \psi_m)} \left( 1 - \frac{\mu_b}{1 - \mu_b \chi_y} \sigma^{-1} + \frac{\mu_b \chi_r}{1 - \mu_b \chi_y} \psi_m \right) u_0.$$

Note that an increase in $u_0$ always creates a negative wealth effect regardless of the sign of the response of interest rates. The present discounted value of transfers satisfies

$$\int_0^\infty e^{-\rho t} \mu_s T_s dt = \int_0^\infty e^{-\rho t} \left[ (\tau - \mu_b T_b'(Y)) y_t - \bar{d}_g (i_t - \pi_t - r_n) \right] dt$$

$$= (\tau - \mu_b T_b'(Y)) \Omega_0 - \bar{d}_g \frac{\epsilon_r u_0}{\rho + \psi_m}.$$
Let $\hat{y}_t$ and $\bar{\pi}_t$ denote the Taylor rule solution, given by

$$
\hat{y}_t = -\frac{\rho + \psi_m}{(\delta + \psi_m)(\rho + \psi_m) - \bar{\sigma}^{-1}\kappa(1 - \phi_T)} \left( \frac{1 - \mu_h}{1 - \mu_h \chi_y} \sigma^{-1} + \frac{\mu_h \chi_r}{1 - \mu_h \chi_y} \phi_m \right) u_t
$$

$$
\bar{\pi}_t = -\frac{\kappa}{(\delta + \psi_m)(\rho + \psi_m) - \bar{\sigma}^{-1}\kappa(1 - \phi_T)} \left( \frac{1 - \mu_h}{1 - \mu_h \chi_y} \sigma^{-1} + \frac{\mu_h \chi_r}{1 - \mu_h \chi_y} \phi_m \right) u_t.
$$

using the fact that $(\bar{\omega}_T + \psi_m)(\bar{\omega}_T + \psi_m) = (\delta + \psi_m)(\rho + \psi_m) - \bar{\sigma}^{-1}\kappa(1 - \phi_T)$.

Our goal is to show that $\hat{y}_t$ coincides with (15) and $\bar{\pi}_t$ coincides with (17), given the path of nominal interest rates and aggregate wealth effects defined above, and that condition (21) is satisfied.

**Output.** We next compute the value of output using the expression

$$
y_t = \hat{y}_t + (\bar{\omega} - \delta) e^{\bar{\omega} t} \Omega_0.
$$

First, note that $\hat{y}_t$ is given by

$$
\hat{y}_t = -\frac{(\bar{\omega} - \rho) e^{-\psi_m t} + (\rho - \omega) e^{\omega t}}{(\bar{\omega} - \omega)(\bar{\omega} + \psi_m)} v_0 + \frac{\bar{\omega} - \delta}{\bar{\omega} - \omega} e^{\omega t} - e^{-\psi_m t} v_0
$$

$$
= -\frac{\rho + \psi_m}{(\bar{\omega} + \psi_m)(\bar{\omega} + \psi_m)} e^{-\psi_m t} v_0 + \frac{\bar{\omega} - \omega}{(\bar{\omega} + \psi_m)(\bar{\omega} + \psi_m)} e^{\omega t} v_0,
$$

where $v_t \equiv \left( \frac{1 - \mu_h}{1 - \mu_h \chi_y} \sigma^{-1} + \frac{\mu_h \chi_r}{1 - \mu_h \chi_y} \phi_m \right) (i_t - r_n)$ and we assumed $\psi_m \neq -\omega$.

The value of $y_t$ is then given by

$$
y_t = -\frac{\rho + \psi_m}{(\bar{\omega} + \psi_m)(\bar{\omega} + \psi_m)} v_t
$$

$$
= -\frac{\rho + \psi_m}{\bar{\sigma}^{-1}\kappa(\phi_T - 1) + (\delta + \psi_m)(\rho + \psi_m)} \left( \frac{1 - \mu_h}{1 - \mu_h \chi_y} \sigma^{-1} + \frac{\mu_h \chi_r}{1 - \mu_h \chi_y} \phi_m \right) u_t,
$$

using the fact that $(\bar{\omega} + \psi_m)(\bar{\omega} + \psi_m) = (\delta + \psi_m)(\rho + \psi_m) - \bar{\sigma}^{-1}\kappa$, the definition of $v_t$, and the expression for the nominal interest rate.

The above derivation assumes $\psi_m \neq |\omega|$. If $\psi_m = |\omega|$, then $i_t = r_n$ and $\hat{y}_t = 0$ for all $t \geq 0$. This implies that $\rho + \psi_m = \bar{\omega} - \delta$, so $y_t = (\bar{\omega} - \delta) e^{\bar{\omega} t} \Omega_0 = (\rho + \psi_m) e^{-\psi_m t} \Omega_0 = \hat{y}_t$.

**Inflation.** Inflation satisfies the condition

$$
\pi_t = \hat{\pi}_t + \kappa e^{\bar{\omega} t} \Omega_0.
$$
where, assuming $\psi_m \neq -\omega$, $\hat{\pi}_t$ is given by

$$\hat{\pi}_t = \frac{\kappa e^{\omega(t)} - e^{-\psi_m t}}{(\omega + \psi_m)(\omega + \psi_m)} v_0.$$  

Inflation is then given by

$$\pi_t = -\frac{\kappa}{\omega + \psi_m} \left( \frac{1 - \mu_b}{1 - \mu_b X_r} \sigma^{-1} + \frac{\mu_b X_r}{1 - \mu_b X_r} \psi_m \right) (i_t - r_n)$$

$$= -\frac{\kappa}{\sigma^{-1} \kappa (\phi_\pi - 1) + (\delta + \psi_m) (\rho + \psi_m)} \left( \frac{1 - \mu_b}{1 - \mu_b X_r} \sigma^{-1} + \frac{\mu_b X_r}{1 - \mu_b X_r} \psi_m \right) u_t.$$  

Therefore, $\pi_t = \hat{\pi}_t$. The derivation above assumes $\psi_m \neq |\omega|$. If $\psi_m = |\omega|$, then $i_t = r_n$ and $\hat{\pi}_t = 0$ for all $t \geq 0$. This implies that $\pi_t = \kappa e^{\omega(t)} \Omega_0 = \hat{\pi}_t$.

**Fiscal Keynesian Cross.** We next verify that condition (21) is satisfied, given the assumed path of fiscal transfers. Solving for $T_{s,t}$ in (21), we obtain

$$\int_0^\infty e^{-\mu_s t} T_{s,t} dt = (\tau - \mu_b T_b(Y)) \Omega_0 + \frac{\kappa}{\omega - \delta} \tilde{d_s} \Omega_0 - \int_0^\infty e^{-\mu_s t} \tilde{d_s} (i_t - \hat{\pi}_t - r_n) dt.$$  

Hence, this condition will be satisfied for the given path of fiscal transfers if

$$\frac{\kappa}{\omega - \delta} \tilde{d_s} \Omega_0 - \int_0^\infty e^{-\mu_s t} \tilde{d_s} (i_t - \hat{\pi}_t - r_n) dt = -\frac{\tilde{d_s}}{\rho + \psi_m} \left( \frac{\rho + \psi_m}{\omega + \psi_m} \right) \left( \frac{\delta + \mu_b X_r}{\sigma^{-1} \kappa (\phi_\pi - 1) + (\delta + \psi_m) (\rho + \psi_m)} \right) u_0.$$  

Rearranging the above expression, we have that

$$\int_0^\infty e^{-\mu_s t} (i_t - \hat{\pi}_t - r_n) dt = -\frac{\kappa}{\omega - \delta} \left( \frac{1 - \mu_b}{1 - \mu_b X_r} \sigma^{-1} + \frac{\mu_b X_r}{1 - \mu_b X_r} \psi_m \right) u_0 + \left( \frac{\delta + \mu_b X_r}{\sigma^{-1} \kappa (\phi_\pi - 1) + (\delta + \psi_m) (\rho + \psi_m)} \right) u_0$$

$$= -\frac{\kappa}{\omega - \delta} \left( \frac{1 - \mu_b}{1 - \mu_b X_r} \sigma^{-1} + \frac{\mu_b X_r}{1 - \mu_b X_r} \psi_m \right) (i_0 - r_n) + \left( \frac{\delta + \mu_b X_r}{\sigma^{-1} \kappa (\phi_\pi - 1) + (\delta + \psi_m) (\rho + \psi_m)} \right) (i_0 - r_n)$$

$$= -\frac{\mu_b X_r}{\omega - \delta} \left( \frac{\rho + \psi_m}{\omega + \psi_m} \right) (i_0 - r_n) + \frac{\mu_b X_r}{1 - \mu_b X_r} \left( \frac{i_0 - r_n}{\omega + \psi_m} \right) + \frac{i_0 - r_n}{\psi_m + \omega}$$

assuming $\psi_m \neq -\omega$.  

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As shown in the proof of Corollary 6, the left-hand side of the above expression is given by

\[
\int_0^\infty e^{-\rho t} (i_t - \hat{\pi}_t - r_n) \, dt = \frac{1}{\psi_m + \omega} \frac{(1 - \mu_b)\sigma^{-1} - \mu_b \chi_r \omega}{(1 - \mu_b)\sigma^{-1} - \mu_b \chi_r \rho} (i_0 - r_n)
\]

\[
= \frac{i_0 - r_n}{\psi_m + \omega} - \frac{1}{\psi_m + \omega} \frac{\mu_b \chi_r \kappa}{1 - \mu_b \chi_y} \frac{(\omega - \rho)}{\kappa \delta^{-1}} (i_0 - r_n)
\]

\[
= \frac{i_0 - r_n}{\psi_m + \omega} - \frac{i_0 - r_n}{\psi_m + \omega} \frac{\mu_b \chi_r \kappa}{1 - \mu_b \chi_y} \frac{1}{\omega - \delta}.
\]

Therefore, condition (21) is satisfied in the case where \( \psi_m \neq -\omega \). An analogous derivation shows that the condition is also satisfied in the case where \( \psi_m = -\omega \). \qed
C Empirical Evidence on the Fiscal Response to Monetary Shocks

We estimate the empirical IRFs using a VAR identified by a recursiveness assumption, as in Christiano et al. (1999), extended to include fiscal variables.

The variables included in the VAR are: real GDP per capita, CPI inflation, real consumption per capita, real investment per capita, capacity utilization, hours worked per capita, real wages, tax revenues over GDP, government expenditures per capita, the federal funds rate, the 5-year constant maturity rate, and the real value of government debt per capita. We estimate a four-lag VAR using quarterly data for the period 1962:1-2007:3. The identification assumption of the monetary shock is as follows: the only variables that are allowed to react contemporaneously to the monetary policy shock are the federal funds rate, the 5-year rate and the value of government debt. All other variables, including government tax revenues and expenditures, are allowed to react with a lag of one quarter. This assumption is the natural extension of Christiano et al. (1999): while agents’ decisions (with agents, in our case, including households and the government) cannot react to the shock contemporaneously, financial variables (in our case, the federal funds rate, the 5-year rate, and the value of government debt) immediately incorporate the information of the shock.

Data sources All the variables are obtained from standard sources (see below), except for the real value of debt, which we construct from the series provided by Hall et al. (2018). These data provide the market value of government debt held by private investors at a monthly frequency from 1776 through 2018. We transform the series into quarterly frequency by keeping the market value of debt in the first month of the quarter. This choice is meant to avoid capturing changes in the market value of debt arising from changes in the quantity of debt after a monetary shock instead of changes in prices.

The data sources are:
- **Nominal GDP**: BEA Table 1.1.5 Line 1
- **Real GDP**: BEA Table 1.1.3 Line 1
- **Consumption Durable**: BEA Table 1.1.3 Line 4
- **Consumption Non Durable**: BEA Table 1.1.3 Line 5
- **Consumption Services**: BEA Table 1.1.3 Line 6
- **Private Investment**: BEA Table 1.1.3 Line 7
- **GDP Deflator**: BEA Table 1.1.9 Line 1
- **Capacity Utilization**: FRED CUMFNS
- **Hours Worked**: FRED HOANBS
- **Nominal Hourly Compensation**: FRED COMPNFB
- **Civilian Labor Force**: FRED CNP16OV

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38For recent work using a similar data construction, see e.g., Cochrane (2019) and Jiang et al. (2019).
**Figure 9**: Estimated IRFs.

Nominal Revenues: BEA Table 3.1 Line 1  
Nominal Expenditures: BEA Table 3.1 Line 21  
Nominal Transfers: BEA Table 3.1 Line 22  
Nominal Gov’t Investment: BEA Table 3.1 Line 39  
Nominal Consumption of Net Capital: BEA Table 3.1 Line 42  
Effective Federal Funds Rate (FF): FRED FEDFUNDS  
5-Year Treasury Constant Maturity Rate: FRED DGS5  

**VAR estimation.** Figure 9 shows the results. As is standard in the literature, we find that a contractionary monetary shock increases the federal funds rate and reduces output and inflation on impact. Moreover, the contractionary monetary shock reduces consumption, investment, and hours worked.

**The Government’s Intertemporal Budget Constraint** The fiscal response in the model corresponds to the present discounted value of fiscal transfers over an infinite horizon, that is, \( \sum_{t=0}^{\infty} \ddot{\beta}^t T_t \), where \( \ddot{\beta} = \frac{1-\lambda}{1+\rho_s} \). We next consider the empirical counterpart of this quantity. First, we calculate a truncated intertemporal budget constraint from period zero to \( T \):

\[
\text{debt revaluation} = \sum_{t=0}^{T} \ddot{\beta}^t \left[ \tau y_t + \tau_t - \ddot{\beta}^{-1} b_y \left( i_{t-1}^{m} - \pi_t - r^{n} \right) \right] = T_{0,T} + \ddot{\beta}^T b_y b_T \tag{39}
\]
The right-hand side of (39) is the present value of the impact of a monetary shock on fiscal accounts. The first term represents the change in revenues that results from the real effects of monetary shocks. If a contractionary monetary shock generates a recession, government revenues will naturally decrease as a consequence, both because output decreases and because the average tax decreases if the tax system is progressive. The second term represents the change in interest payments on government debt that results from change in nominal rates. For example, a contractionary monetary shock increases nominal payments on government debt. The last two terms are adjustments in transfers and other government expenditures, and the final debt position at period $T$, respectively. In particular, $T_{0,T}$ represents the present discounted value of transfers from period 0 through $T$. Provided that $T$ is large enough, such that $(y_t, \tau_t, i_t)$ have essentially converged to the steady state, then the value of debt at the terminal date, $b_T$, equals (minus) the present discounted value of transfers and other expenditures from period $T$ onward. Hence, the last two terms combined can be interpreted as the present discounted value of fiscal transfers from zero to infinity.

The left-hand side represents the revaluation effect of the initial stock of government debt. In the presence of long-term bonds, a contractionary monetary shock reduces the initial value of government bonds. Hence, part of the adjustment in response to the shock comes from a reduction in the value of debt, instead of coming entirely from raising present or future taxes.

Table 1 shows the impact on the fiscal accounts of a monetary policy shock, both in the data and in the estimated model. We start by testing whether our estimate of the fiscal response to a monetary shock is consistent with the government’s intertemporal budget constraint. To test this, we apply equation (39) to the data and check whether the difference between the left-hand side and the right-hand side is different from zero. We decompose the fiscal response in the data into six groups: the present value (PV) of revenues, the PV of interest payments, the PV of transfers and expenditures, the final value of debt, the initial value of debt, and a residual. The residual is calculated as

$$\text{Residual} = \text{Revenues} - \text{Interest Payments} - \text{Transfers} + \text{Debt in } T - \text{Initial Debt}$$

We truncate the calculations to quarter 60, that is, $T = 60$ (15 years) in equation (39). The results

<table>
<thead>
<tr>
<th></th>
<th>(1) Revenues</th>
<th>(2) Interest Payments</th>
<th>(3) Transfers &amp; Expenditures</th>
<th>(4) Debt in $T$</th>
<th>(5) Initial Debt</th>
<th>(1) - (2) - (3) + (4) - (5) Residual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>-26</td>
<td>68.88</td>
<td>-12.09</td>
<td>2.91</td>
<td>-49.74</td>
<td>30.13</td>
</tr>
</tbody>
</table>
reported in Table 1 imply that we cannot reject the possibility that the residual is zero and, therefore, we cannot reject the possibility that the intertemporal budget constraint of the government is satisfied in our estimation.

The adjustment of the fiscal accounts in the data corresponds to the patterns we observed in Figure 2. The contractionary monetary policy shock leads to an increase in the present value of interest payments and of transfers and expenditures. The present value of revenues drops in response to the shock, mostly as a result of the recession generated by the monetary shock. The response of initial debt is quantitatively important, and it accounts for the bulk of the adjustment in the fiscal accounts.

**EBP.** To estimate the response of the corporate spread in the data, we add the EBP measure of Gilchrist and Zakrajšek (2012) into our VAR (ordered after the fed funds rate). Since the EBP is only available starting in 1973, we reduce our sample period to 1973:1-2007:7. The estimated IRFs are in line with those obtained for the longer sample. We find a significant increase of the EBP on impact, of 6.5 bps, in line with the estimates reported in Gertler and Karadi (2015).