Sticky Inflation:

Monetary Policy when Debt Drags Inflation Expectations*

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Abstract

We incorporate the expectation of a potential inflationary-financing event into a standard New Keynesian model. In such an event, monetary policy temporarily resorts to inflating away public debt. The mere anticipation of this possibility links public debt to inflation expectations through the effect of interest rates on the fiscal burden. While disinflation is feasible in the short run, inflation resurges with full force—sticky inflation—as higher debt amplifies the cost of maintaining low inflation. Optimal monetary policy accommodates fiscal shocks with persistently low real interest rates, effectively front-loading inflation and departing from the Taylor principle. We use this framework to interpret the Federal Reserve's "behind the curve" stance following the COVID-19 pandemic.

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1. Introduction

Persistent inflation became globally endemic in the wake of the COVID-19 pandemic. As inflation accelerated, central banks held interest rates low, defying early warnings and drawing harsh criticism. This critique reflects a traditional view: a prompt rate hike would have anchored expectations, signaled policy resolve, and prevented more painful corrections later. But this view overlooks a historical regularity: during major disruptions such as wars, natural disasters, or political crises, inflation often rises alongside surging public debt (see, e.g., Hall and Sargent, 2021, 2022). In such episodes, central banks face an unusual challenge: raising rates may increase the fiscal burden, leading agents to expect inflation as a means of debt stabilization. How monetary policy should respond when inflation is entangled with fiscal solvency remains an open question.

This paper studies optimal monetary policy when agents anticipate the possibility of future inflationary finance. We develop a tractable New Keynesian model in which private agents foresee a future policy shift during which the central bank temporarily tolerates higher inflation to reduce public debt. In this monetary accommodation phase, real interest rates fall while inflation rises, more so when debt is higher. Monetary and fiscal policies eventually return to a stable regime. The key tension arises before the policy shift: although higher interest rates suppress aggregate demand, they also worsen the fiscal outlook, increasing the likelihood of inflationary finance. This feedback loop reshapes monetary policy transmission and calls for a reassessment of optimal policy design.

Our analysis is motivated by recent studies that highlight the role of fiscal expectations in shaping inflation dynamics. Hilscher, Raviv and Reis (2022) show that inflationary disaster expectations rose sharply following the pandemic, potentially reflecting fears of inflationary debt finance.[1] Hazell and Hobler (2024) demonstrate that even a single electoral event, if perceived as fiscally consequential, can significantly shift inflation expectations. Our work is also inspired by recent quantitative models with regime switches in debt-financing strategies, such as Chung, Davig and Leeper (2007), Bianchi and Ilut (2017), and Bianchi and Melosi (2022). These studies show that fiscal regime changes have played an important role in past inflationary episodes. We contribute to this literature with a tractable framework that yields analytical solutions. These allow us to isolate the key parameters governing fiscal-monetary interactions, identify the structural conditions under which specific inflationary dynamics emerge, and derive optimal monetary responses—complementing the insights from simulation-based approaches.

[1] Gomez Cram, Kung and Lustig (2023) find that fiscal news affect Treasury prices through inflation expectations. Li, Fu and Xie (2022) show that inflation expectations respond to fiscal shocks and predict future debt levels. Wiegand (2025) find that fiscal shocks affect breakeven inflation.

Our paper makes three concrete contributions. First, it shows that *sticky inflation* is central to understanding monetary policy in fiscally sensitive environments. Sticky inflation emerges when elevated debt levels raise inflation expectations, as agents anticipate future episodes of inflationary finance. Because increases in primary deficits or interest rates persistently add to the stock of debt, shocks to these variables leave a lasting imprint on expectations. Since inflation is forward-looking, these expectations exert sustained upward pressure on current inflation. This dynamic feedback loop complicates the central bank's task by undermining conventional monetary transmission: although rate hikes may initially suppress inflation by curbing demand, they can ultimately backfire. As the debt burden grows, expectations of future inflation rise, causing inflation to resurge.

Section 2 shows that, prior to an inflationary-finance episode, the equilibrium is characterized by a four-equation system—an extension of the standard three-equation New Keynesian framework. Sticky inflation emerges as a distinct term in both the Phillips curve and the Euler equation. In the Phillips curve, this term appears as an endogenous cost-push shock linked to the debt path; in the Euler equation, it enters as a shifter of the inflation-neutral interest rate.

Section 3 examines how sticky inflation shapes macroeconomic outcomes under alternative monetary policy strategies, setting the stage for the optimal policy analysis that follows. We show that while it is possible to fully stabilize output when the sticky-inflation channel is active, doing so comes at the cost of a prolonged inflationary episode. This inflation is driven by expectations that debt will eventually be monetized.

We also demonstrate that temporary increases in nominal interest rates—intended to curb inflation—may succeed only on impact. For such a policy to be effective in the short run, it must involve a sufficiently persistent contractionary stance. Yet even then, in the absence of a fiscal adjustment that alters the debt path, inflation eventually resurfaces.

These policy experiments highlight the trade-offs faced by the monetary authority: efforts to stabilize one target—whether inflation, output, or debt—inevitably destabilize the others. These trade-offs arise because sticky inflation endogenously breaks the divine coincidence that typically guides monetary policy in New Keynesian frameworks.

We show that it is the beliefs of price-setters—not those of households—that drive these policy trade-offs. This distinction becomes clear in our decomposition of the effects of sticky inflation into two channels: an endogenous cost-push effect, which enters the Phillips curve through firm expectations, and an endogenous interest-rate effect, which reflects household beliefs. If price-setting behavior does not respond to the prospect of future inflationary finance, the cost-push effect vanishes. In that case, monetary policy can fully offset fiscal expectations, just as with standard demand or interest-rate shocks.

When a policy trade-off is present—that is, when price-setting behavior responds to fiscal expectations—a natural question arises: what should the central bank do under the constraints imposed by sticky inflation? Our second contribution provides an answer. We show that what may appear as underreaction relative to standard Taylor-rule prescriptions is, in fact, optimal for a central bank aiming to stabilize both inflation and output. This insight helps rationalize the behavior of many central banks that, despite criticism, deviated from Taylor-style responses in the aftermath of the COVID-19 pandemic.

We formally analyze optimal policies in Section 4. In the presence of sticky inflation, the standard objective of minimizing inflation and output volatility gives rise to an *endogenous debt-stabilization motive*. Because debt influences output and inflation in the event of the inflationary-financing phase, the planner has an incentive to limit debt accumulation preemptively—even if inflationary finance never materializes. The optimal response to a fiscal shock involves deviating from the Taylor principle: nominal interest rates adjust less than one-for-one with inflation and respond negatively to fiscal shocks. This result holds across alternative assumptions about the central bank's degree of commitment.

Optimal policy in our setting departs in important ways from the textbook case. In response to a conventional cost-push shock, optimal policy generates *stagflation*—a rise in inflation accompanied by a contraction in output. In contrast, the optimal response to a fiscal shock involves an initial boom, despite the increase in inflation. This outcome hinges on the debt-stabilization motive: the planner accepts temporarily higher inflation and a wider output gap to moderate the pace of debt accumulation.

A second key distinction concerns the behavior of the price level. In the textbook analysis, the price level is stationary under optimal policy: inflationary episodes are offset by subsequent deflation, returning the price level to its original path. In our setting, however, *price-level targeting* is not optimal. Temporary fiscal shocks instead lead to permanent increases in the price level.

These differences have important implications for evaluating central bank performance in fiscally sensitive environments. For example, if the inflationary episode following the COVID-19 pandemic is interpreted as the result of a conventional cost-push shock, the appropriate policy response would involve a commitment to future deflation to bring the price level back to its pre-shock path. In contrast, such a strategy would be inefficient in the case of a fiscal shockwhere price-level targeting is no longer optimal.

Our normative analysis reveals further subtleties. We examine how central banks with varying degrees of hawkishness should respond to fiscal shocks, interpreting hawkishness as placing greater weight on inflation relative to output deviations. Paradoxically, the more hawkish the central bank, the less it should raise interest rates in response to a surprise fiscal expansion. Rather than resisting inflation in the short run, an optimal hawkish central bank with commitment should front-load inflation to accelerate debt dilution and thereby mitigate the sticky inflation dynamics that would otherwise persist. Understanding sticky inflation is essential: a hawkish central bank cares about the entire path of inflation and is willing to tolerate more of it early on to avoid being trapped with it later. Ironically, a central bank that reacts to an inflation surge with aggressive rate hikes may end up doing precisely the opposite of what its own objectives prescribe.

The third contribution is a policy counterfactual exercise evaluating the role of monetary policy in the post-pandemic U.S. inflation. In Section 5, we confront the theory with data and simulate what would have happened had the Federal Reserve strictly adhered to the Taylor principle. We discipline the calibration using the observed pass-through of fiscal shocks to inflation expectations and decompose the recent inflation surge into contributions from primary deficits, supply shocks, bond-valuation effects, and deviations from the Taylor rule. Using model-implied disturbances that rationalize the observed paths of deficits, interest rates, debt-to-GDP, and inflation, we construct a counterfactual scenario in which the Fed followed a more aggressive Taylor-type response. The model predicts that such a policy would have resulted in higher inflation and debt. Contrary to the conventional view, we find that the Fed's expansionary stance may have dampened medium-term inflation by accelerating debt erosion—an outcome consistent with the logic of sticky inflation and closer to the optimal policy prescription.

Literature review. Interactions between monetary and fiscal policy have been formally studied since Sargent and Wallace (1981) and are now standard material in macroeconomic textbooks, particularly in the context of seigniorage financing (e.g., Ljungqvist and Sargent, 2018). In the canonical New Keynesian model, however, these interactions were largely sidelined, and inflation was modeled as independent of the government's budget constraint. A distinct line of research emerged from the Fiscal Theory of the Price Level (FTPL), which emphasizes that the price level adjusts to ensure the sustainability of nominal debt, given the present value of future primary surpluses(e.g. Leeper, 1991; Woodford, 1998; Cochrane, 1998). The government budget constraint naturally found its way back into New Keynesian models through the FTPL while further adding monetary/fiscal interactions through changes in the real interest rate (e.g., Sims, 2011; Leeper and Leith, 2016; Cochrane, 2018; Caramp and Silva, 2023, among many others).

While our framework shares the FTPL's emphasis on the role of fiscal policy, the mechanism through which government debt influences the economy is fundamentally different. In the FTPL, fiscal variables do not directly enter the Euler equation or the Phillips curve; their effect on output and inflation arises indirectly, through the equilibrium selection mechanism when monetary policy is passive. In contrast, in our setting, fiscal variables—particularly the level of government debt—enter both the Euler equation and the Phillips curve. As a result, they have a direct impact on output and inflation even when monetary policy remains active. This direct dependence of the dynamic system on debt plays a central role in shaping the optimal policy response in our model.

The idea that fiscal variables influence the economy beyond equilibrium selection aligns with recent work on fiscal policy in Heterogeneous Agent New Keynesian (HANK) models (see, e.g., Angeletos, Lian and Wolf 2024). In these models, fiscal variables enter the aggregate Euler equation—an implication of their non-Ricardian features. However, they do not appear in the Phillips curve. In contrast, our framework allows government debt to directly affect price-setting behavior through an expectations channel. This feature is essential for generating deviations from divine coincidence and lies at the heart of the monetary authority's policy trade-offs.

The anticipation of a potential inflationary phase places our work within the broader literature on quantitative models with policy regime switches. This literature has emphasized the risk of fiscal stagflation—a simultaneous rise in inflation and a contraction in output following fiscal shocks (e.g., Bianchi and Ilut 2017; Bianchi and Melosi 2022)—a phenomenon that can also emerge in our framework. However, our optimal policy analysis shows that the planner avoids fiscal stagflation by inducing an initial boom in response to the shock. Moreover, our model with heterogeneous beliefs allows us to disentangle the distinct roles of household and firm expectations in the transmission of fiscal shocks—highlighting a particularly important role for firm expectations.

On the normative front, our work contributes to the literature on optimal fiscal and monetary policy in environments with sticky prices (e.g., Benigno and Woodford 2003, 2007; Schmitt-Grohé and Uribe 2004). This literature typically finds a muted inflation response to fiscal shocks under the optimal policy. In contrast, we derive an endogenous debt-stabilization motive driven by entrenched expectations of future monetary accommodation, which significantly alters the dynamics under the optimal policy.

Our work also relates to recent studies of optimal policy with long-term debt (e.g., Leeper and Zhou 2021) and limited commitment (e.g., Leeper, Leith and Liu 2021), which highlight larger inflation responses to fiscal shocks. Unlike these approaches, we emphasize the role of expectations of monetary accommodation in shaping optimal policy.

Notably, the post-COVID-19 inflation surge has renewed interest in the drivers of inflation from both analytical and quantitative perspectives.[2] On the quantitative side,

[2]On the analytical front, recent work has emphasized channels unrelated to fiscal shocks, such as

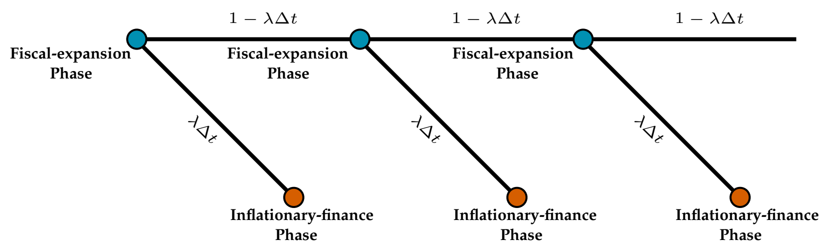


Figure 1: Timeline of events

Note: Over a small time interval Δt, the economy switches to the inflationary-finance phase with probability $\lambda\Delta t$, and stays in the fiscal-expansion phase with the remaining probability.

Blanchard and Bernanke (2023), Gagliardone and Gertler (2023), Shapiro (2024), and Giannone and Primiceri (2024) decompose inflation into labor market and energy shocks, while Benigno and Eggertsson (2023) emphasize nonlinearities in the Phillips curve. This first wave of work follows the New Keynesian tradition, abstracting from debt-financing constraints. In contrast, we explicitly model fiscal-monetary interactions and show how these manifest as cost-push shocks. We see our contribution as part of a second wave of research—alongside Liemen and Posch (2022), Barro and Bianchi (2024), and Smets and Wouters (2024)—that places fiscal dynamics at the center of inflationary analysis.

2. Model

2.1 Environment

We cast the model in continuous time, $t \in [0, \infty)$. The economy starts at a *fiscal-expansion phase* where the government runs primary deficits. With Poisson intensity λ, the economy switches to an *inflationary-finance phase* that lasts for a predetermined amount of time, T^*. In the inflationary-finance phase, government debt is reduced through a mix of fiscal and monetary tools. After the inflationary-finance phase is over, deficits, debt, output, and inflation are stabilized forever. Figure 1 summarizes the timeline of events.

The economy is populated by households, firms, and a government. Each group has possibly different views about the arrival rate of the inflationary-finance phase. Next, we describe the agents' behavior, relegating derivations to Appendix A.

employer-worker tensions Lorenzoni and Werning (2023b,a); Guerreiro, Hazell, Lian and Patterson (2024), hiring frictions Michaillat and Saez (2024), and supply-side constraints Comin, Johnson and Jones (2023).

Notation. We index variables in the inflationary-finance phase using an asterisk ($*$) superscript whereas variables during the fiscal-expansion phase do not carry the superscript. For example, π_t represents inflation at time t of the fiscal-expansion phase whereas π_t^* is inflation at time t since the start of the inflationary-finance phase. Variables in the steady state are denoted by an upper bar. For example, consumption in a steady state is denoted by $\overline{C}$.

Government. The government is comprised of fiscal and monetary authorities. The fiscal authority sends lump-sum transfers T_t—taxes if $T_t < 0$—to households and issues short-term real debt B_t. Given the assumption of sticky prices, this is equivalent to issuing nominal debt, as the price level is predetermined in our continuous-time setting. The monetary authority sets the nominal interest rate i_t.

The government's flow budget constraint is given by

$$\dot{B}_t = (i_t - \pi_t)B_t + T_t, \tag{1}$$

given $B_0 > 0$, where π_t denotes the inflation rate and i_t the nominal interest rate. Fiscal transfers, which equal primary deficits—or surpluses when negative—satisfy the rule:

$$T_t = -\rho B_t - \gamma(B_t - \overline{B}) + \Psi_t, \tag{2}$$

where ρ denotes the interest rate that prevails in a zero-inflation steady state, $\overline{B}$ is the steady-state level of debt, and Ψ_t corresponds to a fiscal shock. Importantly, $\gamma \geq 0$ controls the strength of fiscal responses—primary surpluses—to the level of government debt. If $\gamma > 0$, debt is mean reverting to $\overline{B}$; if $\gamma = 0$, transitory fiscal shocks lead debt to stabilize at different levels.

During the *fiscal-expansion phase*, there are ongoing fiscal pressures, $\Psi_t > 0$. Meanwhile, the monetary authority's instrument, the nominal rate i_t, satisfies a Taylor rule:

$$i_t = \rho + \phi\pi_t + u_t. \tag{3}$$

We focus on the case where the Taylor coefficient ϕ and the fiscal rule coefficient γ are such that the economy is always in an active monetary regime, following the Leeper (1991) terminology. The disturbance u_t allows the monetary authority to respond freely to the fiscal expansion.[3] These choices allow us to analyze an independent monetary authority

[3]The disturbance u_t captures the *response* of the monetary authority to the fiscal expansion, so we refer to it as a disturbance to the policy rule instead of a shock.

that freely chooses interest rates—deviating by u_t from the Taylor rule.

When the economy switches to the inflationary-finance phase, the government sets $\Psi_t = 0$, and the monetary authority commits to set a constant real interest rate for a time interval of length T^*. The rate is set to whatever level is necessary to bring debt to a target level B^n. Once debt reaches B^n, monetary policy implements a zero inflation target, and the economy permanently reaches its steady-state level. T^* is fixed regardless of the debt level. This assumption translates debt levels into a period of future low policy rates, which, in turn, lead to inflationary bursts.

Discussion: monetary accommodation. In our setting, a fiscal shock triggers expectations that a fiscal adjustment may rely on a period of high inflation and low bond returns. This pattern is consistent with the historical evidence on the effects of large fiscal shocks. For instance, Hall and Sargent (2022) showed that low real rates of return on government debt accounted for 45% of the decline in the debt-GDP ratio from 1945 to 1960.[4] A similar dynamic was observed during the COVID-19 pandemic, with low rates of return explaining an even larger fraction of the decline in the debt-GDP ratio since its peak in 2020. The inflationary-finance phase enables us to capture the idea that central banks are unable to credibly signal that the monetary accommodation observed in previous episodes will not recur—it is challenging to convince agents that this time is different. This feature also motivates our assumption that the probability of switching to the inflationary-finance phase does not respond to policy. As in Caballero and Simsek (2022), agents are opinionated and they are not easily persuaded by the monetary authority.

Households and firms. The household block follows the structure of the textbook New Keynesian model. However, the presence of a Poisson event modifies the households' Euler equation to incorporate the uncertainty regarding the policy stance:

$$\frac{\dot{C}_t}{C_t} = \underbrace{(i_t - \pi_t - \rho)}_{\text{standard term}} + \underbrace{\lambda_h \left[\frac{C_t}{C_t^J} - 1\right]}_{\text{policy uncertainty}}, \tag{4}$$

where C_t^J denotes consumption at the instant the economy switches to the inflationary-finance phase and λ_h denotes households' subjective expectation of switching. This Euler equation includes a standard term associated with the gap between real interest rates and the discount rate dictating consumption growth. The second term captures the effect of

[4]Real GDP growth accounted for 30% of the decline in debt-GDP ratio, while an increase in primary surpluses explained only 25% of the reduction in debt during this period.

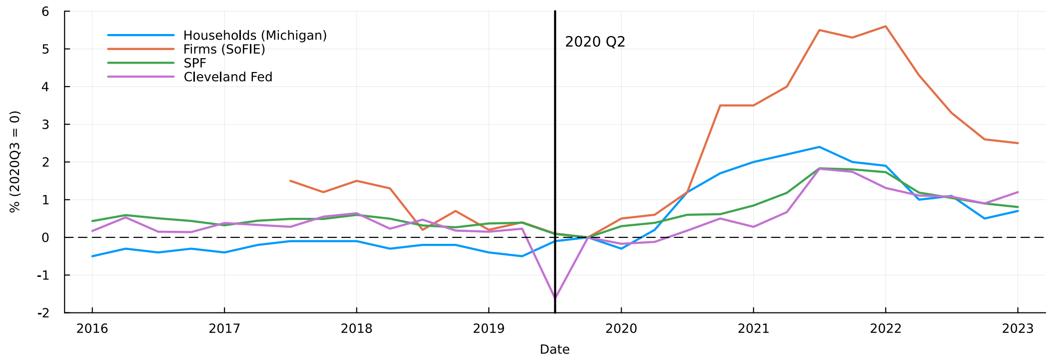


Figure 2: Inflation expectations of households, firms, and professional forecasters

policy uncertainty. The adjustment is given by the jump in marginal utilities the instant the economy enters the inflationary-finance phase.[5]

Likewise, the production side also follows the standard New Keynesian model featuring firms facing sticky prices. The key object of this supply-side block is a modified New Keynesian Phillips curve (NKPC):

$$\dot{\pi}_t = \underbrace{(i_t - \pi_t)\,\pi_t + \epsilon\varphi^{-1}\left((1-\epsilon^{-1}) - \frac{W_t}{P_t}\right)Y_t}_{\text{standard term}} + \underbrace{\lambda_f \frac{\eta_t^J}{\eta_t}\left(\pi_t - \pi_t^J\right)}_{\text{policy uncertainty}}, \tag{5}$$

where λ_f denotes the firms' subjective belief of switching states and η_t denotes the economy's stochastic discount factor (SDF). Like the Euler equation, the firm's Phillips curve features a standard term associated with marginal costs. The second term captures the impact on inflation of policy uncertainty. Firms anticipate that if the economy switches to the inflationary-finance phase, inflation will jump to π_t^J—which we dub the *jump inflation* term. As adjusting prices immediately is costly, firms reduce price-setting costs by raising prices today. The jump in inflation is adjusted by η_t^J, the SDF after switching states, which translates the probability of a reform to a risk-adjusted probability.

Discussion: the role of belief heterogeneity. We allow for households' and firms' beliefs to differ from objective probabilities. Since large fiscal expansions are rare, it can be challenging for any entity—households, firms, monetary authorities—or even modelers—to accurately assess the likelihood of policy changes. Moreover, empirically the behavior of households' and firms' expectations differed during the COVID-19 episode. As seen in Figure 2, firms raised their inflation expectations more aggressively than households during this period, consistent with the assumption of heterogeneous beliefs.

[5] Similar terms appear with other forms of uncertainty, such as the uninsurable idiosyncratic income risk of McKay, Nakamura and Steinsson (2016) or the aggregate disaster risk in Caramp and Silva (2021).

2.2 A 4-equation log-linear representation

Part of the appeal of the standard New Keynesian model is its log-linear representation into a tractable 3-equation system. Here, we present a tractable 4-equation log-linear representation that includes the feedback of fiscal variables on inflation expectations.

Steady state and log-linear deviations. The steady-state corresponds to the case $\Psi_t = 0$ and $u_t = 0$, so $B_t = \overline{B}$, $C_t = \overline{C}$, $i_t = \rho$, and $\pi_t = 0$, where $\overline{B}$ corresponds to the initial condition for government debt and $\overline{C}$ is the steady-state level of consumption.

We denote log-linear deviations from steady state by lower-case variables. We also define $b_t \equiv \frac{B_t - \overline{B}}{\overline{B}}$, and the output gap $x_t \equiv \frac{Y_t - \overline{Y}}{\overline{Y}}$. In turn, $\psi_t \equiv \Psi_t / \overline{B}$ denotes the fiscal shock scaled by steady-state debt.

Dynamics: Inflationary-finance phase. Once the inflationary-finance phase begins, fiscal shocks ψ_t^* and the parameter controlling the fiscal response γ are set to zero. In turn, the monetary authority implements a constant real interest rate r^* for T^* periods, as needed to bring debt to a level that no longer requires a fiscal response to stabilize it. Hence, during the inflationary-finance phase, debt evolves according to $b_t^* = b_0^* + (r^* - \rho)t$ for $t \leq T^*$. To ensure that debt reaches the sustainable level after T^* periods, monetary policy must set the real interest rate to:

$$r^* = \rho - \frac{b_0^* - b^n}{T^*}, \tag{6}$$

where $b^n \equiv \frac{B^n - \overline{B}}{\overline{B}}$ denotes the *natural or neutral debt level*, that is, the debt level for which no fiscal response is needed to keep debt constant. This is also the debt level at which inflation and output would jump to zero at the start of an inflationary-finance phase. Once the target debt level is reached by the end of the reform, the monetary authority implements a zero inflation target, that is, $\{x_{T^*}^*, \pi_{T^*}^*\} = \{0, 0\}$.

Given the terminal condition at the end of the reform, we can roll back the Euler equation and NKPC to obtain:

$$x_t^* = (r^* - \rho)(t - T^*) = (b_0^* - b^n)\left(1 - \frac{t}{T^*}\right), t \in [0, T^*], \tag{7}$$

and

$$\pi_t^* = \kappa(r^* - \rho)\int_t^{T^*} \exp(-\rho(s - t))(s - T^*)ds. \tag{8}$$

Since at any moment t debt does not jump when the economy switches phases, debt at the start of an inflationary-finance phase equals debt at the end of the fiscal-expansion phase, $b_0^* = b_t$. Thus, using the expression for the real rate, given in (6), and using (7) and (8), we can write inflation and the output gap at the instant of the fiscal-monetary reform in terms of the *debt gap* $b_t - b^n$ at the instant of the switch:

$$\pi^*(b_t) \equiv \kappa\Phi(b_t - b^n) \quad \text{and} \quad x^*(b_t) \equiv b_t - b^n, \tag{9}$$

where $\Phi \equiv \int_0^{T^*} \exp(-\rho s)\left(1 - \frac{s}{T^*}\right) ds > 0$.

Inflation and the output gap at the instant of the reform, given by (9), in general, will differ from their values the instant prior to the reform. Thus, these variables jump at the start of the inflationary-finance phase. The jump size depends on the debt gap. The larger the gap, the lower the real interest rate and the higher the inflation rate. The coefficient Φ controls the pass-through from debt to inflation. It captures the increase in inflation required to bring debt to its neutral level during an inflationary-finance phase.

Dynamics: fiscal-expansion phase. The system of linearized Euler equation, NKPC, and government budget constraint is:

$$\dot{x}_t = i_t - \pi_t - \rho + \lambda_h x_t - \lambda_h(b_t - b^n) \tag{10}$$

$$\dot{\pi}_t = (\rho + \lambda_f)\pi_t - \kappa x_t - \lambda_f \kappa\Phi(b_t - b^n) \tag{11}$$

$$\dot{b}_t = i_t - \pi_t - \rho - \gamma(b_t - b^n) + \psi_t. \tag{12}$$

Here, $\kappa > 0$ is the slope of the Phillips curve. The Taylor rule (Eq. 3) completes the 4-equation system.

The dynamic system above nests several important benchmark models. In the absence of regime switching ($\lambda_h = \lambda_f = 0$), the framework reduces to either the textbook New Keynesian model—when $\phi > 1$ and $\gamma \geq 0$—or the FTPL, when $\phi \leq 1$ and $\gamma < 0$. When debt enters only the Euler equation ($\lambda_h > 0$, $\lambda_f = 0$), the model becomes isomorphic to a HANK environment, such as the overlapping-generations setup in Angeletos et al. (2024). The case with homogeneous beliefs corresponds to a tractable version of regime-switching models, akin to those studied in Chung et al. (2007) and Bianchi (2013). Our setting with heterogeneous beliefs generalizes these approaches by allowing debt to enter directly into both the Euler equation and the NKPC with potentially different coefficients.

Determinacy and implementation. Next, we provide the conditions for local determinacy. All proofs are provided in Appendix B.

Proposition 1 (Determinacy and implementability)**.** *Consider a given path of monetary disturbances u_t and fiscal shock ψ_t. Assume that $\gamma \in (0, \rho + \lambda_f + \lambda_h)$. Then,*

I. ***Determinacy.*** *There exists a unique bounded equilibrium if and only if*

$$[\gamma - \lambda_h (1 + \lambda_f \Phi)] (\phi - 1) > -\gamma \frac{\rho + \lambda_f}{\kappa} \lambda_h. \tag{13}$$

II. ***Implementability.*** *Let $\hat{i}_t$ denote a path of nominal interest rates and $(\hat{x}_t, \hat{\pi}_t, \hat{b}_t)$ that satisfies the Euler equation (10), the NKPC (11), and the government's flow budget constraint (12). Suppose $u_t = \hat{i}_t - \rho - \phi\hat{\pi}_t$, with ϕ satisfying (13), such that we can write the policy rule as*

$$i_t = \hat{i}_t + \phi(\pi_t - \hat{\pi}_t). \tag{14}$$

Then, the unique solution to (10)-(12) and (14) is given by $x_t = \hat{x}_t$, $\pi_t = \hat{\pi}_t$, and $b_t = \hat{b}_t$.

Condition (13) generalizes the Taylor principle to our setting.[6] For the rest of the paper, we assume condition (13) is satisfied. An implication is that monetary policy is active in the sense of Leeper (1991). Appendix B further shows that fiscal policy is passive when $\gamma \geq 0$. The second part of Proposition 1 shows how a time-varying inflation target implements any equilibrium allocation. A similar approach can be used to implement the equilibrium outcomes in the inflationary-finance phase by assuming the monetary authority follows the policy rule: $i_t^* = \rho + \phi\pi_t^* + u_t^*$, given the same coefficient ϕ.[7]

Integral representation Given an arbitrary path for the real rate $r_t = i_t - \pi_t$, we can characterize the system in closed form. The path of debt satisfies the following condition:

$$b_t = e^{-\gamma t} b_0 + \int_0^t e^{-\gamma(t-s)} (\psi_s + r_s - \rho) ds. \tag{15}$$

Debt accumulates through two forces: fiscal pressures, ψ_s, and real interest rates that exceed the natural rate ρ. The parameter γ controls the mean reversion in public debt.

Policy uncertainty leads to a *discounted Euler equation*:

$$x_t = -\int_t^\infty e^{-\lambda_h(s-t)} (r_s - \rho) ds + \lambda_h \int_t^\infty e^{-\lambda_h(s-t)} (b_s - b^n) ds. \tag{16}$$

[6] When $\lambda_h = 0$, we recover the standard Taylor principle: equilibrium determinacy requires $\phi > 1$.

[7] Here disturbances to the Taylor rule are regime-dependent, but the coefficients are fixed, in contrast to the literature on regime-dependent rules—see e.g. Farmer, Waggoner and Zha (2009).

This equation states that changes in future interest rates are discounted by λ_h. Moreover, the output gap depends on the present discounted value of future debt gaps.

Integrating the NKPC forward, we obtain the inflation rate

$$\pi_t = \kappa \int_t^\infty e^{-(\rho+\lambda_f)(s-t)} x_s ds + \kappa \Phi \lambda_f \int_t^\infty e^{-(\rho+\lambda_f)(s-t)} (b_s - b^n) ds. \tag{17}$$

As in the standard New Keynesian model, inflation is given by forward-looking components. One component equals the expected present value of output gaps in the fiscal-expansion phase. The second component captures the expectation effects. Several empirical studies document such effects: Hazell, Herreno, Nakamura and Steinsson (2022) estimate a similar NKPC that contains a term capturing long-term inflation expectations and find that most of the variation in inflation comes from that term.[8] Coibion, Gorodnichenko and Weber (2022) argue that news about future debt leads households to anticipate higher inflation, both in the short run and the long run, connecting these expectations effects to the level of public debt, consistent with (17).

The appearance of a backward-looking variable in the model—the level of public debt—has important implications. To compute inflation and output in the textbook model, we need only information on rates from that moment onward. Thus, if the shock vanishes with time, so will its effects. This is not true when the backward-looking behavior of debt is present, as we now need information on the entire history of rates, not only rates going forward. As a result, monetary policy in the past can affect today's outcomes. The sticky inflation phenomenon we highlight in this paper critically depends on this feature.

3. Three policy experiments

This section presents three policy experiments. The results demonstrate that once the expectation of future monetary accommodation is present, monetary policy can no longer stabilize output and inflation simultaneously. This finding is significant, as it implies the failure of *divine coincidence*. This result depends crucially on firms' expectation. If firms do not expect a reform, it is possible to jointly stabilize the output gap and inflation—even if households do expect one. To isolate the role of firm expectations, we assume $\lambda_h = 0$ throughout most of the section. For tractability, we also focus on the case without a fiscal stabilizer ($\gamma = 0$). These assumptions are relaxed at the end of the section.

[8] Hazell et al. (2022) attribute fluctuations in the expectations component to *permanent* changes in monetary policy—permanent changes in output gap targets. Equation (17) shows that *temporary* fiscal shocks can rationalize that evidence since they will provoke movements in the expectation component.

For the rest of the analytical formulations, we assume the fiscal shock is exponentially decaying, the continuous-time analog of AR(1) processes in discrete time: $\psi_t = e^{-\theta_\psi t}\psi_0$.

Policy I: Output gap stabilization. In the first experiment, monetary policy aims to stabilize output during the fiscal-expansion phase. That is, monetary policy implements a zero output gap, $x_t = 0$.

To stabilize the output gap, the real rate must satisfy $r_t = \rho$. Given the fiscal shock, government debt is increasing over time: $b_t = b^{lr} - \psi_t/\theta_\psi$, where $b^{lr} \equiv b_0 + \psi_0/\theta_\psi$ denotes the long-run debt level in the fiscal-expansion phase.

The proposition below shows that the expectation effects induced by the fiscal shock lead to an increasing path of inflation over time.

Proposition 2 (Inflation under output gap stabilization)**.** *Suppose $x_t = 0$ in the fiscal-expansion phase. Then, inflation is*

$$\pi_t = \frac{\kappa\lambda\Phi}{\rho+\lambda}\left[b_t - b^n + \frac{\psi_t}{\rho+\lambda+\theta_\psi}\right]. \tag{18}$$

Moreover, inflation increases over time, $\dot{\pi}_t = \frac{\kappa\lambda\Phi}{\rho+\lambda+\theta_\psi}\psi_t > 0$, and converges to a positive level, $\lim_{t\to\infty}\pi_t = \frac{\kappa\lambda\Phi}{\rho+\lambda}(b^{lr} - b^n) > 0$.

Proposition 2 shows that, to stabilize output, monetary policy must live with an ever-growing inflation. In the fiscal-expansion phase, inflation increases initially in proportion to primary deficits. However, inflation persists even after deficits dissipate. That is, *inflation is sticky*. Sticky inflation occurs because the jump inflation component in the Phillips curve, $\pi_t^J = \kappa\Phi(b_t - b^n)$, reflects the expected burst in inflation that trails the path of debt in an inflationary-finance phase.

Therefore, an independent monetary policy focused on stabilizing output must live with inflation that trails debt. The sole belief, rational or not, of a future compromise to aid debt stabilization is enough to destabilize inflation in a monetary independent regime.

Policy II: Inflation stabilization. In the previous exercise, monetary policy focuses exclusively on stabilizing output. In the next exercise, it attempts to combat inflation by temporarily raising rates, such that $r_t - \rho = e^{-\theta_r t}(r_0 - \rho)$, for a given initial rate $r_0 > \rho$ and persistence parameter $\theta_r > 0$.

It is useful to express results relative to the previous experiment. For that, we use a superscript og to denote variables in the output-gap stabilization exercise. With mean reverting shocks to r_t, the output gap follows:

$$\dot{x}_t = r_t - \rho \Rightarrow x_t = -\frac{1}{\theta_r}(r_t - \rho), \tag{19}$$

where we used the terminal condition $\lim_{t\to\infty} x_t = 0$. As $r_t > \rho$, the output gap is negative during the fiscal-expansion phase. In turn, the path of debt is

$$b_t = b_t^{og} + \frac{1 - e^{-\theta_r t}}{\theta_r}(r_0 - \rho), \tag{20}$$

where $b_t^{og} = b_0 + \frac{1-e^{-\theta_\psi t}}{\theta_\psi}\psi_0$.

We solve for inflation using the NKPC—equation (17). A policy that fights inflation deviates from the output-gap stabilization solution through the sum of two effects: a fight-inflation effect and a jump-inflation effect. Formally:

Lemma 1. *Suppose $r_0 > \rho$. With mean-reverting real interest rates, $\dot{r}_t = -\theta_r(r_t - \rho)$, inflation is given by:*

$$\pi_t - \pi_t^{og} = F_t^\pi + J_t^\pi.$$

where F_t^π and J_t^π are, correspondingly, fight and jump inflation components given by:

$$F_t^\pi = -\frac{\kappa}{\theta_r}\frac{e^{-\theta_r t}}{\rho + \lambda + \theta_r}(r_0 - \rho) < 0 \quad \textit{and} \quad J_t^\pi = \frac{\lambda\kappa\Phi}{\theta_r}\left[\frac{1}{\rho+\lambda} - \frac{e^{-\theta_r t}}{\rho+\lambda+\theta_r}\right](r_0 - \rho) > 0.$$

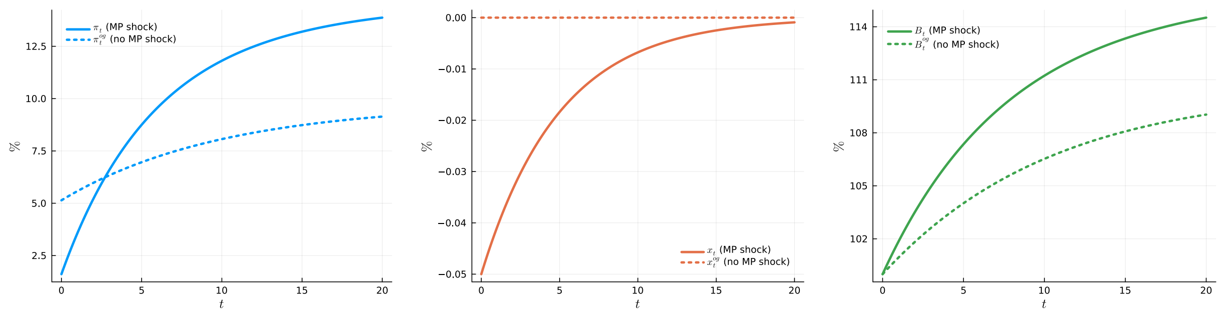
The first term, the fight-inflation term F_t^π, captures the standard effect of contractionary policy through aggregate demand. The term is negative since $r_t > \rho$ and converges to zero as the contractionary effect vanishes. Thus, the increase in r_t has a mitigating effect on inflation, as in standard versions of the New Keynesian model.

The second term, the jump inflation J_t^π, is the expected present value of inflation after monetary accommodation, which depends on the path of debt. Jump inflation is always positive and builds up with time, as debt grows with the accumulation of past interests. Thus, current rate hikes feedback into present inflation through the expectation of a greater burst in inflation in the future.

Whereas the fight-inflation term vanishes over time, the jump-inflation term continues to build up. Hence, which effect dominates depends on the horizon ahead of the monetary stimulus and the persistence of the shocks. Concretely, we have:

Proposition 3 (Stepping on a Rake)**.** *Suppose $r_0 > \rho$. The rate increase reduces inflation on impact, i.e., $\pi_0 < \pi_0^{og}$ iff:*

$$\theta_r < \frac{\rho + \lambda}{\lambda\Phi}.$$



(a) Inflation (b) Output gap c) Debt

Figure 3: Equilibrium paths with and without contractionary monetary shock

However, there always exists a $\hat{T} > 0$ such that $\pi_t > \pi_t^{og}$ for $t > \hat{T}$.

The proposition shows two things. First, to be successful in the present, monetary policy must commit to a sufficiently persistent contractionary policy stance. Monetary policy can fight inflation in the short run, provided the policy is sufficiently persistent.

Second, although monetary policy may succeed in fighting inflation in the short run, it faces an unpleasant "stepping-on-a-rake" result: eventually, inflation will come back and stronger. Once again, *inflation is sticky*. The reason is that the contractionary effect on the output gap eventually fades away, whereas the effect on the government debt builds up over time.

Figure 3 shows the path of inflation, output gap, and debt, for an attempt to fight inflation in the fiscal-expansion phase. In Panel (a), we see two paths of inflation: a baseline (solid) corresponding to a temporary increase in policy rates and a counterfactual (dotted) with real rates equal to its natural level. While the anti-inflationary strategy is successful early on, inflation returns a year into the policy. Panel (b) shows the contractionary effect on the output gap, while Panel (c) shows the path of debt, which accumulates at a faster pace with the contractionary policy. With an expected monetary accommodation lurking, attempts to curtail inflation have standard short-run effects. Unlike the canonical New Keynesian model, they lead to higher inflation in the medium run.

While monetary policy cannot fully stabilize inflation with temporary movements in the output gap, it could do so if it were to induce a permanent decline in output. Hence, full inflation stabilization in the fiscal-expansion phase is feasible for the monetary authority, but it requires to keep the economy persistently depressed.[9]

[9] In this case, the output gap must offset movements in the government debt, $x_t = -\lambda\Phi(b_t - b^n)$. This condition requires that the real rate be $r_t - \rho = -\frac{\lambda\Phi}{1+\lambda\Phi}\psi_t$, in which case debt is given by $b_t = b_0 + \frac{1-e^{-\theta_\psi t}}{\theta_\psi}\frac{\psi_0}{1+\lambda\Phi}$.

Discussion: relation to the literature. Versions of the result in Proposition 3 appear in the literature on fiscal-monetary interactions, though they typically rely on different mechanisms. Sims (2011) coined the term "stepping on a rake" to describe a similar dynamic, which arises in his framework from changes in the valuation of long-term debt—a channel absent in our model. The result also echoes the unpleasant monetarist arithmetic of Sargent and Wallace (1981), where a one-time increase in the money supply leads to persistently higher inflation through seigniorage. In contrast, our mechanism operates through expectations of future inflation tied to the level of debt.

Bianchi and Melosi (2019, 2022) shows that fiscal shocks in an economy with a temporarily inconsistent policy regime—characterized by active monetary and fiscal policies—can trigger a fiscal stagflation. We obtain a similar result in a setting with active monetary and passive fiscal policy, but where agents anticipate a shift toward future monetary accommodation. Importantly, we not only provide an analytical characterization of the conditions under which this "stepping-on-a-rake" behavior emerges, but also examine the implications for optimal monetary policy in such an environment.

Policy III: Debt stabilization. In the third policy experiment, monetary policy attempts to stabilize debt. Stabilizing debt requires the real rate to neutralize the effects of deficits: $r_t - \rho = -\psi_t$, so $b_t = b_0$. Thus, the output gap is: $x_t = \psi_t/\theta_\psi$ given the terminal condition $\lim_{t\to\infty} x_t = 0$. In this case, inflation follows:

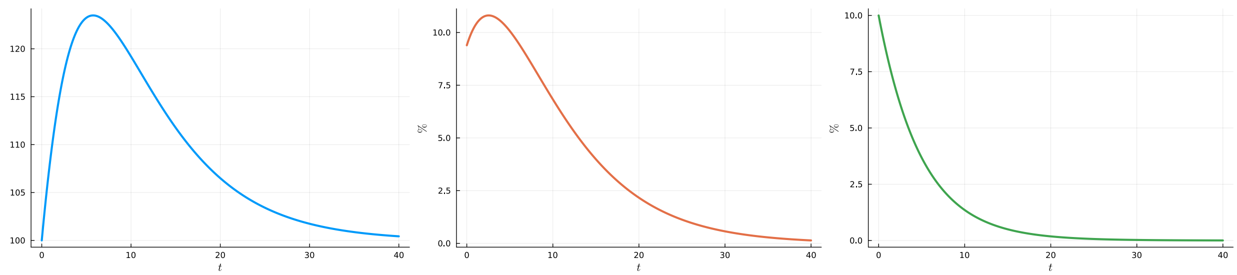
$$\pi_t = \frac{\kappa}{\theta_\psi}\frac{\psi_t}{\rho + \lambda + \theta_\psi}.$$

All in all, to stabilize the debt, the monetary authority must overheat the economy in proportion to the trajectory of primary deficits.

Debt stabilizers and household expectations. The previous exercises abstracted from household expectation effects ($\lambda_h = 0$) and debt stabilizers ($\gamma = 0$). We now examine the role of these two features. For clarity, we return to the first exercise and consider a policy that fully stabilizes output ($x_t = 0$), while setting $b_0 = b^n = 0$ to simplify the expressions.

With households expecting a reform, stabilizing output in the fiscal-expansion phase requires setting the real rate to $r_t - \rho = \lambda_h b_t$. A high interest rate is necessary to offset the expansionary effects of a positive debt gap. Given this real rate, debt will follow:

$$\dot{b}_t = -(\gamma - \lambda_h)b_t + \psi_t \Rightarrow b_t = \frac{e^{-(\gamma-\lambda_h)t} - e^{-\theta_\psi t}}{\theta_\psi + \lambda_h - \gamma}\psi_0,$$



(a) Government debt (b) Inflation (c) Fiscal shock

Figure 4: Equilibrium with households' expectation effects and debt stabilizer

Thus, provided $\lambda_h < \gamma$, debt eventually reverts to its initial level. In turn, inflation is given by the present discounted value of its jump inflation term, which is given by

$$\pi_t = \frac{\psi_0}{\theta_\psi + \lambda_h - \gamma}\left[\frac{e^{-(\gamma-\lambda_h)t}}{\rho+\lambda+\gamma-\lambda_h} - \frac{e^{-\theta_\psi t}}{\rho+\lambda+\theta_\psi}\right] > 0.$$

These expressions clarify the roles of λ_h and γ. A positive λ_h implies that real interest rates rise with the level of debt, which amplifies debt accumulation through higher borrowing costs. In contrast, a positive γ triggers an offsetting response of primary surpluses, helping to reduce debt. When $\lambda_h < \gamma$, inflation converges back to its steady state. In this case, the automatic debt stabilizer is strong enough for primary surpluses to gradually reduce debt. Figure 4 illustrates this mechanism: a fiscal shock causes a sharp rise in government debt and a temporary inflation surge. As the shock fades, the stabilizer mechanism lowers debt, and inflation subsides. The figure also highlights the sticky inflation phenomenon—inflationary effects persist even after ψ_t has nearly returned to zero.

When $\gamma = \lambda_h$, we recover the case discussed at the beginning of this section. To stabilize output, interest rates must rise to counteract the household-expectation effects. In this scenario, the automatic debt stabilizer exactly offsets the impact of higher interest rates on debt. As a result, debt dynamics are identical to the case with $\gamma = \lambda_h = 0$. When $\gamma < \lambda_h$, the feedback loop between debt accumulation and real rates leads to explosive dynamics: debt and inflation spiral upward. In cases where real rates react to the level of debt, fiscal sustainability requires that primary surpluses respond aggressively to debt.

This last exercise shows that adding household expectations and an automatic stabilizer does not change the main message of the output stabilization exercise. Appendix A.2 develops the other experiments and shows that the same lessons carry through in general.

Discussion: endogenous fiscal cost-push shocks. We have shown that efforts to stabilize one variable—output, inflation, or debt—inevitably destabilize the others. As a result, divine coincidence breaks down in our setting, even in the absence of traditional supply shocks. The reason is that expectations of future monetary accommodation by price-setters generate an endogenous *fiscal cost-push shock*.[10] Unlike conventional supply shocks, the fiscal cost-push shock responds to changes in monetary policy. This feature has important implications for the design of optimal policy, which we consider next.

4. Optimal Policy

In this section, we study optimal monetary policy during the fiscal-expansion phase. As discussed above, the expectation of monetary accommodation breaks divine coincidence. Thus, a benevolent monetary authority faces a non-trivial trade-off between stabilizing output, inflation, and debt.

4.1 The optimal policy problem

We consider a standard approximation to households' welfare function in which the planner minimizes the expected present value of squared deviations of output and inflation from their steady-state values. The only policy instrument is the path of nominal interest rates during the fiscal-expansion phase. The planner commits to a path of interest rates.

We present the optimal policy without automatic debt stabilizers, $\gamma = 0$, and no households' expectation effects, $\lambda_h = 0$. We relegate more general solutions to Appendix D. The planner's and firms' beliefs coincide. Hence, we write $\lambda_f = \lambda$.

The planner's objective. Once the inflationary-finance phase initiates, the planner has no control over inflation or output, but we can still compute the value of its welfare objective. Starting with a debt level b_0^*, the value of the planner's objective is proportional to the square deviation of debt from its neutral level:

$$\mathcal{P}^*(b_0^*) = \int_0^{T^*} e^{-\rho t}(\alpha x_t^{*2} + \beta \pi_t^{*2})dt = \Upsilon \cdot (b_0^* - b^n)^2$$

where $\Upsilon \equiv (\alpha + \beta(\kappa\Phi)^2) \int_0^{T^*} e^{-\rho t} \left(1 - \frac{t}{T^*}\right)^2 dt$.[11]

[10] This result is reminiscent of the endogenous cost-push shock in Guerrieri, Lorenzoni, Straub and Werning (2021). While they rely on asymmetric sectorial shocks, we focus on the role of expectation effects.

[11] We use that $x_t^* = (b_0^* - b^n)\left(1 - \frac{t}{T^*}\right)$ and $\pi_t^* = \kappa\Phi(b_0^* - b^n)\left(1 - \frac{t}{T^*}\right)$ to obtain $\mathcal{P}^*(b_0^*) = \Upsilon(b_0^* - b^n)^2$.

At the beginning of the fiscal-expansion phase, the planner's objective function can be written as

$$\mathcal{P} = -\frac{1}{2}\mathbb{E}\left[\int_0^\tau e^{-\rho t}\left(\alpha x_t^2 + \beta \pi_t^2\right) dt + e^{-\rho\tau}\mathcal{P}^*_\tau(b_\tau)\right],$$

where τ denotes the random time the economy switches to an inflationary-finance phase. Given the arrival time is exponentially distributed, we obtain:

$$\mathcal{P} = -\frac{1}{2}\int_0^\infty e^{-(\rho+\lambda)t}\left[\alpha x_t^2 + \beta\pi_t^2 + \lambda\cdot\Upsilon\cdot(b_t - b^n)^2\right] dt.$$

This objective tells us that even though only output and inflation directly affect the planner's objective, the influence of government debt on the inflationary-finance phase creates an endogenous *debt-stabilization motive*. In other words, in addition to inflation and output, the planner wants to minimize deviations of government debt from its natural level; the weight on debt does not come from the planner's concern about budgetary affairs but because debt will affect inflation in the inflationary-finance phase. Debt will also affect inflation in the fiscal-expansion phase through expectation effects.

An important implication of this result is that our setting becomes markedly distinct from the classic analysis of Barro (1979) and its modern extensions (e.g., Aiyagari, Marcet, Sargent and Seppälä, 2002). In those frameworks, fluctuations in government debt are used to optimally smooth distortionary taxes over time. In contrast, deviations of debt from its natural level are costly in our setting, giving the planner an explicit incentive to stabilize the debt path itself.

Competitive equilibria. The planner's problem involves choosing a competitive equilibrium. A competitive equilibrium corresponds to a bounded solution to the system (10)-(12) given b_0, a path of fiscal shock ψ_t, and a path of real interest rates.

For any given initial condition for the output gap, inflation satisfies:

$$\pi_0 = \kappa\frac{x_0 + \lambda\Phi(b_0 - b^n)}{\rho+\lambda} + \frac{\kappa}{\rho+\lambda}\int_0^\infty e^{-(\rho+\lambda)t}\left[(1+\lambda\Phi)(r_t-\rho) + \lambda\Phi\psi_t\right] dt. \tag{21}$$

Thus, the set of competitive equilibria can be indexed by a path of real interest rates $\{r_t\}_0^\infty$ and an initial output gap x_0—the monetary authority can implement a particular equilibrium using the conditions in Proposition 1. While the planner can freely choose the initial output gap, it cannot independently choose *both* the output gap and inflation.

Debt expropriation and lack of a classical solution. As often occurs in optimal Ramsey problems, the planner may have incentives to expropriate private agents at time zero. Debt is real, and prices are sticky, so expropriation cannot occur through a price level jump. Instead, the planner can effectively choose an arbitrarily negative real return r_t on debt for an infinitesimal period, which leads to a downward jump in government debt in period zero. This would amount to an instantaneous debt deflation.

The above observation implies that a classical solution to the planner's problem, one where state variables follow a continuous path, does not exist. The possibility of an "expropriation-like" debt path occurs because the model does not penalize extremely low rates. To avoid the possibility of expropriation, we introduce penalties on past promises, similar to the approach in Marcet and Marimon (2019) and Dávila and Schaab (2023).[12]

In particular, we consider a penalized version of the problem in which the planner faces a penalty associated with the choice of the initial value for each forward-looking variable, namely x_0 and π_0. By appropriately choosing the penalties, we ensure there is no expropriation. The penalty itself does not directly affect the path of inflation and output. Its effect on the optimal solution is entirely mediated by the impact on the initial debt level. The planner's problem can be written as follows:

Problem 1 (Commitment Problem)**.** *The planner's problem is*

$$\max_{[x_t,\pi_t,b_t,r_t]_0^\infty} -\frac{1}{2}\int_0^\infty e^{-(\rho+\lambda)t}\left[\alpha x_t^2+\beta\pi_t^2+\lambda\Upsilon(b_t-b^n)^2\right]dt+\xi_x x_0+\xi_\pi\pi_0, \tag{22}$$

subject to the equilibrium system conditions (10-12) and the initial condition for inflation, (21)*, given b_0 and fiscal shock's path, ψ_t.*

The integral above is the original objective, and the last two terms capture the penalties on the initial output gap, ξ_x, and initial inflation, ξ_π. Absent the penalties, the initial value of the co-states for inflation, output gap, and debt are all zero. In that case, there would be a discontinuous jump in the value of debt at $t=0$. We choose the values of ξ_x and ξ_π such that $\lim_{t\to 0} b_t = b_0$, while the initial value of the co-states on the output gap and inflation is still equal to zero.

Optimal interest rate policy. The following proposition characterizes interest rates under the optimal policy.

[12] We show in the Appendix C.1 that a classical solution with smooth state variables does not exist without a penalty. Formally, it is optimal to have a Dirac mass on interest rates in period zero, and $\lim_{t\to 0} b_t \neq b_0$. See Arutyunov, Karamzin and Pereira (2019) for a discussion of control problems lacking classical solutions.

Proposition 4 (Interest rates.). *The paths of real and nominal interest rates under the optimal policy are:*

$$r_t - \rho = -\beta \frac{\kappa(1+\lambda\Phi)}{\lambda\Upsilon + \alpha}\pi_t - \frac{\lambda\Upsilon}{\lambda\Upsilon+\alpha}\psi_t \quad \text{and } i_t - \rho = \left[1 - \beta\frac{\kappa(1+\lambda\Phi)}{\lambda\Upsilon+\alpha}\right]\pi_t - \frac{\lambda\Upsilon}{\lambda\Upsilon+\alpha}\psi_t.$$

The proposition provides a solution to the optimal path of real rates as a linear rule. An important implication is that it is optimal to *reduce* the real interest rate in response to the fiscal shock. That is, to the extent the shock is inflationary, i.e., if $\pi_t \geq 0$, real rates should fall below the natural rate ρ. Moreover, nominal rates move less than one-to-one with inflation. This result sharply contrasts with the standard prescription based on the Taylor rule.[13] In contrast, Proposition 4 shows that it is optimal to *underreact* to the shock.

To understand the intuition behind this result, consider a perturbation to a path of real rates that raises both x_t and b_t by one unit while keeping their values at other dates constant.[14] The perturbation reduces the planner's objective by

$$\mathcal{I}_t = \alpha x_t + \lambda\Upsilon(b_t - b^n) + \beta\kappa(1+\lambda\Phi)\int_0^t \pi_s ds. \tag{23}$$

The first two terms reflect the direct impact of changing x_t and b_t, while the last term captures the indirect impact through inflation in all past dates. Under the optimal policy, the marginal cost of changing x_t and b_t is equalized across all periods, so $\dot{\mathcal{I}}_t = 0$. This implies that it is optimal to reduce the real rate when inflation is high, so the first two terms offset the welfare impact of having high inflation.[15] Therefore, nominal rates must react less than one-to-one to inflation under the optimal policy.

Dynamics under optimal policy. To characterize the dynamics under the optimal policy, we use the optimal real interest rule, combined with the fact that $x_t = x_0 + b_t - b_0 - \hat{\psi}_t$, and collapse the solution into a bivariate system in π_t and b_t:

$$\begin{bmatrix} \dot{\pi}_t \\ \dot{b}_t \end{bmatrix} = \begin{bmatrix} \rho+\lambda & -\kappa(1+\lambda\Phi) \\ -\hat{\beta} & 0 \end{bmatrix} \begin{bmatrix} \pi_t \\ b_t - b^n \end{bmatrix} + \begin{bmatrix} \kappa(\hat{\psi}_t + b_0 - b^n - x_0) \\ \frac{\alpha}{\lambda\Upsilon+\alpha}\psi_t \end{bmatrix}, \tag{24}$$

[13] Of course, in our solution, the planner still uses the *threat* of reacting to movements in inflation more than one-to-one off the equilibrium to ensure the equilibrium selection.

[14] Given x_0, and under the log utility assumption, an increase in real rates raises the output gap and government debt by the same amount.

[15] Notice that $\dot{\mathcal{I}}_t = \alpha(r_t - \rho) + \lambda\Upsilon(r_t - \rho + \psi_t) + \beta\kappa(1+\lambda\Phi)\pi_t$, using the Euler equation and government's flow budget constraint. Hence, $\dot{\mathcal{I}}_t = 0$ implies the the optimal real rate in Eq. (23) after rearrangement.

where $\hat{\beta} \equiv \frac{\beta\kappa(1+\lambda\Phi)}{\lambda\Upsilon+\alpha}$ and $\hat{\psi}_t = \frac{1-e^{-\theta_\psi t}}{\theta_\psi}\psi_0$. The eigenvalues of this system are:

$$\overline{\omega} = \frac{\rho+\lambda+\sqrt{(\rho+\lambda)^2+4\kappa(1+\lambda\Phi)\hat{\beta}}}{2} > 0, \qquad \underline{\omega} = \frac{\rho+\lambda-\sqrt{(\rho+\lambda)^2+4\kappa(1+\lambda\Phi)\hat{\beta}}}{2} < 0.$$

Because there is one positive and one negative eigenvalue, there is a unique bounded solution for any given x_0.

The optimality condition for the initial output gap is:

$$\int_0^\infty e^{-(\rho+\lambda)t}\left[\alpha x_t + \frac{\beta\kappa}{\rho+\lambda}\pi_t\right] = 0. \tag{25}$$

This condition says that the planner sets the discounted value of a combination of output and inflation to zero, depending on the relative weight of output and inflation on welfare. Therefore, if inflation is, on average positive, it is optimal to choose x_0 such that the present value of the output gap is negative, counteracting the inflationary pressures.

4.2 Hawks vs. doves

It is instructive to consider extreme cases where the planner only cares about inflation or only about output, which we associate with *hawkish* and *dovish* central banks. In both cases, the planner assigns a positive endogenous weight to debt stabilization. To simplify the message, we set $b_0 = b^n = 0$. We characterize optimal policy for generic values of α and β in Appendix C.

Doves. Consider first the case of a dovish central bank, that is, $\beta = 0$.

Proposition 5 (Optimal policy: Doves)**.** *If $\beta = 0$, then,*

(i) Inflation:

$$\pi_t = \frac{\kappa}{\overline{\omega}}\frac{\alpha\lambda\Phi}{\alpha+\lambda\Upsilon}\frac{\psi_0}{\overline{\omega}+\theta_\psi} + \frac{\kappa\lambda(\alpha\Phi-\Upsilon)}{\lambda\Upsilon+\alpha}\frac{1-e^{-\theta_\psi t}}{\theta_\psi(\overline{\omega}+\theta_\psi)}\psi_0. \tag{26}$$

where $\pi_t > 0$ and $\dot{\pi}_t > 0$.

(ii) Output gap:

$$x_t = \frac{\lambda\Upsilon}{\lambda\Upsilon+\alpha}\frac{\psi_0}{\rho+\lambda+\theta_\psi} - \frac{\lambda\Upsilon}{\lambda\Upsilon+\alpha}\frac{\psi_0-\psi_t}{\theta_\psi}, \tag{27}$$

where $x_0 > 0$ and $\dot{x}_t < 0$.

(iii) Government debt:

$$b_t = \frac{\alpha}{\lambda\Upsilon + \alpha}\frac{\psi_0 - \psi_t}{\theta_\psi}. \tag{28}$$

Proposition 5 characterizes the optimal reaction of a dovish central bank to a fiscal shock. The dovish central bank faces a trade-off between stabilizing the output gap in the fiscal-expansion phase and stabilizing the output gap in the inflationary-finance phase, which ultimately requires influencing the government debt. The optimal response of the monetary authority is to partially offset the effects of the fiscal shock on debt:

$$r_t - \rho = -\frac{\lambda\Upsilon}{\lambda\Upsilon + \alpha}\psi_t.$$

This can be seen from the optimality condition (23). When the planner gives no weight to inflation, equalizing the marginal cost of changing interest rates, $\dot{\mathcal{I}}_t = 0$, requires the following condition to be satisfied: $\alpha\dot{x}_t = -\lambda\Upsilon\dot{b}_t < 0$. Hence, it is optimal to front-load aggregate demand such that the decline in output counteracts the increase in government debt. This requires a reduction in real rates in response to the fiscal shock. The magnitude of the adjustment depends on the relative weight of debt stabilization on welfare. When λ is close to zero, it is unlikely the economy will switch to the inflationary-finance phase, and the planner minimally reacts to the shock. In this case, the output gap is close to zero, and debt absorbs most of the fiscal shock. When λ is large, the planner offsets most of the fiscal shock, dampening the debt response. Given the planner only cares about the output gap, there is no attempt to stabilize inflation, which ends up being positive and increasing over time.

Hawks. Consider next the case of a hawkish central bank, that is, $\alpha = 0$.

Proposition 6 (Optimal policy: Hawks)**.** *Suppose $\alpha = 0$. Then,*

(i) Inflation:

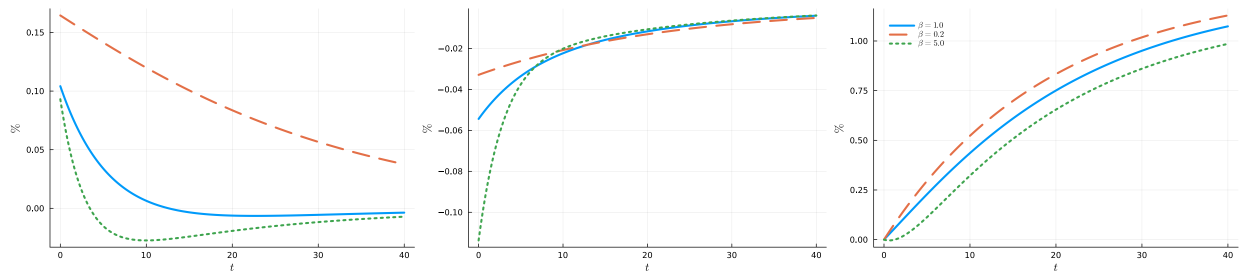
$$\pi_t = \kappa\frac{\psi_t - \frac{\overline{\omega}}{\rho+\lambda+\theta_\psi}e^{\underline{\omega}t}\psi_0}{(\overline{\omega}+\theta_\psi)(\underline{\omega}+\theta_\psi)}, \tag{29}$$

where $\pi_0 > 0$, $\dot{\pi}_0 < 0$, and $\pi_t < 0$ for t sufficiently large.

(ii) Output gap:

$$x_t = \frac{\psi_0}{\rho+\lambda+\theta_\psi}\left[\frac{\overline{\omega}}{\overline{\omega}+\theta_\psi} + \frac{\rho+\lambda+\theta_\psi}{\overline{\omega}+\theta_\psi}\right] - \beta\frac{\kappa(1+\lambda\Phi)}{\lambda\Upsilon}p_t - \frac{\psi_0-\psi_t}{\theta_\psi}, \tag{30}$$

where $p_t = \int_0^t \pi_s ds$ is the price level at date t.



(a) Inflation (b) Real rates (c) Government debt

Figure 5: Equilibrium dynamics under optimal policy

(iii) Government debt

$$b_t = -\beta \frac{\kappa(1+\lambda\Phi)}{\lambda\Upsilon} p_t, \tag{31}$$

where $\dot{b}_0 < 0$ *and* $\lim_{t\to\infty} b_t > 0$.

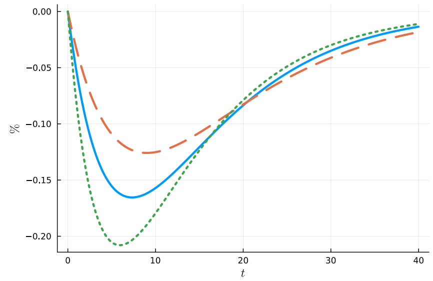
The hawkish central bank faces a trade-off between stabilizing inflation in the fiscal-expansion phase and stabilizing it in the inflationary-finance phase through its effect on government debt. Given $\pi_0 > 0$, it is again optimal to reduce real rates on impact:

$$r_0 - \rho = -\beta \frac{\kappa(1+\lambda\Phi)}{\lambda\Upsilon} \pi_0 - \psi_0 < 0. \tag{32}$$

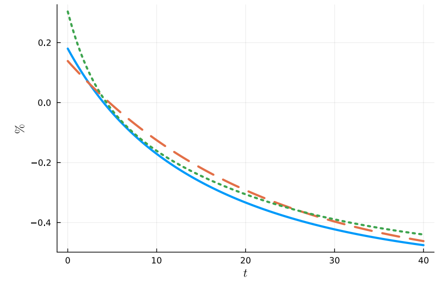
In this case, to equalize the marginal cost of changing interest rates, the following condition must be satisfied: $\lambda\Upsilon\dot{b}_t = -\beta\kappa(1+\lambda\Phi)\pi_t$. Hence, it is optimal to reduce real rates to slow down debt accumulation, partially offsetting inflationary pressures. Interestingly, a hawkish central bank initially reduces real rates more aggressively than its dovish counterpart.[16] Therefore, sticky inflation creates an incentive for the planner to *front-load* inflation. Low real rates initially raise the output gap, creating some short-run inflationary pressures, but it slows down debt accumulation and reduces future inflation.

Discussion: Hawks vs. doves. Figure 5 shows the optimal policy for different values of β, the welfare weight on inflation, for a fixed weight on the output gap, which we normalize to $\alpha = 1$. The case $\beta = 1$ corresponds to a planner who gives equal weight to inflation and the output gap, while the case $\beta > 1$ ($\beta < 1$) corresponds to a planner who gives more weight to inflation (output gap). A striking feature is that the optimal real rate is below its natural level, regardless of β. Therefore, the planner always finds it optimal to move nominal rates less than one-to-one with inflation.

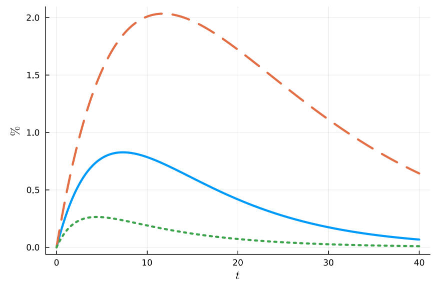
[16] Recall that for a dovish central bank $r_0 - \rho = -\frac{\lambda\Upsilon}{\lambda\Upsilon+\alpha}\psi_0$, which is greater than $-\beta\frac{\kappa(1+\lambda\Phi)}{\lambda\Upsilon}\pi_0 - \psi_0$.



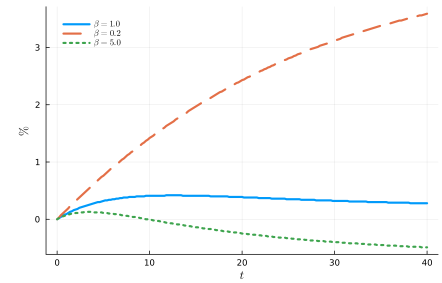
(a) Output gap (Cost-push shock)



(b) Output gap (Fiscal shock)



(c) Price level (Cost-push shock)



(d) Price level (Fiscal shock)

Figure 6: Exogenous cost-push shock vs fiscal shock

Paradoxically, a hawkish central bank achieves lower inflation despite having lower real rates. By reducing the pace of debt accumulation, the planner counteracts the inflationary pressures stemming from de-anchored expectations caused by the fiscal shock.

Automatic debt stabilizers. We focused so far on the case without automatic debt stabilizers ($\gamma = 0$). In this setting, temporary shocks can have permanent effects on government debt. By contrast, when automatic stabilizers are present ($\gamma > 0$), the fiscal authority is assumed to commit to raising taxes over time, ensuring that debt eventually returns to its initial level. Appendix D.1 shows that the optimal policy with automatic debt stabilizers is qualitatively similar to the case with $\gamma = 0$. The main difference regards the long-run behavior debt. With automatic debt stabilizers, debt eventually returns to its initial level, so both output and inflation return to their steady-state values.

4.3 Comparison with textbook analysis

We have shown that sticky inflation in our setting manifests as an endogenous fiscal cost-push shock. While this shock shares similarities with conventional cost-push shocks, the implications for optimal policy differ in important ways.

The first key difference concerns the behavior of the output gap. In standard models, an exogenous cost-push shock leads to stagflation under the optimal policy—that is, inflation rises while the output gap contracts. Panel (a) of Figure 6 illustrates this pattern: the output gap turns negative even when the central bank adopts a dovish stance.

Although our framework allows for fiscal stagflation—as demonstrated in Section 3—Panel (b) shows that optimal policy avoids stagflation on impact. In fact, the output gap is initially positive, even under a highly hawkish central bank. Intuitively, the central bank responds to the fiscal shock by lowering real rates enough to stimulate demand, thereby overheating the economy despite the inflationary impact.

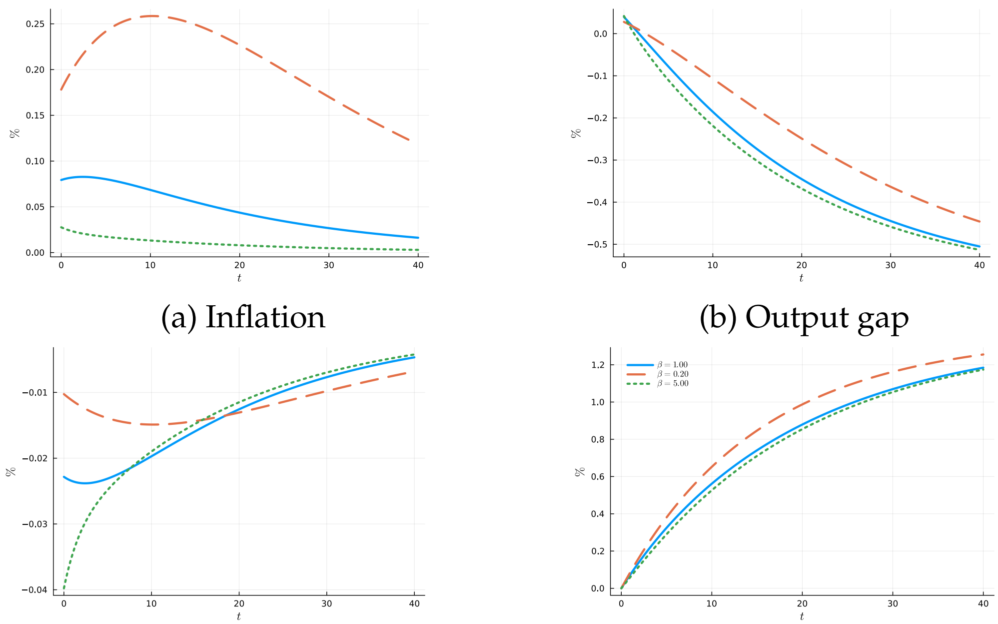
The second key difference concerns the behavior of the price level. In the textbook case, inflationary episodes are followed by deflation, such that the price level eventually returns to its initial level—see Panel (c) of Figure 6. Price-level targeting is a hallmark of optimal monetary policy under commitment (see, e.g., Woodford 2010). In contrast, in our framework, the price level is non-stationary. As shown in Panel (d), it does not revert to its pre-shock level following the inflationary episode.

These differences have important implications for the appropriate monetary response to shocks. In the case of a conventional supply shock, the optimal policy calls for a temporary recession and a commitment to future deflation in order to offset past inflation and return the price level to target. By contrast, when the shock originates from fiscal sources and manifests as a fiscal cost-push shock, the optimal response is fundamentally different: the central bank should initially overheat the economy, and there is no rationale for engineering a period of deflation to reverse the prior rise in the price level.

4.4 Imperfect credibility

In our baseline analysis, both the planner and firms share the belief that fiscal adjustment may eventually come through monetary accommodation. We now turn to the case of imperfect credibility, in which the central bank is fully committed to avoiding monetary accommodation, but firms continue to view it as a possibility.

Formally, we assume that the planner anticipates a fiscal response—such as future tax increases—in Phase II that eliminates the need for lower real interest rates. From the planner's perspective, the economy transitions directly to the steady state after the Poisson event. Firms, however, believe that the Poisson event will trigger monetary accommodation. This divergence in beliefs allows us to study how the central bank should conduct policy when credibility is incomplete–specifically, when firms expect accommodation despite the central bank's full commitment to avoid it.



(a) Inflation (b) Output gap

(c) Real rates (d) Government debt

Figure 7: Optimal monetary policy with imperfect credibility

Figure 7 presents the results. Even when the monetary authority is fully committed to avoiding inflationary finance, it is still optimal to lower real interest rates in response to the shock. The overall pattern closely mirrors the baseline: a hawkish central bank reduces rates more aggressively to slow debt accumulation, which in turn helps anchor inflation expectations.

Thus, the policy of *underreaction* by the central bank remains optimal—even under full commitment to price stability—so long as that commitment lacks full credibility in the eyes of price-setters.

4.5 Robustness to commitment assumptions

We have seen that optimal monetary policy with commitment reduces real rates in response to a fiscal shock. Is this a robust feature, or does it depend on commitment assumptions? To answer this question, we consider two polar opposites of the time-zero commitment case studied. First, we present the solution under *discretion*. Second, we consider optimal policy under the *timeless perspective*. We show that under both scenarios, reducing real rates in response to the fiscal shock is still optimal. This shows that under-reaction is a robust feature of optimal policies under sticky inflation.

Discretion. To capture the idea of discretion in continuous time, we assume that the planner has commitment over a random time interval and takes as given the actions of future planners.[17] Formally, assume that with Poisson intensity $\overline{\lambda}$, the monetary control is surrendered to a new planner. This implies that, in expectation, the planner has control over $\frac{1}{\overline{\lambda}}$ periods. We are interested in the limit as $\overline{\lambda} \to \infty$. This corresponds to the continuous-time analog of discretion in discrete time, where the planner controls policy over a single period. The next proposition characterizes the optimal policy.

Proposition 7 (Discretion)**.** *As $\overline{\lambda} \to \infty$, the real interest rate under the optimal policy is given by $r_t - \rho = -\psi_t$. Moreover, the output gap is $x_t = 0$.*

Proposition 7 shows that, under discretion, the real rate is also below its natural level upon a fiscal shock. With an arbitrarily short planning horizon, the planner cannot directly control inflation, which depends on future decisions and has no incentive to distort the output gap. Hence, the planner fully stabilizes debt to influence future decisions. Behind the scenes, the planner sets the output gap to zero and promises a decline over time, given the low interest rate. Once a new planner arrives, the planner does not keep this promise, and sets the output gap again to zero.[18]

Timeless perspective. Finally, we consider next the case of optimal policy under the timeless perspective, in the sense of Woodford (1999). When the planner commits to a time-zero plan, it sets the value of the co-states for the forward-looking variables equal to zero at $t = 0$. Under the timeless perspective, the initial value of co-states equals the corresponding value for a planner who started its planning in the distant past.[19] The next proposition shows that the timeless perspective and commitment solutions actually coincide when $b_0 = b^n$.

Proposition 8 (Timeless perspective)**.** *Suppose that $b_0 = b^n$, such that government debt is at its natural level when the fiscal shock is announced. Then, the optimal policy when the planner commits to a time-zero plan coincides with the optimal policy under the timeless perspective.*

An implication of Proposition 8 is that the solution to the Ramsey problem satisfies a *self-consistency* property: output and inflation can be described by time-invariant functions of the exogenous shock, ψ_t, a predetermined variable, b_t, and variables describing

[17]For a similar formulation of a problem with discretion in continuous time, see e.g., Harris and Laibson (2013) and Dávila and Schaab (2023).

[18]In Appendix D.2, we consider the case of partial commitment, where the planner takes the initial value of the output gap as given. Optimal policy with partial commitment coincides with the full commitment case for a dovish central bank. In this case, it is also optimal for the real rate to be below the natural level.

[19]For a formal discussion of this procedure, see the discussion in Giannoni and Woodford (2017).

history-dependence, the co-states on the forward-looking variables. From the point of view of a planner who started planning in the distant past, there is no incentive to have output and inflation deviate from these time-invariant functions.[20] Once again, we find that a reduction in real rates after a fiscal shock is a robust feature of optimal policies under sticky inflation.

5. Staying behind the curve?

In this final study, we compare the observed dynamics of the U.S. economy in the post-COVID-19 period with the counterfactual scenario where the Fed follows a Taylor rule. The exercise is motivated by the policy debates ongoing in the aftermath of the COVID-19 pandemic.

5.1 The debate

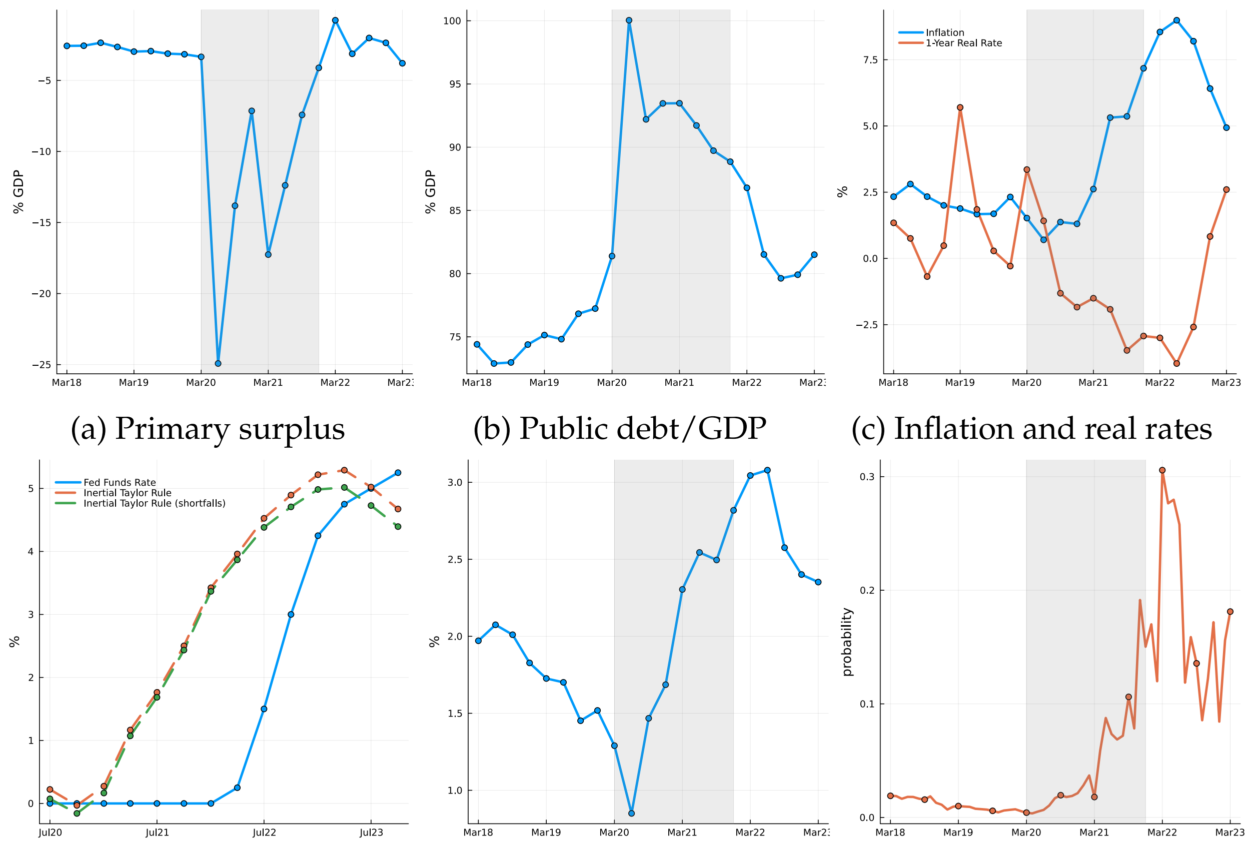
To set the stage, we present some data patterns from the period. In response to the COVID-19 crisis, the United States implemented an unprecedented fiscal expansion, resulting in the highest level of government debt (normalized by GDP) in the post-war era. Panel (a) of Figure 8 shows the large increase in primary deficits in the aftermath of the COVID-19 crisis, reaching 25% of GDP at its peak. Panel (b) shows how large deficits—coupled with disruptions to production—substantially increased the debt-to-GDP ratio. Panel (c) shows a burst in inflation that persisted for two years. The increase in inflation was unlike any other in the last 40 years. An important aspect of this episode was that real interest rates remained remarkably low; Panel (c) also shows that the 1-year (ex-ante) interest rate was negative for over two years after the beginning of the fiscal expansion.

Low real rates reflected the Fed's response to the inflationary pressures: Panel (d) shows that the Fed kept its nominal policy rate target low even after the surge in inflation. Panel (d) also shows the nominal rates dictated by two versions of a Taylor rule.[21] The figure illustrates the extent of the Fed's *underreaction* relative to the Taylor rule.[22] The Fed's underreaction period was also marked by a persistent increase in inflation expectations, as shown in Panels (e) and (f). In particular, Panel (e) shows the increase in the

[20]The assumption that $b_0 = b^n$ is important, as we would observe dynamics under the solution to the Ramsey problem even in the absence of shocks, so the optimal policy would be time-dependent and deviate from the solution under the timeless perspective. This observation motivates our focus on the case $b_0 = b^n$

[21]These two versions of the Taylor exemplify the rules discussed by the Fed's Monetary Policy Report during this period. For an assessment of these rules, see Papell and Prodan-Boul (2024).

[22]Bocola, Dovis, Jørgensen and Kirpalani (2024) provide complementary evidence of the Fed's underreaction based on movements in bond prices.



(a) Primary surplus (b) Public debt/GDP (c) Inflation and real rates

(d) Taylor rules (e) 5-year breakeven inflation (f) 5-year inflation disaster

Figure 8: Pre- and Post-COVID-19 Data

Note: Panel (a) shows the primary surplus to GDP ratio. Panel (b) shows the market debt held by the public plus central bank reserves over GDP. Panel (c) shows year-over-year CPI inflation and the Federal Reserve of Cleveland estimate of the 1-year (ex-ante) real rate. Panel (d) shows the lower limit of the Federal Funds target range and the predicted nominal rate for two specifications of inertial Taylor rules. Panel (e) shows the 5-year breakeven inflation. Panel (f) shows the implied average probability of inflation exceeding 4% over the next five years, inflation disaster risk, as estimated by Hilscher et al. (2022) based on inflation option prices.

5-year breakeven inflation, the difference between the yield on a nominal bond and the yield on an inflation-protected bond (TIPS).[23] Panel (f) shows that the market-implied inflation-disaster probability, as measured by Hilscher et al. (2022), also increased substantially during this period.

The Fed's underreaction when inflation expectations were rising led many commentators to state the Fed was staying "behind the curve." This was a call to a more aggressive stance for fears that the Fed had lost control over inflation expectations and a subsequent

[23] The breakeven inflation is not an unbiased measure of inflation expectation, as it incorporates a risk premium. However, survey-based measures showed patterns similar to the market-based ones.

painful recession would be necessary to get inflation back to target—see e.g. Bordo, Taylor and Cochrane (2023) for an account of the debate. Evidently, the Fed ignored the advice. Did it make a mistake by deviating from the Taylor rule? Did it risk triggering an inflationary spiral? The next exercise investigates whether following the Taylor rule would have been the correct policy response in the context of our model.

5.2 Taylor rules vs. realized policies

We use the model to assess the quantitative relevance of our sticky-inflation channel in shaping the dynamics of debt and inflation following the COVID-19 pandemic. To that end, we provide a historical shock decomposition and counterfactual analysis. We employ a discretized version of the model, which we use to construct a Kalman filter to obtain the series of shocks that best fit the data. We present the details of the model in Appendix E.

We focus on four shocks: a fiscal shock, which captures the exogenous fiscal expansion; a standard cost-push or markup shock, which reflects the sectorial reallocations and bottlenecks experienced in this period; a monetary shock, representing the deviations of monetary policy from our specified rule; and a bond-valuation shock. The bond-valuation shock captures not only unmodeled revaluation effects, as in Bianchi and Melosi (2017), but also the impact of asset purchases by the Fed, other sources of financing, and the approximation error of the linearization, as in Hall and Sargent (2024).[24] We then use the identified shocks to compute the dynamics of an economy subject to the same fiscal and cost-push shocks, but where the monetary authority no longer deviates from the Taylor rule—the "Taylor scenario."

Calibration. For the calibration, we treat variable values during 2009Q4 as a steady-state target. Table 1 summarizes the calibration. We adopt a standard calibration for parameters commonly used in the New Keynesian literature. We set the discount rate, ρ, to reflect the average real interest rate in the U.S. from 1990 to 2019 of 0.88% per year. We set the elasticity of intertemporal substitution, σ, to 0.25, roughly in line with the evidence by Best et al. (2020). We set the slope of the NKPC, κ, to 0.0138, which is the value in the empirical work of Hazell et al. (2022). We find in the literature a range of values for the Taylor rule inflation coefficient, ϕ_π, from 1.2 to 1.5. We set the coefficient to the lower bound of 1.2 to capture a moderate monetary policy response to inflation

[24]The bond-valuation shock is necessary to match the evolution of government debt in the data, given that we take the primary surplus, the policy rate, and the inflation rate as observables.

Table 1: Calibration of the Model

Parameter	Symbol	Value	Description
Discount rate	ρ	0.0022	Real-rate average (1990-2019)
Elasticity of Intertemporal Substitution	σ	0.25	Best, Cloyne, Ilzetzki and Kleven (2020)
Slope of the NKPC	κ	0.0138	Hazell et al. (2022)
Taylor rule inertia	ρ_i	0.90	Bocola et al. (2024)
Taylor coefficient	ϕ_π	1.2	Moderate response calibration
Fiscal rule	γ	0.038	Bianchi, Faccini and Melosi (2023)
Initial debt to quarterly GDP ratio	b^n	0.7683*4	Debt to GDP in 2019Q4
Quarters of high inflation in Phase II	T^*	16	Hazell and Hobler (2024)
Probability of Phase II	λ_f	0.015	Hilscher et al. (2022)

deviations from an inflation target. This choice is to bring the actual and realized interest-rate path coefficients as close as possible. We also include a backward-looking component in the monetary rule, consistent with the evidence on the inertial behavior of the Fed. Following the estimates in Bocola et al. (2024), we set the autoregressive parameter, ρ_i, to 0.90.

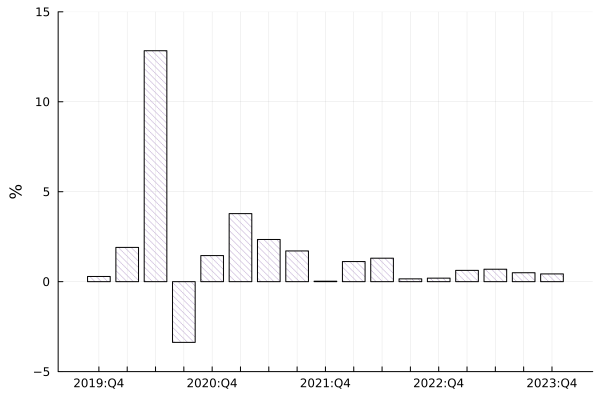
The rest of the parameters merit further discussion. We set the fiscal rule coefficient, γ, to 0.038, following Bianchi et al. (2023). This parameter represents the repayment rate of deficits, which, in turn, governs the mean reversion in public debt. We set the inflation-neutral debt level, b^n, to 0.7683×4 so that the debt-to-quarterly-GDP ratio in 2019Q4 was at its neutral level. Thus, under our calibration, the sticky-inflation channel was muted before the pandemic.

We set the probability of a monetary-fiscal reform, λ, to 0.015 which translates into an annual probability of 6% or, equivalently, a probability of observing a monetary-fiscal reform once every 15 years. This choice is consistent with the inflation disaster risk in Hilscher et al. (2022).[25]

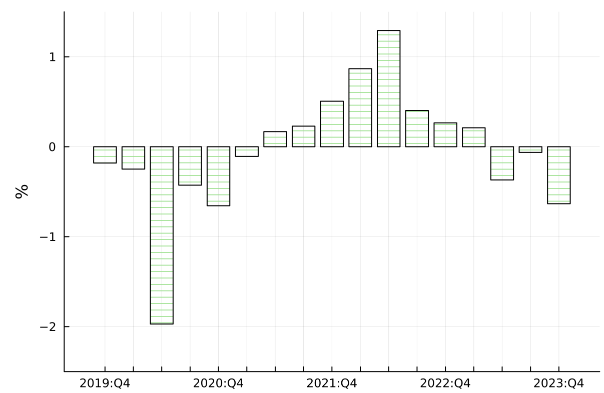
The duration of the fiscal consolidation phase, T^*, governs the pass-through from the debt-gap to inflation. We set T^* to 16 quarters so that the implied pass-through is consistent with the empirical impulse responses to fiscal events in Hazell and Hobler (2024).[26]

[25]Recall that this parameter represents the likelihood that the economy will experience an inflation burst. Using inflation option prices, Hilscher et al. (2022) report probabilities that inflation will exceed $4\% - 6\%$ thresholds.

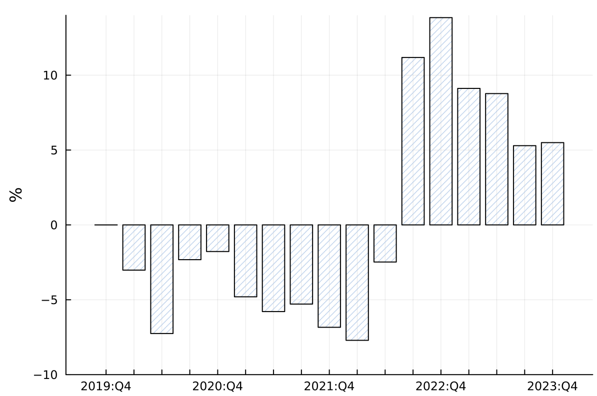
[26]That paper uses electoral outcomes in the senate race in Georgia to proxy for the expectation of the



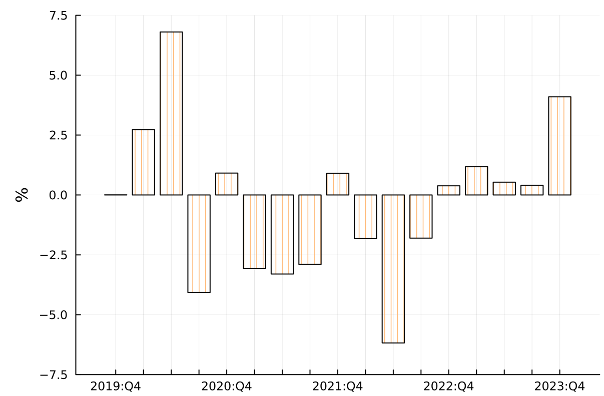
(a) Fiscal Shock



(b) Cost-Push Shock



(c) Monetary Shock



(d) Bond-Valuation Shock

Figure 9: COVID-19 Shock Decomposition: The Shocks

Finally, we assume that the monetary, term premium and fiscal shocks are i.i.d., while the cost-push shock follows an AR(1) process. Notice that even though the monetary shock is i.i.d, the inertial monetary rule induces a high degree of persistence. Assuming that the fiscal shock is i.i.d. captures the fact that the fiscal expansion post-COVID was perceived as a one time event. On the contrary, for the cost-push shock we follow the literature and assume an autoregressive coefficient of 0.83 (see Bocola et al., 2024). We set the standard deviation of the monetary and term-premium shocks to 0.18% per year, the fiscal shock to 1% per year, and the cost-push shock to 2.67% per year. Our results are insensitive to alternative calibrations of the standard deviations.

Biden stimulus plan. Their study shows a pass-through of 0.18% inflation over the next 2 years to a 1% increase in the deficit-to-GDP ratio.

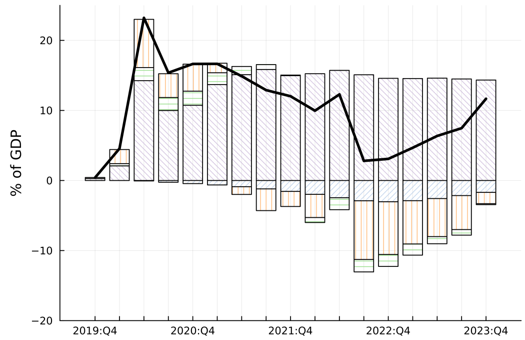
Shock decomposition. Next, we conduct a shock decomposition analysis using historical time series data for the market value of debt to GDP ratio, primary deficits to GDP ratio, inflation, and nominal policy rates—see Figure 8. To understand the exercise, it is useful to discuss how the shocks are identified. The fiscal shock is directly inferred from the path of primary surpluses; the monetary shock is identified as deviations from the Taylor rule; the bond-valuation shock is directly extracted from the government debt path, given the fiscal rule, the primary deficits, and the path of nominal rates; finally, the exogenous cost-push shock is identified through the Phillips curve implied by the model.[27]

Figure 9 presents the identified structural shocks. The fiscal shock (Panel a), measured as a percentage of GDP, exhibited a significant surge in 2020Q2. This was primarily driven by an increase in government spending coupled with a contraction in GDP, resulting in a fiscal shock of almost 13% of GDP. Following this period, the fiscal shock remained mildly positive, except for 2020Q3, which was influenced by a GDP rebound. The cost-push shock (Panel b), measured in units of the annual inflation rate, was negative throughout 2020 but turned positive thereafter, peaking in 2022Q2 when they contributed an additional 5 percentage points to the annualized inflation rate. The monetary shock (Panel c), expressed in units of the annualized policy rate, was mildly positive in 2020, when the policy rate was at the zero lower bound. However, it became significantly negative as the inflation rate began to rise, implying a substantial deviation from the Taylor rule. Lastly, the bond-valuation shock (Panel d) closely mirrored the monetary shock, indicating that it was largely driven by valuation effects of long-term bonds and other Fed operations. Our shock-decomposition results are consistent with the findings in Rubbo (2024). Using disaggregated price data, she concludes that the disinflation in 2020 was mainly driven by industry-specific shocks, while the surge in inflation in 2021 is mainly explained by aggregate factors, such as monetary and fiscal policy.

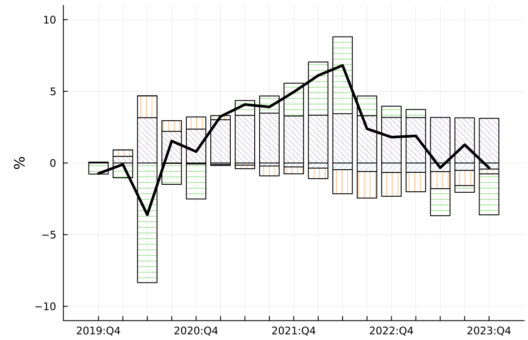
The effect of the shocks on the main macroeconomic variables is illustrated in Figure 10. Panels (a) through (c) display the contribution of shocks to the paths of government debt path, inflation, and nominal policy rates, respectively. We report the debt-to-GDP ratio as deviations from its neutral level (2019Q4), the inflation rate as deviations from a 2% inflation target, and the nominal rate as annualized percentage points. Next, we describe how the shocks influenced the dynamics of each of these variables.

Government Debt: A key feature of the debt-to-GDP path is the sharp increase during the second quarter of 2020, following the onset of the pandemic. This spike is primarily

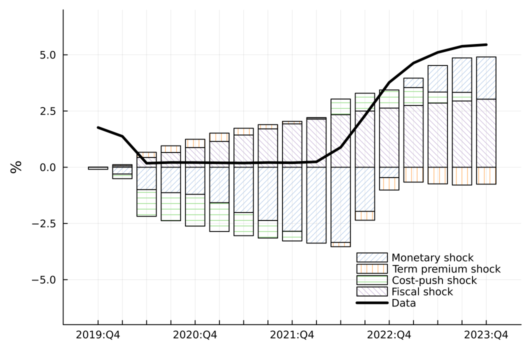
[27] Since shocks are inferred directly from the data series, the Kalman filter optimizes the initial conditions to best fit the shock decomposition—the initial conditions' quantitative contribution is minor.



(a) Government Debt to GDP



(b) Inflation



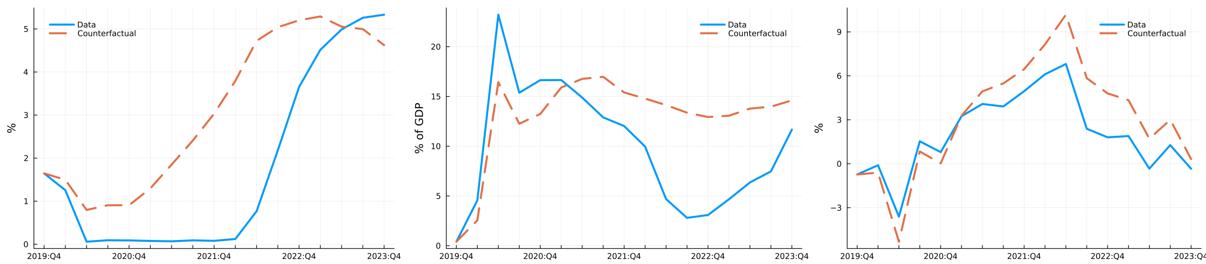
(c) Federal Funds Rate

Figure 10: COVID-19 Shock Decomposition: Government Debt, Inflation, and Fed Funds Rate

driven by abnormally large fiscal shocks, as government spending surged in response to the crisis. Additionally, the bond-valuation shock, which captures the flattening of the yield curve and other changes in the sources of government funding, contributed to the increase in market debt. Cost-push shocks had a smaller impact, predominantly through a reduction in inflation during the early phase of the pandemic.

Monetary policy played a limited role in the initial stages of the crisis. In 2020Q2, monetary policy was constrained by the zero lower bound in its ability to offset deflationary pressures. As the pandemic progressed, fiscal shocks continued to expand, especially during the quarters following 2021Q1, which coincided with the fiscal stimulus measures implemented by the Biden administration. The contribution of fiscal shocks to debt levels remained significant throughout this period. By 2021Q3, monetary policy shocks substantially contributed to reducing government debt. How so becomes clear from the decomposition of inflation and policy rates.

Inflation: Panel (b) presents the decomposition of inflation. The cost-push shock had a significant deflationary impact in 2020Q2, resulting in an annualized inflation rate of -5%.



(a) Federal Funds Rate (b) Public Debt to GDP (c) Inflation

Figure 11: COVID-19 Counterfactual Monetary Policy

This deflationary trend persisted until the end of 2020. By 2021, the effect of these shocks reversed, likely arising from supply bottlenecks and the Ukranian war, as suggested by other studies.[28] By 2023Q4, the cost-push shocks began to dissipate. During this period, the sticky-inflation component (represented by the backslash bars in the figure) becomes more prominent. Sticky inflation arose due to the persistent fiscal deficits, contributing to debt accumulation. As debt deviated further from its target, inflation expectations gained momentum, amplifying inflationary pressures. According to our model, the fiscal shock, though short-lived, contributed almost 5% to annual inflation throughout the period. That inflation started to stabilize in 2022 can be attributed to the deviations of monetary policy from the prescriptions of the Taylor rule.

Monetary Policy Rates: Starting from 2020Q4, the Fed deviated from the Taylor rule and stayed "behind the curve." Indeed, Panel (c) indicates that nominal rates should have been much higher given the cost-push and fiscal shocks. However, the most notable aspect of the decomposition is that despite the expansionary stance of monetary policy—see Figure 9, Panel (c)—, the effect on inflation was deflationary (see the slash bars in Panel b). This apparent paradox is nothing but the sticky-inflation channel at work. Because the lower policy rates eased the debt burden, as shown in Figure 10, Panel (a), the Fed indirectly mitigated the sticky-inflation component in the Phillips curve. Lower debt levels helped temper inflation expectations, thus counteracting the inflationary pressures from fiscal and cost-push shocks.

Counterfactuals. Using the filtered shocks, we simulate a counterfactual scenario in which the Fed would have followed the Taylor rule, turning off the deviations from it as well as the bond-valuation shock, and keeping the fiscal shock and the cost-push shock

[28] Guerrieri, Lorenzoni, Straub and Werning (2022) show that shocks that asymmetric sectorial shocks can manifest as cost-push shocks in the New Keynesian model.

on. This counterfactual analysis allows us to explore what would have happened to debt and inflation if the Fed had responded more aggressively during the inflation surge, as advised by its critics. The results are shown in Figure 11.

Panel (a) illustrates the actual (solid) and counterfactual (dashed) nominal interest rate paths. Had the Fed adhered to the Taylor principle, nominal rates would have been reduced more rapidly in the early phases of the pandemic and increased much more aggressively, given the rising inflation starting in 2020Q4. This more forceful response, while countering inflation, would have significantly increased the burden of government debt, as shown in Panel (b). The more aggressive stance would have led to a more persistent increase in the debt-to-GDP ratio due to the higher debt servicing cost.

Interestingly, the counterfactual inflation path shows that, despite the more aggressive anti-inflationary policy, inflation would have actually been significantly *higher*. Again, the apparent paradox arises from the interaction of two opposing forces: the reduction in inflation through lower demand stimulus and the countervailing effect of the sticky-inflation component, amplified by the larger debt burden. Thus, while the Taylor rule would have curbed demand-driven inflation, the resulting increase in debt would have fueled inflationary pressures through the fiscal channel, ultimately offsetting the benefits of the rule's tighter policy.

Given the optimal policy prescriptions derived in the previous section, we conclude that the Fed's decision to stay "behind the curve" was appropriate. Our counterfactual shows that adhering to the Taylor rule would have resulted in suboptimal outcomes, with higher debt and moderate inflation. All in all, the exercise shows that admitting the possibility that inflation expectations can be dragged by debt alters the conventional wisdom regarding the optimal response to monetary policy.

It is important to note that the Taylor rule does not necessarily indicate that the Fed intentionally acted to ease the debt burden. Rather, the Fed's gradualism doctrine, characterized by measured responses to unfolding events, aligned well with the optimal policy in this context. Our model indicates that the Fed's decision to resist calls for a faster tightening of monetary policy was ultimately the right course of action.

6. Conclusion

This paper offers new insights regarding fiscal-monetary interactions in New Keynesian models. First, we demonstrated that in an environment where monetary accommodation is anticipated, attempts to curb inflation can backfire, as inflation expectations linked to debt levels create "sticky inflation." Second, we showed that due to this stickiness, optimal policy should balance inflation and debt objectives, often keeping real interest rates low after fiscal shocks.

In our analysis, we assumed that economic agents are opinionated and they are not easily persuaded by the monetary authority. In particular, we did not explore the possibility that early anti-inflation efforts signal that monetary policy will resist future inflationary financing. However, without signaling effects, such efforts are futile. Understanding how medium-term inflation expectations respond to signaling, possibly by incorporating policy stance attention as in Bassetto and Miller (2022), is essential to complete the story.

Understanding sticky inflation is particularly relevant in today's high-debt environment. Sticky inflation rationalizes the repeated failures to curb inflation in countries like Argentina, Brazil, and Turkey, where orthodox central bankers often raised real interest rates with limited long-term success. These episodes suggest that temporary measures are unlikely to overcome sticky inflation unless expectations of monetary accommodation dissipate. We hope developed economies pay attention to this lesson.

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A. Derivations

A.1 Derivations for Section 2

Households. The household problem is given by

$$V_t(B_t) = \max_{[C_s,N_s]_{s\geq t}} \mathbb{E}_t \left[\int_t^{t^*} e^{-\rho(s-t)} \left[u(C_s) - h(N_s) \right] ds + e^{-\rho(t^*-t)} V^*_{t^*}(B^*_t) \right], \tag{33}$$

subject to

$$\dot{B}_t = (i_t - \pi_t)B_t + \frac{W_t}{P_t}N_t + D_t + T_t - C_t, \tag{34}$$

and a No-Ponzi condition, where t^* denotes the arrival time for a Poisson process with intensity $\lambda \geq 0$, B_t denotes the real valued of bonds held by households, W_t is the nominal wage, P_t is the price level, D_t are dividends payed by firms, T_t denotes fiscal transfers.

The HJB equation for this problem is given by

$$\rho V = u(C) - h(N) + \dot{V} + V_B \left[(i-\pi)B + \frac{W}{P}N + T - C \right] + \lambda [V^* - V], \tag{35}$$

where $\dot{V}$ denotes the time derivative of the value function conditional on no-switching.

The first-order conditions are given by

$$u'(C) = V_B, \qquad h'(N) = V_B \frac{W}{P}. \tag{36}$$

The envelope condition is given by

$$\rho V_B = V_B(i-\pi) + \dot{V}_B + V_{BB} \left[(i-\pi)B + \frac{W}{P}N + T - C \right] + \lambda [V^*_B - V_B]. \tag{37}$$

Combining the envelope condition with the optimality condition for consumption, we obtain

$$0 = (i - \pi - \rho) + \frac{u''(C)C}{u'(C)} \frac{\dot{C}_t}{C_t} + \lambda \left[\frac{u'(C^*)}{u'(C)} - 1 \right] \Rightarrow \frac{\dot{C}}{C} = \sigma^{-1}(i - \pi - \rho) + \frac{\lambda}{\sigma} \left[\frac{u'(C^*)}{u'(C)} - 1 \right], \tag{38}$$

where $\sigma = -\frac{u''(C)C}{u'(C)}$.

The optimality condition for labor can be written as

$$\frac{h'(N)}{u'(C)} = \frac{W}{P}. \tag{39}$$

Firms. There are two types of firms in the economy: final-goods producers and intermediate-goods producers. Final goods are produced by competitive firms according to the production function $Y_t = \left(\int_0^1 Y_{i,t}^{\frac{\epsilon}{\epsilon-1}} di\right)^{\frac{\epsilon-1}{\epsilon}}$, where $Y_{i,t}$ denotes the output of intermediate $i \in [0,1]$. The demand for intermediate i is given by $Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} Y_t$, where $P_{i,t}$ is the price of intermediate i, $P_t = \left(\int_0^1 P_{i,t}^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}$ is the price level, and Y_t is the aggregate output.

Intermediate-goods producers have monopoly over their variety and operate the technology $Y_{i,t} = A_t N_{i,t}$, where $N_{i,t}$ denotes labor input. Firms are subject to quadratic adjustment costs on price changes, so the problem of intermediate i is given by

$$Q_{i,t}(P_i) = \max_{[\pi_{i,s}]_{s\geq t}} \mathbb{E}_t \left[\int_t^{t^*} \frac{\eta_s}{\eta_t}\left(\frac{P_{i,s}}{P_{i,t}} Y_{i,s} - \frac{W_s}{P_s}\frac{Y_{i,s}}{A_s} - \frac{\varphi}{2}\pi_{i,s}^2\right) ds + \frac{\eta_{t^*}}{\eta_t} Q_{i,t}^*(P_{i,t}^*)\right], \tag{40}$$

subject to $Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} Y_t$ and $\dot{P}_{i,t} = \pi_{i,t} P_{i,t}$, given $P_{i,t} = P_i$ and $\eta_t = e^{-\rho t} u'(C_t)$, where φ is the adjustment cost parameter.

The HJB equation for this problem is

$$0 = \max_{\pi_{i,t}} \eta_t \left(\frac{P_{i,t}}{P_t} Y_{i,t} - \frac{W_t}{P_t}\frac{Y_{i,t}}{A} - \frac{\varphi}{2}\pi_{i,t}^2\right) dt + \mathbb{E}_t[d(\eta_t Q_{i,t})], \tag{41}$$

where $\frac{\mathbb{E}_t[d(\eta_t Q_{i,t})]}{\eta_t dt} = -(i_t - \pi_t) Q_{i,t} + \frac{\partial Q_{i,t}}{\partial P_{i,t}} \pi_{i,t} P_{i,t} + \frac{\partial Q_{i,t}}{\partial t} + \lambda \frac{\eta_t^*}{\eta_t}\left[Q_{i,t}^* - Q_{i,t}\right]$.

The first-order condition is given by

$$\frac{\partial Q_{i,t}}{\partial P_i} P_{i,t} = \varphi \pi_{i,t}.$$

The change in π_t conditional on no switching in state is then given by

$$\left(\frac{\partial^2 Q_{i,t}}{\partial t \partial P_i} + \frac{\partial^2 Q_{i,t}}{\partial P_i^2}\pi_{i,t} P_{i,t}\right) P_{i,t} + \frac{\partial Q_{i,t}}{\partial P_i}\pi_{i,t} P_{i,t} = \varphi \dot{\pi}_{i,t}. \tag{42}$$

The envelope condition with respect to $P_{i,t}$ is given by

$$0 = \left((1-\epsilon)\frac{P_{i,t}}{P_t} + \epsilon \frac{W_t}{P_t A}\right)\left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} \frac{Y_t}{P_{i,t}} + \frac{\partial^2 Q_{i,t}}{\partial t \partial P_i} + \frac{\partial^2 Q_{i,t}}{\partial P_i^2}\pi_{i,t} P_{i,t} + \\ \frac{\partial Q_{i,t}}{\partial P_i}\pi_{i,t} - (i_t - \pi_t)\frac{\partial Q_{i,t}}{\partial P_i} + \lambda \frac{\eta_t^*}{\eta_t}\left(\frac{\partial Q_{i,t}^*}{\partial P_i} - \frac{\partial Q_{i,t}}{\partial P_i}\right). \tag{43}$$

Multiplying the expression above by $P_{i,t}$ and using Equation (42), we obtain

$$0 = \left((1-\epsilon)\frac{P_{i,t}}{P_t} + \epsilon \frac{W_t}{P_t A}\right)\left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} Y_t + \varphi \dot{\pi}_t - (i_t - \pi_t)\varphi \pi_{i,t} + \lambda \varphi \frac{\eta_t^*}{\eta_t}\left(\pi_{i,t}^* - \pi_{i,t}\right).$$

Rearranging the expression above, we obtain the non-linear New Keynesian Phillips curve

$$\dot{\pi}_t = (i_t - \pi_t)\,\pi_t + \lambda\frac{\eta^*}{\eta_t}\,(\pi_t - \pi_t^*) - \frac{\epsilon\varphi^{-1}}{A}\left(\frac{W_t}{P_t} - (1-\epsilon^{-1})A\right)Y_t.$$

Government and market clearing. The government flow budget constraint is given by

$$\dot{B}_t^g = (i_t - \pi_t)B_t^g + T_t, \tag{44}$$

where B_t^g denotes the real value of government debt. The government must also satisfy the No-Ponzi condition $\lim_{T\to\infty}\mathbb{E}_t[\eta_T B_T^g] = 0$.

The market clearing condition is given by

$$C_t = Y_t, \qquad N_t = \int_0^1 N_{i,t}di, \qquad B_t = B_t^g. \tag{45}$$

A.2 Derivations for Section 3

In this section, we revisit our three policy experiments in the context of the more general version of the model, which includes households' expectation effects and a debt-stabilization term. In this case, the dynamic system describing the evolution of output, inflation, and debt is given by

$$\dot{x}_t = r_t - \rho + \lambda_h x_t - \lambda_h(b_t - b^n) \tag{46}$$

$$\dot{\pi}_t = (\rho + \lambda_f)\pi_t - \kappa x_t - \lambda_f\kappa\Phi(b_t - b^n) \tag{47}$$

$$\dot{b}_t = r_t - \rho - \gamma(b_t - b^n) + \psi_t. \tag{48}$$

Output gap stabilization. Consider first the case of output-gap stabilization, so $x_t = 0$ for all t. This requires that the interest rate is given by

$$r_t - \rho = \lambda_h(b_t - b^n). \tag{49}$$

The law of motion of debt is then given by

$$\dot{b}_t = -\left(\gamma - \lambda_h\right)(b_t - b^n) + \psi_t. \tag{50}$$

Solving the differential equation above, we obtain

$$b_t - b^n = e^{-(\gamma-\lambda_h)t}(b_0 - b^n) + \int_0^t e^{-(\gamma-\lambda_h)(t-s)}\psi_s ds. \tag{51}$$

Assuming $\psi_t = e^{-\theta_\psi t}\psi_0$, we obtain

$$b_t - b^n = e^{-(\gamma-\lambda_h)t}(b_0 - b^n) + \frac{e^{-(\gamma-\lambda_h)t} - e^{-\theta_\psi t}}{\theta_\psi - (\gamma - \lambda_h)}\psi_0. \tag{52}$$

Notice that b_t converges back to the steady state if $\gamma > \lambda_h$.

Inflation is given by

$$\pi_t = \lambda_f \kappa \Phi \int_t^\infty e^{-(\rho+\lambda_f)(s-t)}(b_s - b^n)ds. \tag{53}$$

Plugging the value of b_t into the expression above, we obtain

$$\pi_t = \lambda_f \kappa \Phi \left[\frac{e^{-(\gamma-\lambda_h)t}}{\rho + \lambda_f + \gamma - \lambda_h}\left(b_0 - b^n + \frac{\psi_0}{\theta_\psi - (\gamma - \lambda_h)}\right) - \frac{e^{-\theta_\psi t}}{\rho + \lambda_f + \theta_\psi}\frac{\psi_0}{\theta_\psi - (\gamma - \lambda_h)}\right]. \tag{54}$$

Notice that debt and inflation depend on γ and λ_h only through their difference. Hence, if we assume that $\gamma = \lambda_h > 0$, we obtain the same values of b_t and π if we assume $\gamma = \lambda_h = 0$, which corresponds to the case in Section 3. If $\gamma > \lambda_h$, then eventually debt and inflation eventually return to their steady-state levels.

Inflation stabilization. Next, we will consider the case of inflation stabilization. Suppose the real rate is given by $r_t - \rho = e^{-\theta_r t}(r_0 - \rho)$. In this case, debt is given by:

$$b_t - b^n = \underbrace{e^{-\gamma t}(b_0 - b^n) + \frac{e^{-\gamma t} - e^{-\theta_\psi t}}{\theta_\psi - \gamma}\psi_0}_{b_t^p} + \underbrace{\frac{e^{-\gamma t} - e^{-\theta_r t}}{\theta_r - \gamma}(r_0 - \rho)}_{b_t^r}. \tag{55}$$

The first term corresponds to the level of debt if the monetary authority implements a passive policy of setting the real rate equal to its natural level at all periods, $r_t = \rho$, and the second term captures the impact on debt of changing the real rate.

The output gap is given by

$$x_t = -\int_t^\infty e^{-\lambda_h(s-t)}(r_s - \rho)ds + \lambda_h \int_t^\infty e^{-\lambda_h(s-t)}(b_s - b^n)ds. \tag{56}$$

The output gap can be expressed as follows

$$x_t = x_t^p - \frac{r_t - \rho}{\lambda_h + \theta_r} + \frac{\lambda_h}{\theta_r - \gamma}\left[\frac{e^{-\gamma t}}{\lambda_h + \gamma} - \frac{e^{-\theta_r t}}{\lambda_h + \theta_r}\right](r_0 - \rho), \tag{57}$$

where

$$x_t^p = \lambda_h \left[\frac{e^{-\gamma t}}{\lambda_h + \gamma}\left(b_0 - b^n + \frac{\psi_0}{\theta_\psi - \gamma}\right) - \frac{\psi_t}{(\lambda_h + \theta_\psi)(\theta_\psi - \gamma)}\right]. \tag{58}$$

There are now two opposing effects on the output gap. Higher rates reduce the output gap through the usual intertemporal substitution channel. However, higher real rates push debt up, which creates a positive output gap in the inflationary-finance phase. This expectation tends to increase the output gap today. Therefore, the presence of this expectation attenuates the response of output to higher real rates.

Inflation is given by

$$\pi_t = \kappa \int_t^\infty e^{-(\rho+\lambda_f)(s-t)} x_s ds + \lambda_f \kappa \Phi \int_t^\infty e^{-(\rho+\lambda_f)(s-t)} (b_s - b^n) ds. \tag{59}$$

Inflation is given by

$$\pi_t = \pi_t^p + F_t + J_t^x + J_t^b, \tag{60}$$

where $\pi_t^p \equiv \kappa \int_t^\infty e^{-(\rho+\lambda_f)(s-t)} x_s^p ds + \lambda_f \kappa \Phi \int_t^\infty e^{-(\rho+\lambda_f)(s-t)} b_s^p ds$, and

$$F_t \equiv -\frac{\kappa}{\lambda_h + \theta_r} \frac{r_t - \rho}{\rho + \lambda_f + \theta_r} < 0 \tag{61}$$

$$J_t^x \equiv \frac{\kappa \lambda_h}{\theta_r - \gamma} \left[\frac{e^{-\gamma t}}{(\lambda_h + \gamma)(\rho + \lambda_f + \gamma)} - \frac{e^{-\theta_r t}}{(\lambda_h + \theta_r)(\rho + \lambda_f + \theta_r)} \right] (r_0 - \rho) > 0 \tag{62}$$

$$J_t^b \equiv \frac{\kappa \lambda_f \Phi}{\theta_r - \gamma} \left[\frac{e^{-\gamma t}}{\rho + \lambda_f + \gamma} - \frac{e^{-\theta_r t}}{\rho + \lambda_f + \theta_r} \right] (r_0 - \rho) > 0. \tag{63}$$

The fight-inflation term dominates at period zero if the following condition is satisfied

$$\frac{1}{\rho + \lambda_f + \theta_r} > \frac{\lambda_f \Phi}{\rho + \lambda_f + \gamma} \frac{\lambda_h + \theta_r}{\rho + \lambda_f + \theta_r} + \frac{\lambda_h}{\theta_r - \gamma} \left[\frac{\lambda_h + \theta_r}{(\lambda_h + \gamma)(\rho + \lambda_f + \gamma)} - \frac{1}{\rho + \lambda_f + \theta_r} \right]. \tag{64}$$

We can write the expression above as follows

$$\theta_r < \frac{\rho + \lambda_f + \gamma - \lambda_h \left[\lambda_f \Phi + \frac{\lambda_h - \gamma + \rho + \lambda_f}{\lambda_h + \gamma} \right]}{\lambda_f \Phi + \frac{\lambda_h}{\lambda_h + \gamma}}. \tag{65}$$

Notice that we recover the condition for the fight inflation to dominate at $t = 0$ when $\lambda_h = \gamma = 0$.

If $\theta_r > \gamma$, such that the response of taxes to government debt is not too strong, then the jump inflation term eventually dominates, consistent with the stepping-on a-rake result.

Debt stabilization. We consider next the case where the monetary authority stabilizes government debt, $b_t = 0$. For simplicity, we focus on the case $b^n = 0$. The real interest rate is then given by $r_t - \rho = -\psi_t$. This corresponds to the previous case with $r_0 - \rho = -\psi_0$ and $\theta_r = \theta_\psi$. Given the low real rate, for λ_h sufficiently small, we have a positive output gap and inflation on impact.

B. Proofs

Proof of Proposition 1

Proof. We first show that fiscal policy is passive, that is, for any Lebesgue integrable path for (x_t, π_t, i_t), government debt is bounded if and only if $\gamma \geq 0$. Note that in the fiscal consolidation phase and the inflationary-finance phase, government debt is bounded by construction. In the fiscal-expansion phase, from equation (15) we get

$$\lim_{t\to\infty} b_t = \lim_{t\to\infty} e^{-\gamma t} b_0 + \lim_{t\to\infty} \int_0^t e^{-\gamma(t-s)} (i_s - \pi_s - \rho + \psi_s) ds.$$

Notice that since $\gamma \geq 0$, $e^{-\gamma t} \leq 1$ for all $t \geq 0$. Then

$$\lim_{t\to\infty} b_t = \lim_{t\to\infty} e^{-\gamma t} b_0 + \lim_{t\to\infty} \int_0^t e^{-\gamma(t-s)} (i_s - \pi_s - \rho + \psi_s) ds \leq \lim_{t\to\infty} b_0 + \lim_{t\to\infty} \int_0^t (i_s - \pi_s - \rho + \psi_s) ds < \infty,$$

where the last inequality follows from (i_t, π_t) being Lebesgue integrable.

For **I.**, notice that the dynamic system is given by

$$\begin{bmatrix} \dot{\pi}_t \\ \dot{x}_t \\ \dot{b}_t \end{bmatrix} = \begin{bmatrix} (\rho + \lambda_f) & -\kappa & -\lambda_f \kappa \Phi \\ (\phi - 1) & \lambda_h & -\lambda_h \\ (\phi - 1) & 0 & -(\gamma - \rho) \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \\ b_t \end{bmatrix} + \begin{bmatrix} \lambda_f \kappa \Phi b^n \\ u_t + \lambda_h b^n \\ u_t + \psi_t \end{bmatrix}.$$

The equilibrium is uniquely determined if the matrix above has two eigenvalues with positive real components and an eigenvalue with a non-positive real component. The eigenvalues of the system above satisfies the characteristic equation:

$$f(\lambda) \equiv \lambda^3 + \underbrace{[\gamma - (\rho + \lambda_f + \lambda_h)]}_{\equiv a} \lambda^2 + \underbrace{[(\phi - 1)\kappa(1 + \lambda_f \Phi) + \lambda_h(\rho + \lambda_f) - \gamma(\rho + \lambda_f + \lambda_h)]}_{\equiv b} \lambda + \underbrace{[\gamma(\rho + \lambda_f)\lambda_h + (\phi - 1)\kappa[\gamma - (\lambda_h + \lambda_h \lambda_f \Phi)]]}_{\equiv c} = 0.$$

Using Descartes' rule of signs, we get that $c > 0$ is a necessary condition for determinacy. To see this, suppose $c < 0$. Then, two options exist for the number of sign changes of $f(\lambda)$: one and three. This implies that there can be either 1 or 3 roots with a positive real part. Since we need two roots with positive real part for determinacy, we can rule out those cases.

Next, we show that $c > 0$ is a sufficient condition for determinacy. Because $\gamma < \rho + \lambda_f + \lambda_h$, $a < 0$. Then, we are guaranteed two sign changes. Using the Routh-Hurwitz criterion, not all roots of f are negative, completing the proof.

Part **II.** is immediately true by construction.

Proof of Proposition 2

Proof. From equation (17), given $x_t = 0$ and $\pi_t^J = \kappa\Phi(b_t - b^n)$, inflation is given by

$$\pi_t = \kappa\lambda\Phi \int_t^\infty e^{-(\rho+\lambda)(s-t)}(b_s - b^n)ds. \tag{66}$$

Debt is given by $b_s = b_0 + \frac{1-e^{-\theta_\psi s}}{\theta_\psi}\psi_0 = b_t + \frac{1-e^{-\theta_\psi (s-t)}}{\theta_\psi}\psi_t$. We can then write inflation as follows:

$$\pi_t = \frac{\kappa\lambda\Phi}{\rho+\lambda}\left[b_t - b^n + \frac{\psi_t}{\rho+\lambda+\theta_\psi}\right]. \tag{67}$$

The limit of the expression above as $t \to \infty$ is $\lim_{t\to\infty}\pi_t = \frac{\kappa\lambda\Phi}{\rho+\lambda}(b^{lr} - b^n)$. Differentiating the expression above with respect to time, we obtain

$$\dot{\pi}_t = \frac{\kappa\lambda\Phi}{\rho+\lambda}\left[\psi_t - \frac{\theta_\psi\psi_t}{\rho+\lambda+\theta_\psi}\right] = \frac{\kappa\lambda\Psi}{\rho+\lambda+\theta_\psi}\psi_t > 0. \tag{68}$$

□

Proof of Lemma 1.

Proof. From equation (17), inflation is given by

$$\pi_t = \kappa\int_t^\infty e^{-(\rho+\lambda)(s-t)}x_s ds + \lambda\kappa\Phi\int_t^\infty e^{-(\rho+\lambda)(s-t)}(b_s - b^n)ds, \tag{69}$$

where $x_t = -\frac{1}{\theta_r}(r_t - \rho)$ and $b_t = b_t^{og} + \frac{1-e^{-\theta_r t}}{\theta_r}(r_0 - \rho)$.

We can then write inflation as follows:

$$\pi_t = \pi_t^{og} \underbrace{-\frac{\kappa(r_t-\rho)}{\theta_r(\rho+\lambda+\theta_r)}}_{F_t} + \underbrace{\frac{\kappa\lambda\Phi}{\theta_r}\left[\frac{1}{\rho+\lambda} - \frac{e^{-\theta_r t}}{\rho+\lambda+\theta_r}\right](r_0-\rho)}_{J_t}, \tag{70}$$

where $\pi_t^{og} = \kappa\lambda\Phi\int_t^\infty e^{-(\rho+\lambda)(s-t)}(b_s^{og} - b^n)ds$.

□

Proof of Proposition 3.

Proof. The fight-inflation strategy is successful at bringing inflation down at $t = 0$ if:

$$-F_0^\pi > J_0^\pi \iff \frac{\kappa(r_0-\rho)}{\theta_r(\rho+\lambda+\theta_r)} > \frac{\kappa\lambda\Phi}{\theta_r}\left[\frac{1}{\rho+\lambda} - \frac{1}{\rho+\lambda+\theta_r}\right](r_0-\rho).$$

We can write the inequality above as follows:

$$1 > \frac{\lambda\Phi}{\rho+\lambda}\theta_r \iff \theta_r < \frac{\rho+\lambda}{\lambda\Phi}. \tag{71}$$

Notice that $\lim_{t\to\infty} F_t^\pi = 0$ and $\lim_{t\to\infty} J_t^\pi = \frac{\kappa\lambda\Phi}{\theta_r(\rho+\lambda)}(r_0-\rho) > 0$. Hence, there exits $\hat{T} > 0$ such that for $t > \hat{T}$ the following inequality holds:

$$-F_t^\pi < J_t^\pi. \tag{72}$$

Hence, $\pi_t > \pi_t^{og}$ for $t > \hat{T}$. □

Proof of Proposition 4

Proof. The Hamiltonian for problem 1 is given by

$$\begin{aligned}\mathcal{H}_t = & -\frac{1}{2}\left[\alpha x_t^2 + \beta\pi_t^2 + \lambda\Upsilon(b_t-b^n)^2\right] + \mu_{\pi,t}\left[(\rho+\lambda)\pi_t - \kappa x_t - \lambda\kappa\Phi(b_t-b^n)\right] + \mu_{b,t}\left[r_t-\rho+\psi_t\right] \\ & + \mu_{x,t}[r_t-\rho] + (\mu_{x,0}+\xi_x)(\rho+\lambda)x_0 + (\mu_{\pi,0}+\xi_\pi)\left[\kappa x_0 + \frac{\kappa(1+\lambda\Phi)}{\rho+\lambda}(r_t-\rho)\right],\end{aligned} \tag{73}$$

The dynamics of the co-states are given by

$$\dot{\mu}_{\pi,t} - (\rho+\lambda)\mu_{\pi,t} = \beta\pi_t - \mu_{\pi,t}(\rho+\lambda) \tag{74}$$

$$\dot{\mu}_{b,t} - (\rho+\lambda)\mu_{b,t} = \lambda\Upsilon(b_t-b^n) + \kappa\lambda\Phi\mu_{\pi,t} \tag{75}$$

$$\dot{\mu}_{x,t} - (\rho+\lambda)\mu_{x,t} = \alpha x_t + \kappa\mu_{\pi,t}. \tag{76}$$

The optimality condition for the real interest rate is

$$\mu_{b,t} + \mu_{x,t} = -\xi, \tag{77}$$

where $\xi \equiv \frac{\kappa(1+\lambda\Phi)}{\rho+\lambda}(\mu_{\pi,0}+\xi_\pi)$.

The optimality condition for the initial output gap:

$$(\rho+\lambda)(\mu_{x,0}+\xi_x) + \kappa(\mu_{\pi,0}+\xi_\pi) = 0. \tag{78}$$

We will choose $\xi_x = -\kappa\frac{\mu_{\pi,0}+\xi_\pi}{\rho+\lambda}$, such that $\mu_{x,0} = 0$. We show below that we can set $\mu_{\pi,0} = 0$ without loss of generality.

The optimality condition for interest rates imply that $\dot{\mu}_{b,t} + \dot{\mu}_{x,t} = 0$. From the law of motion of the co-states, we obtain

$$\alpha x_t + \lambda \Upsilon (b_t - b^n) = \kappa(1 + \lambda\Phi)\left(\mu_{\pi,0} - \mu_{\pi,t}\right) + (\rho + \lambda)\xi. \tag{79}$$

Differentiating the expression above with respect to time, we obtain

$$\alpha(r_t - \rho) + \lambda\Upsilon(r_t - \rho + \psi_t) = -\kappa(1 + \lambda\Phi)\beta\pi_t. \tag{80}$$

Rearranging the expression above, we obtain the real interest rate

$$r_t - \rho = -\beta\frac{\kappa(1 + \lambda\Phi)}{\lambda\Upsilon + \alpha}\pi_t - \frac{\lambda\Upsilon}{\lambda\Upsilon + \alpha}\psi_t, \tag{81}$$

and the nominal interest rate is given by

$$i_t = \rho + \left[1 - \beta\frac{\kappa(1 + \lambda\Phi)}{\lambda\Upsilon + \alpha}\right]\pi_t - \frac{\lambda\Upsilon}{\lambda\Upsilon + \alpha}\psi_t. \tag{82}$$

□

Proof of Propositions 5 and 6

Proof. The proof of Proposition 9 derives the solution to the optimal policy problem for arbitrary values of α and β. Here, we specialize the general formulas to the case of doves, $\beta = 0$, and hawks $\alpha = 0$.

Doves. Suppose $\beta = 0$. In this case, initial inflation is given by

$$\pi_0 = \frac{\kappa}{\overline{\omega}}\left[\frac{\alpha\Phi}{\alpha + \lambda\Upsilon}\frac{\lambda\psi_0}{\overline{\omega} + \theta_\psi} + \frac{\lambda\Upsilon}{\lambda\Upsilon + \alpha}\frac{|\underline{\omega}|\psi_0}{(\rho + \lambda + \theta_\psi)(\overline{\omega} + \theta_\psi)} + \lambda\Phi(b_0 - b^n)\right], \tag{83}$$

Using the fact that $b_0 = b^n$ and that $\underline{\omega} = 0$, the expression above simplifies to

$$\pi_0 = \frac{\kappa}{\overline{\omega}}\frac{\alpha\lambda\Phi}{\alpha + \lambda\Upsilon}\frac{\psi_0}{\overline{\omega} + \theta_\psi}. \tag{84}$$

Inflation is then given by

$$\pi_t = \frac{\kappa\lambda(\alpha\Phi - \Upsilon)}{\lambda\Upsilon + \alpha}\frac{1 - e^{-\theta_\psi t}}{(\theta_\psi + \overline{\omega})\theta_\psi}\psi_0 + \pi_0. \tag{85}$$

The initial value of the output gap is given by

$$x_0 = \frac{\lambda\Upsilon}{\lambda\Upsilon + \alpha}\frac{\psi_0}{\rho + \lambda + \theta_\psi}. \tag{86}$$

The real rate is given by $r_t - \rho = -\frac{\lambda\Upsilon}{\lambda\Upsilon+\alpha}\psi_t$, then output gap is given by

$$x_t = \frac{\lambda\Upsilon}{\lambda\Upsilon + \alpha}\frac{\psi_0}{\rho + \lambda + \theta_\psi} - \frac{\lambda\Upsilon}{\lambda\Upsilon + \alpha}\frac{\psi_0 - \psi_t}{\theta_\psi}. \tag{87}$$

The government debt is given by

$$b_t = \frac{\alpha}{\lambda\Upsilon + \alpha}\frac{\psi_0 - \psi_t}{\theta_\psi}. \tag{88}$$

Hawks. Suppose $\alpha = 0$. In this case, initial inflation is given by

$$\pi_0 = \frac{\kappa}{\theta_\psi + \overline{\omega}}\frac{\psi_0}{\rho + \lambda + \theta_\psi} > 0. \tag{89}$$

Inflation at t is given by

$$\pi_t = -\kappa\frac{e^{\underline{\omega}t} - e^{-\theta_\psi t}}{(\theta_\psi + \overline{\omega})(\theta_\psi + \underline{\omega})}\psi_0 + e^{\underline{\omega}t}\pi_0. \tag{90}$$

Combining the previous two expressions, we obtain

$$\pi_t = \kappa\frac{\psi_t - \frac{\overline{\omega}}{\rho+\lambda+\theta_\psi}e^{\underline{\omega}t}\psi_0}{(\theta_\psi + \overline{\omega})(\theta_\psi + \underline{\omega})}. \tag{91}$$

Suppose $\theta_\psi > |\underline{\omega}|$, then the numerator is negative for t sufficiently large, and the denominator is positive, so $\lim_{t\to\infty} < 0$. If $\theta_\psi < |\underline{\omega}|$, then the numerator is positive for t sufficiently large, and the denominator is negative, so again $\lim_{t\to\infty} < 0$.

The derivative of inflation with respect to time is given by

$$\dot{\pi}_t = -\kappa\frac{\underline{\omega}e^{\underline{\omega}t} + \theta_\psi e^{-\theta_\psi t}}{(\theta_\psi + \overline{\omega})(\theta_\psi + \underline{\omega})}\psi_0 + e^{\underline{\omega}t}\frac{\kappa}{\theta_\psi + \overline{\omega}}\frac{\underline{\omega}\psi_0}{\rho + \lambda + \theta_\psi} \tag{92}$$

$$= -\frac{\kappa}{\theta_\psi + \overline{\omega}}\left[\frac{\theta_\psi}{\theta_\psi + \underline{\omega}}\psi_t - \frac{|\underline{\omega}|}{\theta_\psi + \underline{\omega}}\frac{\overline{\omega}e^{\underline{\omega}t}\psi_0}{\rho + \lambda + \theta_\psi}\right]. \tag{93}$$

The term in brackets is always positive, so $\dot{\pi}_0 < 0$. Notice that inflation is decreasing at $t = 0$. If $\theta_\psi < |\underline{\omega}|$, so the fiscal shock is very persistent, then inflation is eventually increasing. If $\theta_\psi > |\underline{\omega}$, then inflation is decreasing even for large t.

The initial output gap

$$x_0 = \frac{\overline{\omega}}{\theta_\psi + \overline{\omega}} \left[\frac{1}{\rho + \lambda + \theta_\psi} + \frac{1}{\overline{\omega}} \right] \psi_0. \tag{94}$$

The real interest rate is given by $r_t - \rho = -\frac{\beta}{\lambda \Upsilon} \kappa (1 + \lambda \Phi) \pi_t - \psi_t$. Output gap is given by

$$x_t = x_0 - \beta \frac{\kappa(1+\lambda\Phi)}{\lambda \Upsilon} p_t - \frac{\psi_0 - \psi_t}{\theta_\psi}. \tag{95}$$

The government debt is given by

$$b_t = -\beta \frac{\kappa(1+\lambda\Phi)}{\lambda\Upsilon} p_t. \tag{96}$$

Debt is initially decreasing, as $\dot{b}_t = -\beta \frac{\kappa(1+\lambda\Phi)}{\lambda\Upsilon} \pi_0 < 0$. The price level is given by

$$p_t = \kappa \frac{\frac{\psi_0 - \psi_t}{\theta_\psi} - \frac{\overline{\omega}}{\rho + \lambda + \theta_\psi} \frac{1 - e^{\omega t}}{|\underline{\omega}|} \psi_0}{(\theta_\psi + \overline{\omega})(\theta_\psi + \underline{\omega})}. \tag{97}$$

Taking the limit as $t \to \infty$, we obtain

$$\lim_{t \to \infty} p_t = \kappa \frac{\frac{1}{\theta_\psi} + \frac{\overline{\omega}}{\rho + \lambda + \theta_\psi} \frac{1}{\underline{\omega}}}{(\theta_\psi + \overline{\omega})(\theta_\psi + \underline{\omega})} = \kappa \frac{\rho + \lambda}{(\theta_\psi + \overline{\omega})(\theta_\psi + \rho + \lambda)\theta_\psi} \frac{1}{\underline{\omega}} < 0. \tag{98}$$

Therefore, $\lim_{t \to \infty} b_t > 0$. □

Proof of Proposition 7.

Proof. The planner's objective is given by

$$\mathcal{P}_0(b_0) = -\frac{1}{2} \int_0^\infty e^{-(\rho + \lambda + \overline{\lambda})t} \left[\alpha x_t^2 + \beta \pi_t^2 + \lambda \Upsilon (b_t - b^n)^2 \right] dt + \int_0^\infty e^{-(\rho + \lambda + \overline{\lambda})t} \overline{\lambda} \mathcal{P}_t(b_t) dt. \tag{99}$$

The planner's problem consists of maximizing the objective above subject to the constraints

$$\dot{\pi}_t = (\rho + \lambda)\pi_t - \kappa x_t - \lambda \kappa \Phi (b_t - b^n), \qquad \dot{b}_t = r_t - \rho + \psi_t, \qquad \dot{x}_t = r_t - \rho.$$

We also include a penalty on π_0 and x_0, as in the case with full commitment.

Optimality conditions The optimality conditions are given by

$$\dot{\mu}_{\pi,t} - (\rho + \lambda + \overline{\lambda})\mu_{\pi,t} = \beta\pi_t - (\rho + \lambda)\mu_{\pi,t} \tag{100}$$

$$\dot{\mu}_{b,t} - (\rho + \lambda + \overline{\lambda})\mu_{b,t} = \lambda\Upsilon(b_t - b^n) - \overline{\lambda}\mathcal{P}_{b,t}(b_t) + \lambda\kappa\Phi\mu_{\pi,t} \tag{101}$$

$$\dot{\mu}_{x,t} - (\rho + \lambda + \overline{\lambda})\mu_{x,t} = \alpha x_t + \kappa\mu_{\pi,t}, \tag{102}$$

where $\mathcal{P}_{b,t}(b_t)$ denotes the partial derivative of $\mathcal{P}_t(b_t)$ with respect to debt.

The optimality condition for the interest rate is given by

$$\mu_{x,t} + \mu_{b,t} = -\xi, \tag{103}$$

where $\xi \equiv \frac{\kappa(1+\lambda\Phi)}{\rho+\theta}\xi_\pi$.

The optimality condition for x_0 is given by

$$\mu_{x,0} = 0. \tag{104}$$

Standard envelope arguments imply that

$$\mu_{b,t} = \mathcal{P}_{b,t}(b_t). \tag{105}$$

The discretion limit. Consider the limit as $\overline{\lambda} \to \infty$, so each planner has commitment only over an infinitesimal amount of time. In the limit, the co-states on π_t and x_t are given by

$$\mu_{\pi,t} = 0, \qquad \mu_{x,t} = 0. \tag{106}$$

Integrating the expression for $\mu_{x,t}$ forward, we obtain

$$\mu_{x,t} = -\int_t^\infty e^{-(\rho+\lambda+\overline{\lambda})(s-t)}\left[\alpha x_s + \kappa\mu_{\pi,s}\right] ds \Rightarrow \lim_{\overline{\lambda}\to\infty} \overline{\lambda}\mu_{x,t} = -\alpha x_t, \tag{107}$$

using the fact that $\lim_{\overline{\lambda}\to\infty} \mu_{\pi,t} = 0$. Hence, from the optimality condition for x_0, we obtain $x_0 = 0$. Differentiating the optimality condition for the interest rate with respect to time, we obtain

$$(\rho + \lambda + \overline{\lambda})\xi = \alpha x_t + \lambda\Upsilon(b_t - b^n) - \overline{\lambda}\mu_{b,t} + \kappa(1 + \lambda\Phi)\mu_{\pi,t}, \tag{108}$$

where we used the envelope condition for b_t

Given $\mu_{b,t} = -\xi - \mu_{x,t}$, and combining the previous two expressions, we obtain

$$(\rho + \lambda)\xi = \lambda\Upsilon(b_t - b^n). \tag{109}$$

Therefore, the interest rate is given by

$$r_t - \rho = -\psi_t. \tag{110}$$

□

Proof of Proposition 8.

Proof. The dynamics under the optimal policy are characterized by the conditions:

$$\dot{\pi}_t = (\rho + \lambda)\pi_t - \kappa x_t - \lambda\kappa\Phi(b_t - b^n) \tag{111}$$
$$\dot{b}_t = r_t - \rho - \gamma(b_t - b^n) + \psi_t \tag{112}$$
$$\dot{x}_t = r_t - \rho + \theta_h x_t - \theta_h^*(b_t - b^n) \tag{113}$$
$$\dot{\mu}_{\pi,t} = \beta\pi_t \tag{114}$$
$$\dot{\mu}_{b,t} = (\rho + \lambda)\mu_{b,t} + \lambda\Upsilon(b_t - b^n) + \kappa\lambda\Phi\mu_{\pi,t} \tag{115}$$
$$\dot{\mu}_{x,t} = (\rho + \lambda)\mu_{x,t} + \alpha x_t + \kappa\mu_{\pi,t}, \tag{116}$$

where the real rate is given by

$$r_t - \rho = -\beta\frac{\kappa(1 + \lambda\Phi)}{\lambda\Upsilon + \alpha}\pi_t - \frac{\lambda\Upsilon}{\lambda\Upsilon + \alpha}\psi_t, \tag{117}$$

given the initial value of debt, b_0, and the boundary conditions $\mu_{x,0} = \mu_{\pi,0} = 0$.

Consider the case without a fiscal shock, $\psi_t = 0$, and denote the co-states in this case with no shocks by $\mu_{x,t}^{ns}$ and $\mu_{\pi,t}^{ns}$. The optimal policy under the timeless perspective corresponds to the solution to the system above when we replace the initial conditions by the long-run values of these multipliers: $\mu_{x,0} = \lim_{t\to\infty}\mu_{x,t}^{ns}$ and $\mu_{\pi,0} = \lim_{t\to\infty}\mu_{\pi,t}^{ns}$ (see Giannoni and Woodford (2017) for a discussion in the context a general model). This is equivalent to the problem of a planner who started its planning in a distant past, so the multipliers had time to converge to their long-run values.

Even without shocks, the limits $\lim_{t\to\infty}\mu_{x,t}^{ns}$ and $\lim_{t\to\infty}\mu_{\pi,t}^{ns}$ will not be equal to zero, provided that $b_0 \neq b^n$. However, in the case $b_0 = b^n$, the solution to the system above in the absence of shocks is simply $\pi_t = x_t = b_t = \mu_{\pi,t} = \mu_{x,t} = \mu_{b,t} = 0$. Hence, we have that $\lim_{t\to\infty}\mu_{x,t}^{ns} = 0$ and $\lim_{t\to\infty}\mu_{\pi,t}^{ns} = 0$, so the boundary conditions for the problem under the timeless perspective coincide with the time-zero commitment solution. □

Proof of Proposition 9.

Proof. The matrix of eigenvectors and its inverse are given by

$$V = \begin{bmatrix} \frac{\kappa(1+\lambda\Phi)}{\underline{\omega}} & \frac{\kappa(1+\lambda\Phi)}{\overline{\omega}} \\ 1 & 1 \end{bmatrix}, \qquad V^{-1} = \frac{\overline{\omega}|\underline{\omega}|}{(\overline{\omega}-\underline{\omega})\kappa(1+\lambda\Phi)} \begin{bmatrix} -1 & \frac{\kappa(1+\lambda\Phi)}{\overline{\omega}} \\ 1 & \frac{\kappa(1+\lambda\Phi)}{|\underline{\omega}|} \end{bmatrix}. \tag{118}$$

Let $Z_t = [\pi_t, b_t]'$ denote the vector of endogenous variables, A the matrix of coefficients, and U_t the vector of coefficients. We can then write the dynamic system as $\dot{Z}_t = AZ_t + U_t$. We can write the matrix of coefficients as $A = V\Lambda V^{-1}$, where Λ is a diagonal matrix with the eigenvalues. Using the matrix eigendecomposition, we can decouple the system using the transformation: $z_t \equiv V^{-1}Z_t$ and $u_t \equiv V^{-1}U_t$. This gives us the system of decoupled differential equations:

$$\dot{z}_{1,t} = \overline{\omega} z_{1,t} + u_{1,t}, \qquad \dot{z}_{2,t} = \underline{\omega} z_{2,t} + u_{2,t}. \tag{119}$$

Integrating the first equation forward and the second backwards, we obtain

$$z_{1,t} = -\int_t^\infty e^{-\overline{\omega}(s-t)} u_{1,s} ds, \qquad z_{2,t} = e^{\underline{\omega}t} z_{2,0} + \int_0^t e^{\underline{\omega}(t-s)} u_{2,s} ds. \tag{120}$$

Rotating the system back to its original coordinates, we obtain

$$\pi_t = \frac{\kappa(1+\lambda\Phi)}{|\underline{\omega}|} \int_t^\infty e^{-\overline{\omega}(s-t)} u_{1,s} ds + \frac{\kappa(1+\lambda\Phi)}{\overline{\omega}} \left[e^{\underline{\omega}t} z_{2,0} + \int_0^t e^{\underline{\omega}(t-s)} u_{2,s} ds \right], \tag{121}$$

and

$$b_t - b^n = -\int_t^\infty e^{-\overline{\omega}(s-t)} u_{1,s} ds + e^{\underline{\omega}t} z_{2,0} + \int_0^t e^{\underline{\omega}(t-s)} u_{2,s} ds. \tag{122}$$

The disturbances $u_{1,t}$ and $u_{2,t}$ are given by

$$u_{1,t} = \frac{|\underline{\omega}|}{\overline{\omega}-\underline{\omega}} \left[\frac{\alpha}{\lambda\Upsilon+\alpha}\psi_t - \frac{\overline{\omega}}{(1+\lambda\Phi)}(\hat{\psi}_t + b_0 - b^n - x_0) \right] \tag{123}$$

$$u_{2,t} = \frac{\overline{\omega}}{\overline{\omega}-\underline{\omega}} \left[\frac{\alpha}{\lambda\Upsilon+\alpha}\psi_t + \frac{|\underline{\omega}|}{(1+\lambda\Phi)}(\hat{\psi}_t + b_0 - b^n - x_0) \right], \tag{124}$$

where $\hat{\psi}_t = \frac{1-e^{-\theta_\psi t}}{\theta_\psi}\psi_0$ if $\theta_\psi > 0$ and $\hat{\psi}_t = \psi_0 t$ if $\theta_\psi = 0$.

The forward integral of $u_{1,t}$ is given by

$$\int_t^\infty e^{-\overline{\omega}(s-t)} u_{1,s} ds = \frac{|\underline{\omega}|}{\overline{\omega}-\underline{\omega}} \left[\left(\frac{\alpha}{\lambda\Upsilon+\alpha} - \frac{\overline{\omega}}{(1+\lambda\Phi)} \frac{1}{\theta_\psi} \right) \frac{\psi_t}{\overline{\omega}+\theta_\psi} - \frac{\frac{\psi_0}{\theta_\psi} + b_0 - b^n - x_0}{1+\lambda\Phi} \right]. \tag{125}$$

The backward integral of $u_{2,t}$ is given by

$$\int_0^t e^{\underline{\omega}(t-s)}u_{2,s}ds = \frac{\overline{\omega}}{\overline{\omega}-\underline{\omega}}\left[\left(\frac{\alpha}{\lambda\Upsilon+\alpha}-\frac{|\underline{\omega}|}{(1+\lambda\Phi)}\frac{1}{\theta_\psi}\right)\frac{e^{\underline{\omega}t}-e^{-\theta_\psi t}}{\theta_\psi+\underline{\omega}}\psi_0+\frac{|\underline{\omega}|}{(1+\lambda\Phi)}(\frac{\psi_0}{\theta_\psi}+b_0-b^n-x_0)\frac{1-e^{\underline{\omega}t}}{|\underline{\omega}|}\right] \tag{126}$$

From the expression for $z_{1,0}$, we obtain

$$\begin{aligned}\pi_0 &= \frac{\kappa(1+\lambda\Phi)}{\overline{\omega}}\left[(b_0-b^n)+\frac{\overline{\omega}-\omega}{|\underline{\omega}|}\int_0^\infty e^{-\overline{\omega}t}u_{1,t}dt\right]\\ &= \frac{\kappa(1+\lambda\Phi)}{\overline{\omega}}\left[(b_0-b^n)+\left(\frac{\alpha}{\lambda\Upsilon+\alpha}+\frac{\overline{\omega}}{(1+\lambda\Phi)}\frac{1}{\theta_\psi}\right)\frac{\psi_0}{\overline{\omega}+\theta_\psi}-\frac{\frac{\psi_0}{\theta_\psi}+b_0-b^n-x_0}{1+\lambda\Phi}\right]\end{aligned} \tag{127}$$

We can then write initial inflation as follows:

$$\pi_0 = \frac{\kappa}{\overline{\omega}}\left[\lambda\Phi(b_0-b^n)+x_0+\frac{\alpha\Phi-\Upsilon}{\alpha+\lambda\Upsilon}\frac{\lambda\psi_0}{\overline{\omega}+\theta_\psi}\right].$$

The initial value for $z_{2,t}$ is given by

$$z_{2,0} = \frac{\overline{\omega}}{\overline{\omega}-\underline{\omega}}\left[\frac{|\underline{\omega}|}{\kappa(1+\lambda\Phi)}\pi_0+b_0-b^n\right].$$

Inflation is then given by

$$\pi_t = \frac{\kappa(1+\lambda\Phi)}{\overline{\omega}-\underline{\omega}}\left[\left(\frac{\alpha}{\lambda\Upsilon+\alpha}+\frac{\overline{\omega}}{(1+\lambda\Phi)}\frac{1}{\theta_\psi}\right)\frac{\psi_t}{\overline{\omega}+\theta_\psi}-\frac{\frac{\psi_0}{\theta_\psi}+b_0-b^n-x_0}{1+\lambda\Phi}\right] \tag{128}$$

$$+\frac{\kappa(1+\lambda\Phi)}{\overline{\omega}-\underline{\omega}}\left[e^{\underline{\omega}t}\left[\frac{|\underline{\omega}|}{\kappa(1+\lambda\Phi)}\pi_0+b_0-b^n\right]+\left(\frac{\alpha}{\lambda\Upsilon+\alpha}-\frac{|\underline{\omega}|}{(1+\lambda\Phi)}\frac{1}{\theta_\psi}\right)\frac{e^{\underline{\omega}t}-e^{-\theta_\psi t}}{\theta_\psi+\underline{\omega}}\psi_0\right] \tag{129}$$

$$+\frac{\kappa(1+\lambda\Phi)}{\overline{\omega}-\underline{\omega}}\left[\frac{1-e^{\underline{\omega}t}}{(1+\lambda\Phi)}(\frac{\psi_0}{\theta_\psi}+b_0-b^n-x_0)\right]. \tag{130}$$

After some rearrangement, we obtain

$$\pi_t = \frac{\kappa\lambda(\alpha\Phi-\Upsilon)}{\lambda\Upsilon+\alpha}\frac{e^{\underline{\omega}t}-e^{-\theta_\psi t}}{(\theta_\psi+\overline{\omega})(\theta_\psi+\underline{\omega})}\psi_0+e^{\underline{\omega}t}\pi_0. \tag{131}$$

Boundary conditions. The optimality condition for x_0 involves the co-states for x and π. Solving the equation for $\mu_{\pi,t}$ backward, we obtain

$$\mu_{\pi,t} = \mu_{\pi,0}+\beta\int_0^t \pi_s ds. \tag{132}$$

Solving the equation for $\mu_{x,t}$ forward, we obtain

$$\mu_{x,0} = -\int_0^\infty e^{-(\rho+\theta)t}\left[\kappa\mu_{\pi,t} + \alpha x_t\right]dt \tag{133}$$

$$= -\frac{\kappa}{\rho+\theta}\mu_{\pi,0} - \frac{\kappa\beta}{\rho+\theta}\int_0^\infty e^{-(\rho+\theta)t}\pi_t dt - \int_0^\infty e^{-(\rho+\theta)t}\alpha x_t dt. \tag{134}$$

The optimality condition for x_0 is given by

$$0 = \mu_{x,0} + \frac{\kappa}{\rho+\theta}\mu_{\pi,0} = -\int_0^\infty e^{-(\rho+\theta)t}\left[\frac{\beta}{\rho+\theta}\pi_t + \alpha x_t\right]. \tag{135}$$

Using the fact that $x_t = x_0 + \hat{r}_t$, we obtain

$$\int_0^\infty e^{-(\rho+\theta)t}x_t dt = \frac{x_0}{\rho+\theta} + \frac{1}{\rho+\theta}\int_0^\infty e^{-(\rho+\theta)t}(r_t - \rho)dt. \tag{136}$$

The optimality condition for x_0 can then be written as

$$0 = \frac{\alpha}{\rho+\theta}x_0 + \frac{1}{\rho+\theta}\int_0^\infty e^{-(\rho+\theta)t}\left[\kappa\beta\pi_t + \alpha\left(-\beta\frac{\kappa(1+\lambda\Phi)}{\lambda\Upsilon+\alpha}\pi_t - \frac{\lambda\Upsilon}{\lambda\Upsilon+\alpha}\psi_t\right)\right]dt. \tag{137}$$

Rearranging the expression above, we obtain

$$\alpha x_0 = \beta\frac{\kappa\lambda(\alpha\Phi-\Upsilon)}{\lambda\Upsilon+\alpha}\int_0^\infty e^{-(\rho+\theta)t}\pi_t dt + \frac{\alpha\lambda\Upsilon}{\lambda\Upsilon+\alpha}\int_0^\infty e^{-(\rho+\theta)t}\psi_t dt. \tag{138}$$

The present discounted value of inflation is given by

$$\int_0^\infty e^{-(\rho+\theta)t}\pi_t dt = \frac{\kappa\lambda(\alpha\Phi-\Upsilon)}{(\lambda\Upsilon+\alpha)(\theta_\psi+\overline{\omega})}\frac{\psi_0}{(\rho+\theta+|\lambda|)(\rho+\theta+\theta_\psi)} + \frac{\pi_0}{\rho+\theta+|\underline{\omega}|}. \tag{139}$$

Combining the previous two equations, we obtain

$$\alpha x_0 = \frac{\beta}{\theta_\psi+\overline{\omega}}\left(\frac{\kappa\lambda(\alpha\Phi-\Upsilon)}{\lambda\Upsilon+\alpha}\right)^2\frac{\psi_0}{(\rho+\theta+|\lambda|)(\rho+\theta+\theta_\psi)} + \beta\frac{\kappa\lambda(\alpha\Phi-\Upsilon)}{\lambda\Upsilon+\alpha}\frac{\pi_0}{\rho+\theta+|\underline{\omega}|} \tag{140}$$

$$+ \frac{\alpha\lambda\Upsilon}{\lambda\Upsilon+\alpha}\frac{\psi_0}{\rho+\theta+\theta_\psi}. \tag{141}$$

Using the fact that $\pi_0 = \frac{\kappa}{\overline{\omega}} \left[\lambda\Phi(b_0 - b^n) + x_0 + \frac{\alpha\Phi - \Upsilon}{\alpha + \lambda\Upsilon} \frac{\lambda\psi_0}{\overline{\omega} + \theta_\psi} \right]$, we obtain

$$x_0 = \frac{\frac{\beta}{\theta_\psi + \overline{\omega}} \left(\frac{\kappa\lambda(\alpha\Phi - \Upsilon)}{\lambda\Upsilon + \alpha} \right)^2 \frac{\psi_0}{(\rho + \theta + |\underline{\omega}|)} \left[\frac{1}{\rho + \theta + \theta_\psi} + \frac{1}{\overline{\omega}} \right] + \frac{\alpha\lambda\Upsilon}{\lambda\Upsilon + \alpha} \frac{\psi_0}{\rho + \theta + \theta_\psi} + \beta \frac{\kappa\lambda(\alpha\Phi - \Upsilon)}{\lambda\Upsilon + \alpha} \frac{\kappa\lambda\Phi(b_0 - b^n)}{\overline{\omega}(\rho + \theta + |\underline{\omega}|)}}{\alpha - \frac{\kappa\beta}{\overline{\omega}(\rho + \theta + |\underline{\omega}|)} \frac{\kappa\lambda(\alpha\Phi - \Upsilon)}{\lambda\Upsilon + \alpha}}. \tag{142}$$

Initial inflation is then given by

$$\pi_0 = \frac{\kappa}{\overline{\omega}} \left[\frac{\frac{\beta}{\theta_\psi + \overline{\omega}} \left(\frac{\kappa\lambda(\alpha\Phi - \Upsilon)}{\lambda\Upsilon + \alpha} \right)^2 \frac{\psi_0}{(\rho + \theta + |\underline{\omega}|)} \frac{1}{\rho + \theta + \theta_\psi} + \frac{\alpha^2\Phi}{\alpha + \lambda\Upsilon} \frac{\lambda\psi_0}{\overline{\omega} + \theta_\psi} + \frac{\alpha\lambda\Upsilon}{\lambda\Upsilon + \alpha} \frac{|\underline{\omega}|\psi_0}{(\rho + \theta + \theta_\psi)(\overline{\omega} + \theta_\psi)} + \alpha\lambda\Phi(b_0 - b^n)}{\alpha - \frac{\kappa\beta}{\overline{\omega}(\rho + \theta + |\underline{\omega}|)} \frac{\kappa\lambda(\alpha\Phi - \Upsilon)}{\lambda\Upsilon + \alpha}} \right]. \tag{143}$$

Notice that the numerator is positive. The denominator is positive for α large or β large. In these cases, a fiscal shock leads to more inflation and higher output gap.

Output gap. The output gap is given by

$$x_t = x_0 - \beta \frac{\kappa(1 + \lambda\Phi)}{\lambda\Upsilon + \alpha} p_t - \frac{\lambda\Upsilon}{\lambda\Upsilon + \alpha} \frac{\psi_0 - \psi_t}{\theta_\psi}, \tag{144}$$

where $p_t = \int_0^t \pi_s ds$.

Government debt. Government debt is given by

$$b_t = -\beta \frac{\kappa(1 + \lambda\Phi)}{\lambda\Upsilon + \alpha} p_t + \frac{\alpha}{\lambda\Upsilon + \alpha} \frac{\psi_0 - \psi_t}{\theta_\psi}. \tag{145}$$

□

□

C. Optimal policy

C.1 The planner's problem

Planner's problem. We can write the planner's problem as follows:

$$\max_{\{[x_t,\pi_t,b_t,r_t]_0^\infty\}} -\frac{1}{2}\int_0^\infty e^{-(\rho+\lambda)t}\left[\alpha x_t^2+\beta\pi_t^2+\lambda\Upsilon(b_t-b^n)^2\right]dt, \tag{146}$$

subject to

$$\dot{\pi}_t = (\rho+\lambda)\pi_t - \kappa x_t - \lambda\kappa\Phi(b_t-b^n) \tag{147}$$

$$\dot{b}_t = r_t - \rho + \psi_t \tag{148}$$

$$\dot{x}_t = r_t - \rho, \tag{149}$$

given b_0 and the initial value for inflation.

The lack of a classical solution. It turns out that a classical solution, where the states are continuous functions of time, does not exist. The issue of non-existence of a solution can be seen more clearly in the case $\beta = 0$, where inflation drops out of the problem. For simplicity, assume that $b_0 = b_n = 0$. The optimality condition for r_t is given by

$$\alpha x_t + \lambda\Upsilon b_t = 0, \tag{150}$$

for all $t \geq 0$. The optimality condition for x_0 is given by

$$\mu_{x,0} = 0 \iff -\alpha\int_0^\infty e^{-(\rho+\lambda)t}x_t dt = 0. \tag{151}$$

Let (x_t^*, b_t^*) denote a candidate solution, where b_t^* is a differentiable function of time satisfying $b_0^* = 0$. Differentiating the optimality condition for r_t with respect to time, we obtain

$$r_t - \rho = -\frac{\lambda\Upsilon}{\alpha+\lambda\Upsilon}\psi_t \Rightarrow \hat{r}_t = -\frac{\lambda\Upsilon}{\alpha+\lambda\Upsilon}\hat{\psi}_t. \tag{152}$$

As $x_t = x_0 + \hat{r}_t$, the optimality condition for x_0 implies that the following condition must hold:

$$\frac{x_0}{\rho+\lambda} + \int_0^\infty e^{-(\rho+\lambda)t}\hat{r}_t dt = 0 \Rightarrow x_0 = \frac{\lambda\Upsilon}{\alpha+\lambda\Upsilon}\int_0^\infty e^{-(\rho+\lambda)t}\psi_t dt > 0. \tag{153}$$

However, from the optimality condition for the interest rate at $t = 0$, we obtain:[29]

$$\alpha x_0 + \lambda \Upsilon b_0 = 0 \Rightarrow x_0 = 0, \tag{154}$$

which contradicts the fact that $x_0 > 0$.

Incentive for expropriation. While a classical solution to this problem does not exist, a generalized solution with discontinuous states exists. In a classical solution, b_t is given by

$$b_t = \int_0^t (r_s - \rho + \psi_s) ds \tag{155}$$

The integral above is equal to zero at $t = 0$, so $b_0 = 0$. Following the approach in optimal impulsive control, consider the following generalization:[30]

$$b_t = \int_0^t (r_s - \rho + \psi_s) ds + \int_{[0,t]} \overline{r}_s d\mu, \tag{156}$$

where μ denotes a Borel measure on $\mathbb{R}_+$. For example, if μ is a Dirac measure with weight on zero, then b_t is given by

$$b_t = \int_0^t (r_s - \rho + \psi_s) ds + \overline{r}_0. \tag{157}$$

In this case, government debt can immediately jump at zero, provided $\overline{r}_0 \neq 0$.

Define $\hat{r}_t \equiv \int_0^\infty (r_s - \rho) ds + \int_{[0,t]} \overline{r}_s d\mu$, so $x_t = x_0 + \hat{r}_t$ and $b_t = \hat{r}_t + \hat{\psi}_t$. In a classical solution, $\hat{r}_t$ must be an absolutely continuous function satisfying $\hat{r}_0 = 0$, while it is a bounded variation function in the context of optimal impulsive control, where $\hat{r}_0$ can take any value. Without the constraint that $\hat{r}_0 = 0$, the planner's problem becomes particularly simple:

$$\max_{\{x_0, [\hat{r}_t]_0^\infty\}} -\frac{1}{2} \int_0^\infty e^{-(\rho+\lambda)t} \left[\alpha \left(x_0 + \hat{r}_t \right)^2 + \lambda \Upsilon \left(\hat{r}_t + \hat{\psi}_t \right)^2 \right] dt, \tag{158}$$

with optimality conditions

$$\alpha x_t + \lambda \Upsilon b_t = 0, \qquad -\alpha \int_0^\infty e^{-(\rho+\lambda)t} x_t dt = 0. \tag{159}$$

[29]Notice that the optimality condition for the interest rate must hold at $t = 0$. From continuity of x_t and b_t, if $\alpha x_t + \lambda \Upsilon b_t > 0$ for $t = 0$, there exists $t_1 > 0$ such that this inequality holds for $t \in [0, t_1)$. By reducing interest rates in this interval, we can improve the planner's objective.

[30]See Arutyunov et al. (2019) for a discussion of optimal impulsive control theory.

The solution in this case takes the form:

$$r_t - \rho = -\frac{\lambda\Upsilon}{\alpha + \lambda\Upsilon}\psi_t, \qquad x_0 = \frac{\lambda\Upsilon}{\alpha + \lambda\Upsilon}\int_0^\infty e^{-(\rho+\lambda)t}\psi_t dt, \qquad b_0 = -\frac{\alpha}{\lambda\Upsilon}x_0. \tag{160}$$

Hence, government debt jumps immediately down on impact, which requires $\bar{r}_0 = -\frac{\alpha}{\lambda\Upsilon}x_0$ and μ to be a Dirac measure with weight in zero. Intuitively, the planner has an incentive to expropriate part of the debt by having the real interest rate be very negative over a small period (the impulse from the Dirac measure).

C.2 Characterization of the optimal policy

The penalized planner's problem. To deal with the incentive to expropriate, we introduce a penalty associated with the initial value of each forward-looking variable:

$$\max_{\{[\pi_t, b_t, x_t, r_t]_0^\infty\}} -\frac{1}{2}\int_0^\infty e^{-(\rho+\lambda)t}\left[\alpha x_t^2 + \beta\pi_t^2 + \lambda\Upsilon(b_t - b^n)^2\right]dt + \xi_x x_0 + \xi_\pi \pi_0, \tag{161}$$

subject to

$$\dot{\pi}_t = (\rho+\lambda)\pi_t - \kappa x_t - \lambda\kappa\Phi(b_t - b^n), \qquad \dot{b}_t = r_t - \rho + \psi_t, \qquad \dot{x}_t = r_t - \rho, \tag{162}$$

given b_0 and the initial value for inflation. We will choose the penalty ξ_x and ξ_π such that there is no discontinuity in b_t at $t = 0$, and the co-state for the output gap is equal to zero at $t = 0$.

Optimality conditions. The Hamiltonian to this problem is given by

$$\begin{aligned}\mathcal{H}_t = &-\frac{1}{2}\left[\alpha x_t^2 + \beta\pi_t^2 + \lambda\Upsilon(b_t - b^n)^2\right] + \mu_{\pi,t}\left[(\rho+\lambda)\pi_t - \kappa x_t - \lambda\kappa\Phi(b_t - b^n)\right] + \mu_{b,t}\left[r_t - \rho + \psi_t\right] \\ &+ \mu_{x,t}[r_t - \rho] + (\mu_{x,0} + \xi_x)(\rho+\lambda)x_0 + (\mu_{\pi,0} + \xi_\pi)\left[\kappa x_0 + \frac{\kappa(1+\lambda\Phi)}{\rho+\lambda}(r_t - \rho)\right],\end{aligned} \tag{163}$$

The dynamics of the co-states are given by

$$\dot{\mu}_{\pi,t} - (\rho+\lambda)\mu_{\pi,t} = \beta\pi_t - \mu_{\pi,t}(\rho+\lambda) \tag{164}$$

$$\dot{\mu}_{b,t} - (\rho+\lambda)\mu_{b,t} = \lambda\Upsilon(b_t - b^n) + \kappa\lambda\Phi\mu_{\pi,t} \tag{165}$$

$$\dot{\mu}_{x,t} - (\rho+\lambda)\mu_{x,t} = \alpha x_t + \kappa\mu_{\pi,t}. \tag{166}$$

The optimality condition for the real interest rate is

$$\mu_{b,t} + \mu_{x,t} = -\xi, \tag{167}$$

where $\xi \equiv \frac{\kappa(1+\lambda\Phi)}{\rho+\lambda}(\mu_{\pi,0} + \xi_\pi)$.

The optimality condition for the initial output gap:

$$(\rho + \lambda)(\mu_{x,0} + \xi_x) + \kappa(\mu_{\pi,0} + \xi_\pi) = 0. \tag{168}$$

We will choose $\xi_x = -\kappa\frac{\mu_{\pi,0}+\xi_\pi}{\rho+\lambda}$, such that $\mu_{x,0} = 0$. We show below that we can set $\mu_{\pi,0} = 0$ without loss of generality.

Real and nominal rates. The optimality condition for interest rates imply that $\dot{\mu}_{b,t} + \dot{\mu}_{x,t} = 0$. From the law of motion of the co-states, we obtain

$$\alpha x_t + \lambda\Upsilon(b_t - b^n) = \kappa(1 + \lambda\Phi)\left(\mu_{\pi,0} - \mu_{\pi,t}\right) + (\rho + \lambda)\xi. \tag{169}$$

Differentiating the expression above with respect to time, we obtain

$$\alpha(r_t - \rho) + \lambda\Upsilon(r_t - \rho + \psi_t) = -\kappa(1 + \lambda\Phi)\beta\pi_t. \tag{170}$$

Rearranging the expression above, we obtain the real interest rate

$$r_t - \rho = -\beta\frac{\kappa(1 + \lambda\Phi)}{\lambda\Upsilon + \alpha}\pi_t - \frac{\lambda\Upsilon}{\lambda\Upsilon + \alpha}\psi_t, \tag{171}$$

and the nominal interest rate is given by

$$i_t = \rho + \left[1 - \beta\frac{\kappa(1 + \lambda\Phi)}{\lambda\Upsilon + \alpha}\right]\pi_t - \frac{\lambda\Upsilon}{\lambda\Upsilon + \alpha}\psi_t. \tag{172}$$

Dynamics under the optimal policy. Using the expression for $x_t = x_0 + b_t - b_0 - \hat{\psi}_t$, we can write a dynamic system for π_t and b_t

$$\begin{bmatrix} \dot{\pi}_t \\ \dot{b}_t \end{bmatrix} = \begin{bmatrix} \rho + \lambda & -\kappa(1 + \lambda\Phi) \\ -\hat{\beta} & 0 \end{bmatrix} \begin{bmatrix} \pi_t \\ b_t - b^n \end{bmatrix} + \begin{bmatrix} \kappa(\hat{\psi}_t + b_0 - b^n - x_0) \\ \frac{\alpha}{\lambda\Upsilon+\alpha}\psi_t \end{bmatrix}, \tag{173}$$

where $\hat{\beta} \equiv \frac{\beta\kappa(1+\lambda\Phi)}{\lambda\Upsilon+\alpha}$ and $\hat{\psi}_t = \frac{1-e^{-\theta_\psi t}}{\theta_\psi}\psi_0$. As b_0 is given and π_0 can jump, there is a unique bounded solution to the system above if the system has a positive eigenvalue and a negative eigenvalue. The eigenvalues of the system satisfy the condition

$$(\rho+\lambda-\omega)(-\omega)-\hat{\beta}\kappa(1+\lambda\Phi)=0 \Rightarrow \omega^2-[\rho+\lambda]\,\omega-\kappa(1+\lambda\Phi)\hat{\beta}=0.$$

Denote the eigenvalues of the system by $\overline{\omega}>0$ and $\underline{\omega}<0$, where

$$\overline{\omega}=\frac{\rho+\lambda+\sqrt{(\rho+\lambda)^2+4\kappa(1+\lambda\Phi)\hat{\beta}}}{2}, \qquad \underline{\omega}=\frac{\rho+\lambda-\sqrt{(\rho+\lambda)^2+4\kappa(1+\lambda\Phi)\hat{\beta}}}{2}. \tag{174}$$

We provide next a characterization of inflation and output gap under the optimal policy.

Proposition 9 (Optimal policy: general case)**.** *Suppose the planner implements the optimal policy given welfare weights $\alpha\geq 0$ and $\beta\geq 0$. Then,*

1. *Inflation is given by*

$$\pi_t=\frac{\kappa\lambda(\alpha\Phi-\Upsilon)}{\lambda\Upsilon+\alpha}\frac{e^{\underline{\omega}t}-e^{-\theta_\psi t}}{(\theta_\psi+\overline{\omega})(\theta_\psi+\underline{\omega})}\psi_0+e^{\underline{\omega}t}\pi_0, \tag{175}$$

where initial inflation is given by

$$\pi_0=\frac{\kappa}{\overline{\omega}}\left[\frac{\frac{\beta}{\theta_\psi+\overline{\omega}}\left(\frac{\kappa\lambda(\alpha\Phi-\Upsilon)}{\lambda\Upsilon+\alpha}\right)^2\frac{\psi_0}{(\rho+\theta+|\underline{\omega}|)}\frac{1}{\rho+\theta+\theta_\psi}+\frac{\alpha^2\Phi}{\alpha+\lambda\Upsilon}\frac{\lambda\psi_0}{\overline{\omega}+\theta_\psi}+\frac{\alpha\lambda\Upsilon}{\lambda\Upsilon+\alpha}\frac{|\underline{\omega}|\psi_0}{(\rho+\theta+\theta_\psi)(\overline{\omega}+\theta_\psi)}+\alpha\lambda\Phi(b_0-b^n)}{\alpha-\frac{\kappa\beta}{\overline{\omega}(\rho+\theta+|\underline{\omega}|)}\frac{\kappa\lambda(\alpha\Phi-\Upsilon)}{\lambda\Upsilon+\alpha}}\right]. \tag{176}$$

2. *Output gap is given by*

$$x_t=x_0-\beta\frac{\kappa(1+\lambda\Phi)}{\lambda\Upsilon+\alpha}p_t-\frac{\lambda\Upsilon}{\lambda\Upsilon+\alpha}\frac{\psi_0-\psi_t}{\theta_\psi}, \tag{177}$$

where $p_t=\int_0^t\pi_s ds$, and the initial output gap is given by

$$\begin{aligned}\alpha x_0=&\frac{\beta}{\theta_\psi+\overline{\omega}}\left(\frac{\kappa\lambda(\alpha\Phi-\Upsilon)}{\lambda\Upsilon+\alpha}\right)^2\frac{\psi_0}{(\rho+\theta+|\lambda|)(\rho+\theta+\theta_\psi)}\\&+\beta\frac{\kappa\lambda(\alpha\Phi-\Upsilon)}{\lambda\Upsilon+\alpha}\frac{\pi_0}{\rho+\theta+|\underline{\omega}|}+\frac{\alpha\lambda\Upsilon}{\lambda\Upsilon+\alpha}\frac{\psi_0}{\rho+\theta+\theta_\psi}.\end{aligned} \tag{178}$$

3. *The government debt is given by*

$$b_t=-\beta\frac{\kappa(1+\lambda\Phi)}{\lambda\Upsilon+\alpha}p_t+\frac{\alpha}{\lambda\Upsilon+\alpha}\frac{\psi_0-\psi_t}{\theta_\psi}. \tag{179}$$

We provide the proof of this proposition in Appendix B.

D. Extensions

D.1 Optimal policy with debt stabilizers and cost-push shocks

In this section, we generalize the optimal policy problem in two dimensions. First, we allow for a positive debt stabilizer, $\gamma > 0$. Second, we introduce an exogenous cost-push shock v_t to the NKPC.

Implementability. Suppose that $\lambda_h = 0$, so the equilibrium dynamics is described by the dynamic system:

$$\dot{\pi}_t = (\rho + \theta)\pi_t - \kappa x_t - \kappa\Phi\lambda b_t - v_t, \qquad \dot{x}_t = r_t - \rho, \qquad \dot{b}_t = r_t - \rho - \gamma b_t + \psi_t, \tag{180}$$

for a given path of real rates, the initial condition for the output gap x_0, the evolution of the fiscal shock ψ_t, and a cost-push shock v_t.

Proposition 10 (Implementability). *Given a path of real rates $[r_t]_0^\infty$ and an initial condition for the output gap, x_0, and for government debt, b_0, then initial inflation is given by*

$$\pi_0 = \kappa \left[\frac{x_0}{\rho + \lambda} + \frac{\lambda \Phi b_0}{\rho + \lambda + \gamma} + \int_0^\infty e^{-(\rho+\lambda)t} \left(\frac{r_t - \rho}{\rho + \lambda} + \frac{\lambda \Phi}{\rho + \lambda + \gamma}(r_t - \rho + \psi_t) + v_t \right) dt \right]. \tag{181}$$

Proof. Integrating the law of motion of debt, we obtain

$$b_t = e^{-\gamma t} b_0 + \hat{r}_{\gamma,t} + \hat{\psi}_{\gamma,t}, \tag{182}$$

where $\hat{r}_{\gamma,t} \equiv \int_0^t e^{-\gamma(t-s)}(r_s - \rho)ds$ and $\hat{\psi}_{\gamma,t} \equiv \int_0^t e^{-\gamma(t-s)}\psi_s ds$.

Output gap is given by

$$x_t = x_0 + \hat{r}_t, \tag{183}$$

where $\hat{r}_t \equiv \hat{r}_{0,t}$. Initial inflation is given by

$$\pi_0 = \kappa \left[\frac{x_0}{\rho + \lambda} + \frac{\lambda \Phi b_0}{\rho + \lambda + \gamma} + \int_0^\infty e^{-(\rho+\lambda)t} \left(\hat{r}_t + \lambda\Phi(\hat{r}_{\gamma,t} + \hat{\psi}_{\gamma,t}) + v_t \right) dt \right]. \tag{184}$$

Applying integration by parts, we obtain the expression for initial inflation. □

Optimal policy. The optimal policy problem is given by

$$\max_{[\pi_t,b_t,x_t,r_t]_0^\infty} -\frac{1}{2}\int_0^\infty e^{-(\rho+\lambda)t}\left[\alpha(x_t-x^*)^2+\beta\pi_t^2+\lambda\Upsilon(b_t-b^n)^2\right]dt+\xi_x x_0+\xi_\pi\pi_0. \tag{185}$$

subject to

$$\dot{\pi}_t=(\rho+\lambda)\pi_t-\kappa x_t-\kappa\Phi\lambda(b_t-b^n)-v_t, \qquad \dot{x}_t=r_t-\rho, \qquad \dot{b}_t=r_t-\rho-\gamma b_t+\psi_t, \tag{186}$$

given the initial condition for inflation, where ξ_x and ξ_π denote the penalty on the initial value of output and inflation.

The Hamiltonian for this problem is given by

$$\mathcal{H}_t=-\frac{1}{2}\left[\alpha(x_t-x^*)^2+\beta\pi_t^2+\lambda\Upsilon(b_t-b^n)^2\right]+(\rho+\lambda)\xi_x x_0+\mu_{x,t}\left[r_t-\rho\right]+\mu_{x,0}(\rho+\lambda)x_0 \tag{187}$$

$$+\mu_{b,t}\left[r_t-\rho-\gamma b_t+\psi_t\right]+\mu_{\pi,t}\left[(\rho+\lambda)\pi_t-\kappa x_t-\lambda\kappa\Phi(b_t-b^n)-v_t\right]$$

$$+\left(\mu_{\pi,0}+\xi_\pi\right)\kappa\left[x_0+\left(1+\frac{(\rho+\lambda)\lambda\Phi}{\rho+\lambda+\gamma}\right)\frac{r_t-\rho}{\rho+\lambda}\right]. \tag{188}$$

Optimality conditions. The dynamics of the co-states are given by

$$\dot{\mu}_{\pi,t}-(\rho+\lambda)\mu_{\pi,t}=\beta\pi_t-(\rho+\lambda)\mu_{\pi,t} \tag{189}$$

$$\dot{\mu}_{b,t}-(\rho+\lambda)\mu_{b,t}=\lambda\Upsilon(b_t-b^n)+\lambda\kappa\Phi\mu_{\pi,t}+\gamma\mu_{b,t} \tag{190}$$

$$\dot{\mu}_{x,t}-(\rho+\lambda)\mu_{x,t}=\alpha(x_t-x^*)+\kappa\mu_{\pi,t}. \tag{191}$$

The optimality condition for the interest rate is given by

$$\mu_{x,t}+\mu_{b,t}=-\frac{\kappa\left[\mu_{\pi,0}+\xi_\pi\right]}{\rho+\lambda}\left(1+\frac{(\rho+\lambda)\lambda\Phi}{\rho+\lambda+\gamma}\right). \tag{192}$$

The optimality condition for the initial value of the output gap is given by

$$(\rho+\lambda)(\mu_{x,0}+\xi_x)+\kappa(\mu_{\pi,0}+\xi_\pi)=0. \tag{193}$$

Real interest rates. The next proposition gives the real interest rate.

Proposition 11 (Real interest rate)**.** *The real interest rate is given by*

$$r_t-\rho=-\beta\frac{\kappa(1+\lambda\Phi)}{\lambda\Upsilon+\alpha}\pi_t-\frac{\lambda\Upsilon}{\lambda\Upsilon+\alpha}\psi_t-\frac{\gamma}{\lambda\Upsilon+\alpha}\left[(\rho+\lambda+\gamma)\mu_{b,t}+\lambda\kappa\Phi\mu_{\pi,t}-\lambda\Upsilon b^n\right]. \tag{194}$$

Proof. The optimality condition for the interest rate implies that $\dot{\mu}_{b,t}+\dot{\mu}_{x,t}=0$. From the law of motion of the co-states, we obtain

$$\alpha(x_t - x^*) + \lambda\Upsilon(b_t - b^n) = -\kappa(1+\lambda\Phi)\mu_{\pi,t} - \gamma\mu_{b,t} + \kappa\left(1 + \frac{(\rho+\lambda)\lambda\Phi}{\rho+\lambda+\gamma}\right)(\mu_{\pi,0} + \xi_\pi). \quad (195)$$

Rearranging the expression above, we obtain

$$\alpha(x_t - x^*) + \lambda\Upsilon(b_t - b^n) = \kappa(1+\lambda\Phi)\left(\mu_{\pi,0} - \mu_{\pi,t}\right) - \gamma\left[\mu_{b,t} + \frac{\kappa\lambda\Phi}{\rho+\lambda+\gamma}\mu_{\pi,0}\right] + \kappa\left(1 + \frac{(\rho+\lambda)\lambda\Phi}{\rho+\lambda+\gamma}\right)\xi_\pi. \quad (196)$$

Differentiating the expression above, we obtain

$$\alpha(r_t - \rho) + \lambda\Upsilon(r_t - \rho + \psi_t - \gamma b_t) = -\beta\kappa(1+\lambda\Phi)\pi_t - \gamma\dot{\mu}_{b,t}. \quad (197)$$

Rearranging the expression above, and using the dynamics for $\mu_{b,t}$, we obtain the real interest rate. □

Dynamic system. Equilibrium dynamics under the optimal policy satisfies the dynamic system:

$$\begin{bmatrix} \dot{\pi}_t \\ \dot{x}_t \\ \dot{b}_t \\ \dot{\mu}_{b,t} \\ \dot{\mu}_{\pi,t} \end{bmatrix} = \begin{bmatrix} \rho+\lambda & -\kappa & -\kappa\Phi\lambda & 0 & 0 \\ -\hat{\beta} & 0 & 0 & -\frac{\gamma(\rho+\lambda+\gamma)}{\lambda\Upsilon+\alpha} & -\frac{\gamma\kappa\lambda\Phi}{\lambda\Upsilon+\alpha} \\ -\hat{\beta} & 0 & -\gamma & -\frac{\gamma(\rho+\lambda+\gamma)}{\lambda\Upsilon+\alpha} & -\frac{\gamma\kappa\lambda\Phi}{\lambda\Upsilon+\alpha} \\ 0 & 0 & \lambda\Upsilon & \rho+\lambda+\gamma & \kappa\lambda\Phi \\ \beta & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \\ b_t \\ \mu_{b,t} \\ \mu_{\pi,t} \end{bmatrix} + \begin{bmatrix} 0.0 \\ -\frac{\lambda\Upsilon}{\lambda\Upsilon+\alpha} \\ \frac{\alpha}{\lambda\Upsilon+\alpha} \\ 0.0 \\ 0.0 \end{bmatrix} \psi_t + \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} v_t + \begin{bmatrix} \kappa\lambda\Phi \\ \frac{\gamma\lambda\Upsilon}{\lambda\Upsilon+\alpha} \\ \frac{\gamma\lambda\Upsilon}{\lambda\Upsilon+\alpha} \\ -\lambda\Upsilon \\ 0 \end{bmatrix} b^n, \quad (198)$$

where $\hat{\beta} \equiv \beta\frac{\kappa(1+\lambda\Phi)}{\lambda\Upsilon+\alpha}$, given the boundary conditions:

$$\mu_{x,0} + \xi_x = -\frac{\kappa}{\rho+\lambda}(\mu_{\pi,0} + \xi_\pi), \qquad \mu_{b,0} - \xi_x = -\frac{\kappa\lambda\Phi}{\rho+\lambda+\gamma}(\mu_{\pi,0} + \xi_\pi). \quad (199)$$

Proposition 12 (Dynamic system)**.** *Let V and Λ denote the matrix of eigenvectors and a diagonal matrix with the eigenvalues of the dynamic system* (198)*, respectively, and denote the vector of endogenous variables by $Z_t = [\pi_t, x_t, b_t, \mu_{b,t}, \mu_{\pi,t}]'$. Assume that V is diagonalizable with real eigenvalues. Then, Z_t is given by*

$$Z_t = V_1 z_{1,t} + V_2 z_{2,t}, \quad (200)$$

where $V = [V_1\ V_2]$, $\Lambda = diag(\Lambda_1, \Lambda_2)$, Λ_1 is a diagonal matrix with positive eigenvalues, Λ_2 is a *diagonal matrix with non-positive eigenvalues, and*

$$z_{1,t} = -\int_t^{\infty} \exp\left(-\Lambda_1(s-t)\right) \left[u_1^{\psi}\psi_s + u_1^{v}v_s + u_1^{n}b^{n}\right] ds, \tag{201}$$

and

$$z_{2,t} = \exp\left(\Lambda_2 t\right) z_{2,0} + \int_0^t \exp\left(\Lambda_2(t-s)\right) \left[u_2^{\psi}\psi_s + u_2^{v}v_s + u_2^{n}b^{n}\right] ds. \tag{202}$$

Proof. Let $Z_t = [\pi_t, x_t, b_t, \mu_{b,t}, \mu_{\pi,t}]'$, so we can write the system above in matrix form:

$$\dot{Z}_t = AZ_t + U^{\psi}\psi_t + U^{v}v_t + U^{n}b^{n}. \tag{203}$$

Assuming the matrix A is diagonalizable, we can write the eigendecomposition $A = V\Lambda V^{-1}$ and obtain a decoupled system under new coordinates:

$$\dot{z}_t = \Lambda z_t + u^{\psi}\psi_t + u^{v}v_t + u^{n}b^{n}, \tag{204}$$

where $z_t = V^{-1}Z_t$ and $u^j = V^{-1}U^j$, for $j \in \{\psi, v, n\}$. Let $z_t = [z_{1,t}', z_{2,t}']'$, where $z_{1,t}$ is associated with the positive eigenvalues, and $z_{2,t}$ is associated with the non-positive eigenvalues (assuming the eigenvalues are real-valued). Solving forward the differential equation for $z_{1,t}$, we obtain

$$z_{1,t} = -\int_t^{\infty} \exp\left(-\Lambda_1(s-t)\right) \left[u_1^{\psi}\psi_s + u_1^{v}v_s + u_1^{n}b^{n}\right] ds. \tag{205}$$

When ψ_t is exponentially decaying, we obtain

$$z_{1,t} = -\left[\Lambda_1 + \theta_{\psi}I\right]^{-1} u_1^{\psi}\psi_t - \left[\Lambda_1 + \theta_{v}I\right]^{-1} u_1^{v}v_t - \Lambda_1^{-1}u_1^{n}b^{n}. \tag{206}$$

Solving backward the differential equation for $z_{2,t}$, we obtain

$$z_{2,t} = \exp\left(\Lambda_2 t\right) z_{2,0} + \int_0^t \exp\left(\Lambda_2(t-s)\right) \left[u_2^{\psi}\psi_s + u_2^{v}v_s + u_2^{n}b^{n}\right] ds. \tag{207}$$

When ψ_t is exponentially decaying, we obtain

$$\begin{aligned} z_{2,t} = {} & \exp\left(\Lambda_2 t\right) z_{2,0} + \left[\Lambda_2 + \theta_{\psi}I\right]^{-1} \left[\exp\left(\Lambda_2 t\right) - \exp\left(-\theta_{\psi}It\right)\right] u_2^{\psi}\psi_0 \\ & + \left[\Lambda_2 + \theta_{v}I\right]^{-1} \left[\exp\left(\Lambda_2 t\right) - \exp\left(-\theta_{v}It\right)\right] u_2^{v}v_0 + \Lambda_2^{-1}\left[\exp\left(\Lambda_2 t\right) - I\right] u_2^{n}b^{n}. \end{aligned} \tag{208}$$

Rotating the system back to the original coordinates, we obtain

$$Z_t = V_1 z_{1,t} + V_2 z_{2,t}. \tag{209}$$

The vector $z_{1,t}$ captures the dependence on the exogenous shocks, while $z_{2,0}$ captures the effect of past promises.

□

Boundary conditions. The next proposition characterizes the boundary conditions

Proposition 13 (Boundary conditions)**.** *The optimality condition for x_0 and for the interest rate evaluated at zero are given:*

$$\frac{\kappa\xi_\pi}{\rho+\lambda} + \xi_x = \int_0^\infty e^{-(\rho+\lambda)t}\left[\alpha(x_t - x^*) + \frac{\kappa\beta}{\rho+\lambda}\pi_t\right] dt, \tag{210}$$

$$\frac{\kappa\lambda\Phi\xi_\pi}{\rho+\lambda+\gamma} - \xi_x = \int_0^\infty e^{-(\rho+\lambda+\gamma)t}\left[\lambda\Upsilon(b_t - b^n) + \frac{\kappa\lambda\Phi\beta\pi_t}{\rho+\lambda+\gamma}\right] dt. \tag{211}$$

Proof. The boundary conditions can be written as

$$\frac{\kappa}{\rho+\lambda}(\mu_{\pi,0}+\xi_\pi) + \xi_x = \int_0^\infty e^{-(\rho+\lambda)t}\left[\alpha(x_t - x^*) + \kappa\mu_{\pi,t}\right] dt = -\mu_{x,0} \tag{212}$$

$$\frac{\kappa\lambda\Phi}{\rho+\lambda+\gamma}(\mu_{\pi,0}+\xi_\pi) - \xi_x = \int_0^\infty e^{-(\rho+\lambda+\gamma)t}\left[\lambda\Upsilon(b_t - b^n) + \kappa\lambda\Phi\mu_{\pi,t}\right] dt = -\mu_{b,0}, \tag{213}$$

Using the fact that $\mu_{\pi,t} = \mu_{\pi,0} + \int_0^t \beta\pi_s ds$, we obtain the two boundary conditions. □

To obtain $\mu_{x,0} = 0$, the value of the co-state in the timeless perspective, the following condition must be satisfied:

$$\xi_x = -\frac{\kappa\xi_\pi}{\rho+\lambda}. \tag{214}$$

This implies that $-\mu_{b,0}$ is given by

$$\left[\frac{\kappa\lambda\Phi}{\rho+\lambda+\gamma} + \frac{\kappa}{\rho+\lambda}\right]\xi_\pi = \int_0^\infty e^{-(\rho+\lambda+\gamma)t}\left[\lambda\Upsilon(b_t - b^n) + \frac{\kappa\lambda\Phi\beta\pi_t}{\rho+\lambda+\gamma}\right] dt. \tag{215}$$

Irrelevance of $\mu_{\pi,0}$. We show next that the system is independent of $\mu_{\pi,0}$, which will allow us to normalize it to zero. Define the adjusted co-states:

$$\tilde{\mu}_{\pi,t} \equiv \mu_{\pi,t} - \mu_{\pi,0}, \qquad \tilde{\mu}_{x,t} \equiv \mu_{x,t} + \frac{\kappa}{\rho+\lambda}\mu_{\pi,0}, \qquad \tilde{\mu}_{b,t} \equiv \mu_{b,t} + \frac{\kappa\lambda\Phi}{\rho+\lambda+\gamma}\mu_{\pi,0}. \tag{216}$$

The law of motion of the adjusted co-states is given by

$$\dot{\tilde{\mu}}_{\pi,t} = \beta\pi_t \tag{217}$$

$$\dot{\tilde{\mu}}_{b,t} - (\rho+\lambda+\gamma)\tilde{\mu}_{b,t} = \lambda\Upsilon(b_t - b^n) + \lambda\kappa\Phi\tilde{\mu}_{\pi,t} \tag{218}$$

$$\dot{\tilde{\mu}}_{x,t} - (\rho+\lambda)\tilde{\mu}_{x,t} = \alpha(x_t - x^*) + \kappa\tilde{\mu}_{\pi,t}. \tag{219}$$

The optimality condition for the interest rate is then given by

$$\tilde{\mu}_{x,t} + \tilde{\mu}_{b,t} = -\frac{\kappa\xi_\pi}{\rho+\lambda}\left(1 + \frac{(\rho+\lambda)\lambda\Phi}{\rho+\lambda+\gamma}\right). \tag{220}$$

The optimality condition for the initial value of the output gap is given by

$$(\rho+\lambda)(\tilde{\mu}_{x,0} + \xi_x) + \kappa(\tilde{\mu}_{\pi,0} + \xi_\pi) = 0. \tag{221}$$

The dynamic system for the equilibrium variables can be equivalently written in terms of the adjusted co-states $(\tilde{\mu}_{\pi,t}, \tilde{\mu}_{x,t}, \tilde{\mu}_{b,t})$. As $\tilde{\mu}_{\pi,0} = 0$, we can assume that $\mu_{\pi,0} = 0$ without loss of generality.

Determination of initial conditions. We have three initial conditions for the system above: $\mu_{x,0} = 0$, $\mu_{\pi,0} = 0$, and the initial value of debt b_0. it remains to write $\mu_{x,0}$ in terms of the remaining variables. The output gap can be written as

$$x_t = V_{x,1}z_{1,t} + V_{x,2}z_{2,t} \tag{222}$$

The average value of $z_{1,t}$ is given by

$$\overline{z}_1 = (\rho+\lambda)\int_0^\infty e^{-(\rho+\lambda)t}z_{1,t}dt = -\left[\Lambda_1 + \theta_\psi I\right]^{-1} u_1^\psi \frac{(\rho+\theta)\psi_0}{\rho+\theta+\theta_\psi} - \left[\Lambda_1 + \theta_v I\right]^{-1} u_1^v \frac{(\rho+\theta)v_t}{\rho+\theta+\theta_v} - \Lambda_1^{-1}u_1^n b^n. \tag{223}$$

The average value of $z_{2,t}$ is given by

$$\overline{z}_2 = (\rho+\theta)\int_0^\infty e^{-(\rho+\theta)t}z_{2,t}dt = \left[I - \frac{1}{\rho+\theta}\Lambda_2\right]^{-1} z_{2,0} + \tilde{z}_2, \tag{224}$$

where

$$\tilde{z}_{2,0} = [\Lambda_2 + \theta_\psi I]^{-1} \left[\left[I - \frac{1}{\rho+\theta}\Lambda_2 \right]^{-1} - \frac{\rho+\theta}{\rho+\theta+\theta_\psi} I \right] u_2^\psi \psi_0$$
$$+ [\Lambda_2 + \theta_v I]^{-1} \left[\left[I - \frac{1}{\rho+\theta}\Lambda_2 \right]^{-1} - \frac{\rho+\theta}{\rho+\theta+\theta_v} I \right] u_2^v v_0 + \Lambda_2^{-1} \left[\left[I - \frac{1}{\rho+\theta}\Lambda_2 \right]^{-1} - I \right] u_2^n b^n \tag{225}$$

The optimality condition for x_0 can be written as follows:

$$0 = \alpha(\overline{x} - x^*) + \frac{\kappa\beta}{\rho+\theta}\overline{\pi}, \tag{226}$$

where $\overline{x} = (\rho+\theta)\int_0^\infty e^{-(\rho+\theta)t} x_t dt$ and $\overline{\pi} = (\rho+\theta)\int_0^\infty e^{-(\rho+\theta)t}\pi_t dt$. The other boundary conditions can be written as

$$0 = V_{\mu_\pi 1} z_{1,0} + V_{\mu_\pi 2} z_{2,0}, \qquad b_0 = V_{b1} z_{1,0} + V_{b2} z_{2,0}. \tag{227}$$

Let's assume that the system has two positive eigenvalues and three non-positive eigenvalues. Then, the $z_{2,0}$ is a three-dimensional vector that can be determined using the initial conditions for $\mu_{x,t}$, $\mu_{\pi,t}$, and b_t:

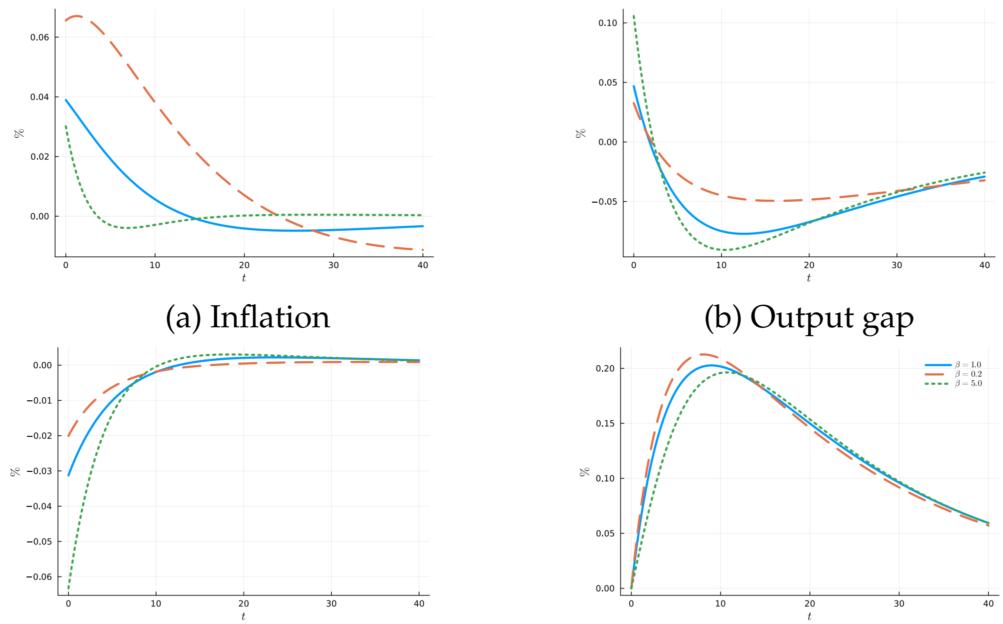
$$\underbrace{\begin{bmatrix} \alpha x^* \\ 0 \\ b_0 \end{bmatrix}}_{d_0} = \underbrace{\begin{bmatrix} \left(\alpha V_{x2} + \frac{\kappa\beta}{\rho+\theta} V_{\pi 2}\right) \left[I - \frac{1}{\rho+\theta}\Lambda_2 \right]^{-1} \\ V_{\mu_\pi 2} \\ V_{b2} \end{bmatrix}}_{D} z_{2,0} + d_1, \tag{228}$$

where

$$d_1 = \begin{bmatrix} \alpha V_{x1} + \frac{\kappa\beta}{\rho+\theta} V_{\pi 1} \\ V_{\mu_\pi 1} \\ V_{b1} \end{bmatrix} z_{1,0} + \begin{bmatrix} \left(\alpha V_{x2} + \frac{\kappa\beta}{\rho+\theta} V_{\pi 2}\right) \tilde{z}_{2,0} \\ 0 \\ 0 \end{bmatrix}. \tag{229}$$

The initial condition for $z_{2,t}$ is then given by

$$z_{2,0} = D^{-1}\left[d_0 - d_1\right]. \tag{230}$$



(a) Inflation (b) Output gap

(c) Real rates (d) Government debt

Figure 12: Optimal policy with a positive debt stabilizer ($\gamma > 0$)

Numerical illustration. We consider next a numerical illustration of the optimal policy with a positive debt stabilizer, $\gamma > 0$. Figure 12 shows the equilibrium dynamics under the optimal policy. In the short-run, the behavior of output, inflation, debt, and real rates mimic the behavior observed in the case $\gamma = 0$ (see Figure 5). Output gap and inflation are initially positive, and debt increases over time. Moreover, a hawkish central bank initially has lower levels of inflation, real rates, and debt, with a higher output gap. Eventually, the debt stabilizer starts to bring government debt back to its steady-state level, causing the output gap to also revert to the steady state. Therefore, the version of the model with a debt stabilizer generates similar results to the case discussed in Section 4, except that variables eventually revert to the steady state.

D.2 Optimal policy with discretion

Optimal policy with finite planning horizon. Consider a planning with a finite planning horizon. We assume that a new planner takes over with a Poisson intensity $\overline{\lambda}$. The current planner takes the actions of future decision-makers as given. This ensures that the Euler equation is satisfied even after a new planner takes over. Let $\mathcal{P}_t(b_t)$ denote the value of a planner at period t with a given level of government debt, and $\mathcal{P}^*(b^*)$ denotes the value of a planner in the inflationary-finance phase. The planner's objective is given by

$$\mathcal{P}_0(b_0) = \mathbb{E}_0 \left[-\frac{1}{2} \int_0^\tau e^{-\rho t} \left[\alpha x_t^2 + \beta \pi_t^2 \right] dt + e^{-\rho \tau} \tilde{P}_\tau(b_\tau) \right], \tag{231}$$

where τ denotes the random time the economy switches to either the inflationary-finance phase, so the planner's value becomes $\tilde{P}_\tau(b_\tau) = \mathcal{P}^*(b_\tau)$, or a new planner's take over, so the planner's value is $\tilde{\mathcal{P}}_\tau(b_\tau) = \mathcal{P}_\tau(b_\tau)$. The density of τ is given by $(\lambda + \overline{\lambda})e^{-(\lambda+\overline{\lambda})t}$ and, conditional on switching, the probability of moving to the inflationary-finance phase is $\frac{\lambda}{\lambda+\overline{\lambda}}$, while the probability of a new planner taking over is given by $\frac{\overline{\lambda}}{\lambda+\overline{\lambda}}$ (see e.g. Cox and Miller (1977) for a derivation).

Using the density of τ, we can then express $\mathcal{P}_0(x_0, b_0)$ as follows:

$$\mathcal{P}_0(b_0) = -\frac{1}{2} \int_0^\infty e^{-(\rho+\lambda+\overline{\lambda})t} \left[\alpha x_t^2 + \beta \pi_t^2 + \lambda \Upsilon (b_t - b^n)^2 \right] dt + \int_0^\infty e^{-(\rho+\lambda+\overline{\lambda})t} \overline{\lambda} \mathcal{P}_t(b_t) dt. \tag{232}$$

The planner's problem consists of maximizing the objective above subject to the constraints

$$\dot{\pi}_t = (\rho + \lambda)\pi_t - \kappa x_t - \lambda \kappa \Phi (b_t - b^n), \qquad \dot{b}_t = r_t - \rho + \psi_t, \qquad \dot{x}_t = r_t - \rho.$$

We also include a penalty on π_0 and x_0, as in the case with full commitment.

Optimality conditions The optimality conditions are given by

$$\dot{\mu}_{\pi,t} - (\rho + \lambda + \overline{\lambda})\mu_{\pi,t} = \beta \pi_t - (\rho + \lambda)\mu_{\pi,t} \tag{233}$$

$$\dot{\mu}_{b,t} - (\rho + \lambda + \overline{\lambda})\mu_{b,t} = \lambda \Upsilon (b_t - b^n) - \overline{\lambda} \mathcal{P}_{b,t}(b_t) + \lambda \kappa \Phi \mu_{\pi,t} \tag{234}$$

$$\dot{\mu}_{x,t} - (\rho + \lambda + \overline{\lambda})\mu_{x,t} = \alpha x_t + \kappa \mu_{\pi,t}, \tag{235}$$

where $\mathcal{P}_{b,t}(b_t)$ denotes the partial derivative of $\mathcal{P}_t(b_t)$ with respect to debt.

The optimality condition for the interest rate is given by

$$\mu_{x,t} + \mu_{b,t} = -\xi, \tag{236}$$

where $\xi \equiv \frac{\kappa(1+\lambda\Phi)}{\rho+\theta} \xi_\pi$.

The optimality condition for x_0 is given by

$$\mu_{x,0} = 0. \tag{237}$$

Standard envelope arguments imply that

$$\mu_{b,t} = \mathcal{P}_{b,t}(b_t). \tag{238}$$

The discretion limit. Consider the limit as $\overline{\lambda} \to \infty$, so each planner has commitment only over an infinitesimal amount of time. In the limit, the co-states on π_t and x_t are given by

$$\mu_{\pi,t} = 0, \qquad\qquad \mu_{x,t} = 0. \tag{239}$$

Integrating the expression for $\mu_{x,t}$ forward, we obtain

$$\mu_{x,t} = -\int_t^\infty e^{-(\rho+\lambda+\overline{\lambda})(s-t)}\left[\alpha x_s + \kappa\mu_{\pi,s}\right] ds \Rightarrow \lim_{\overline{\lambda}\to\infty} \overline{\lambda}\mu_{x,t} = -\alpha x_t, \tag{240}$$

using the fact that $\lim_{\overline{\lambda}\to\infty} \mu_{\pi,t} = 0$. Hence, from the optimality condition for x_0, we obtain $x_0 = 0$. Differentiating the optimality condition for the interest rate with respect to time, we obtain

$$(\rho + \lambda + \overline{\lambda})\xi = \alpha x_t + \lambda\Upsilon(b_t - b^n) - \overline{\lambda}\mu_{b,t} + \kappa(1 + \lambda\Phi)\mu_{\pi,t}, \tag{241}$$

where we used the envelope condition for b_t

Given $\mu_{b,t} = -\xi - \mu_{x,t}$, and combining the previous two expressions, we obtain

$$(\rho + \lambda)\xi = \lambda\Upsilon(b_t - b^n). \tag{242}$$

Therefore, the interest rate is given by

$$r_t - \rho = -\psi_t. \tag{243}$$

The case of partial commitment. In the case of discretion, planner's do not take into account promises made by prior planners. Hence, each planner sets a new value of x_t as they take control, and promise that output gap will evolve according to the Euler equation in the future. As we reduce the planning horizon to zero, each planner chooses the value of the output gap regardless of the path of interest rates. We consider next the case of partial commitment, where the planner has to respect past promises made about the output gap. In this case, the output gap must satisfy the Euler equation at every point in time, except at $t = 0$ when news about the shock arrives.

In this case, the planner's objective is given by

$$\mathcal{P}_0(x_0, b_0) = -\frac{1}{2}\int_0^\infty e^{-(\rho+\lambda+\bar{\lambda})t}\left[\alpha x_t^2 + \beta \pi_t^2 + \lambda\Upsilon(b_t - b^n)^2\right]dt + \int_0^\infty e^{-(\rho+\lambda+\bar{\lambda})t}\bar{\lambda}\mathcal{P}_t(x_t, b_t)dt, \tag{244}$$

and we impose a penalty on π_0, but not on x_0, as the initial output gap is not free.

The optmality conditions are now given by

$$\dot{\mu}_{\pi,t} - (\rho+\lambda+\bar{\lambda})\mu_{\pi,t} = \beta\pi_t - (\rho+\lambda)\mu_{\pi,t} \tag{245}$$

$$\dot{\mu}_{b,t} - (\rho+\lambda+\bar{\lambda})\mu_{b,t} = \lambda\Upsilon(b_t - b^n) - \bar{\lambda}\mathcal{P}_{b,t}(x_t, b_t) + \lambda\kappa\Phi\mu_{\pi,t} \tag{246}$$

$$\dot{\mu}_{x,t} - (\rho+\lambda+\bar{\lambda})\mu_{x,t} = \alpha x_t + \kappa\mu_{\pi,t} - \bar{\lambda}\mathcal{P}_{x,t}(x_t, b_t). \tag{247}$$

The optimality condition for the interest rate is the same as under discretion, and the envelope conditions for output gap and debt are given by

$$\mu_{x,t} = \mathcal{P}_{x,t}(x_t, b_t), \qquad \mu_{b,t} = \mathcal{P}_{b,t}(x_t, b_t). \tag{248}$$

Differentiating the optimality condition for the interest rate with respect to time, we obtain

$$(\rho+\lambda+\bar{\lambda})\xi = \alpha x_t + \lambda\Upsilon(b_t - b^n) - \bar{\lambda}(\mu_{b,t} + \mu_{x,t}) + \kappa(1+\lambda\Phi)\mu_{\pi,t}, \tag{249}$$

where we used the envelope conditions.

Taking the limit as $\bar{\lambda} \to \infty$, we obtain

$$(\rho+\lambda)\xi = \alpha x_t + \lambda\Upsilon(b_t - b^n) \Rightarrow r_t - \rho = -\frac{\lambda\Upsilon}{\lambda\Upsilon + \alpha}\psi_t. \tag{250}$$

In period $t = 0$, the planner is allowed to choose x_0, which must satisfy the condition:

$$\mu_{x,0} = 0 \Rightarrow 0 = \int_0^\infty e^{-(\rho+\lambda)t}\alpha x_t dt = 0, \tag{251}$$

where we used the fact that $\mu_{\pi,t} = 0$ as $\bar{\lambda} \to \infty$. Therefore, optimal policy with partial commitment coincides with the optimal policy with commitment for a dovish central bank, that is, when $\beta = 0$.

Taking the limit of a discrete-time economy. Welfare is measured by

$$\sum_{t=0}^{\infty}\left(e^{-\rho\Delta t}\right)^t\left[\alpha x_t^2 + \beta\pi_t^2\right]\Delta t. \tag{252}$$

The NKPC is given by

$$\pi_t = e^{-\rho\Delta t}\mathbb{E}_t\left[\pi_{t+\Delta t}\right] + (\kappa x_t + u_t)\,\Delta t. \tag{253}$$

Under discretion, the planner's problem is given by

$$\max_{x_t,\pi_t} -\frac{1}{2}\left[\alpha x_t^2 + \beta\pi_t^2\right]\Delta t, \tag{254}$$

subject to

$$\pi_t = e^{-\rho\Delta t}\mathbb{E}_t\left[\pi_{t+\Delta t}\right] + (\kappa x_t + u_t)\,\Delta t, \tag{255}$$

taking as given $\mathbb{E}_t\pi_{t+\Delta t}$.

The optimal solution is given by

$$x_t = -\frac{\kappa\beta}{\alpha}\pi_t\Delta t. \tag{256}$$

D.3 Optimal policy under the timeless perspective

The dynamics under the optimal policy are characterized by the following conditions:

$$\dot{\pi}_t = (\rho+\lambda)\pi_t - \kappa x_t - \lambda\kappa\Phi(b_t - b^n) \tag{257}$$

$$\dot{b}_t = r_t - \rho - \gamma(b_t - b^n) + \psi_t \tag{258}$$

$$\dot{x}_t = r_t - \rho + \theta_h x_t - \theta_h^*(b_t - b^n) \tag{259}$$

$$\dot{\mu}_{\pi,t} = \beta\pi_t \tag{260}$$

$$\dot{\mu}_{b,t} = (\rho+\lambda)\mu_{b,t} + \lambda\Upsilon(b_t - b^n) + \kappa\lambda\Phi\mu_{\pi,t} \tag{261}$$

$$\dot{\mu}_{x,t} = (\rho+\lambda)\mu_{x,t} + \alpha x_t + \kappa\mu_{\pi,t}, \tag{262}$$

where the real rate is given by

$$r_t - \rho = -\beta\frac{\kappa(1+\lambda\Phi)}{\lambda\Upsilon+\alpha}\pi_t - \frac{\lambda\Upsilon}{\lambda\Upsilon+\alpha}\psi_t, \tag{263}$$

given the initial value of debt, b_0, and the boundary conditions $\mu_{x,0} = \mu_{\pi,0} = 0$.

Consider the case without a fiscal shock, $\psi_t = 0$, and denote the co-states in this case with no shocks by $\mu_{x,t}^{ns}$ and $\mu_{\pi,t}^{ns}$. The optimal policy under the timeless perspective corresponds to the solution to the system above when we replace the initial conditions by the long-run values of these multipliers: $\mu_{x,0} = \lim_{t\to\infty}\mu_{x,t}^{ns}$ and $\mu_{\pi,0} = \lim_{t\to\infty}\mu_{\pi,t}^{ns}$ (see Giannoni and Woodford (2017) for a discussion in the context a general model). This is equivalent to the problem of a planner who started its planning in a distant past, so the multipliers had time to converge to their long-run values.

Even without shocks, the limits $\lim_{t\to\infty}\mu^{ns}_{x,t}$ and $\lim_{t\to\infty}\mu^{ns}_{\pi,t}$ will not be equal to zero, provided that $b_0 \neq b^n$. However, in the case $b_0 = b^n$, the solution to the system above in the absence of shocks is simply $\pi_t = x_t = b_t = \mu_{\pi,t} = \mu_{x,t} = \mu_{b,t} = 0$. Hence, we have that $\lim_{t\to\infty}\mu^{ns}_{x,t} = 0$ and $\lim_{t\to\infty}\mu^{ns}_{\pi,t} = 0$, so the boundary conditions for the problem under the timeless perspective coincide with the time-zero commitment solution.

E. Historical Shock Decomposition and Taylor Counterfactual

For the historical shock decomposition, we construct the discrete version analogue of the model in Section 2. In addition to the fiscal shock already analyzed, we incorporate three additional shocks into the model. First, we include a monetary shock that allows the monetary authority to deviate from the prescriptions of the interest rate rule. We calibrate the Taylor coefficient on the lower end of its plausible range to minimize the importance of these shocks in the conclusion. Second, we include a standard-cost push shock, to capture movements in the inflation rate that are orthogonal to the evolution of debt. It represents sectoral reallocations and supply bottlenecks, as experienced during the pandemic. This shock is identified through the Phillips curve implied by the model. Third, we add a "bond-valuation" shock to the return on government debt. This shock is meant to capture the effect on the one-period holding return on government debt of revaluation effects, risk premium movements, changes in the maturity structure, and other unmodeled dimensions that affect the government's budget constraint. The shock is directly extracted from the government debt path, given the fiscal rule, the primary deficits and the path of nominal rates. Since shocks are inferred directly from the data series, the Kalman filter optimizes the initial conditions to best fit the shock decomposition—the initial conditions' quantitative contribution is minor.

E.1 The Model

The model can be characterized by the following equations:

1. IS curve

$$x_t = x_{t+1} - \sigma\left(i_t - \pi_{t+1} - \rho\right)$$

2. New Keynesian Phillips Curve (NKPC)

$$\pi_t = \beta\left[\left(1-\lambda_f\right)\pi_{t+1} + \lambda_f \kappa \Phi b_{t+1}\right] + \kappa x_t + \mu_t$$

3. Interest rate rule

$$i_t = \rho + \rho_i(i_{t-1} - \rho) + \phi_\pi \pi_t + u_t^m$$

4. Government debt evolution

$$b_t = \left(1-\gamma\right) b_{t-1} + r_t b_n + u_t^f$$

5. Primary surplus

$$\psi_t = u_t^f - (\rho + \gamma)\, b_{t-1}$$

6. One-period holding return on government debt

$$r_t = (i_t - \pi_t - \rho) + u_t^r,$$

where μ_t denotes the cost-push shock, u_t^m denotes the monetary shock, u_t^f denotes the fiscal shock, and u_t^r denotes the bond-valuation shock. Note that in the data we do not observe the fiscal shock directly but through its effect on the primary surplus. Thus, we denote ψ_t the primary surplus, which includes the fiscal shock and the automatic adjustment of transfers to changes in the stock of debt. Finally, we assume that all disturbances follow an AR(1) process.

E.2 The Data

For the exercise, we take the dynamics of inflation, the primary surplus, the stock of debt, and the nominal rate as observables. The inflation rate is measured as the growth rate of the GDP deflator (NIPA Table 1.1.7 line 1). The primary surplus is the difference between government receipts (NIPA Table 3.1 line 1) and total expenditures (NIPA Table 3.1 line 20) net of interest payments (NIPA Table 3.1 line 12 - NIPA Table 3.1 line 27), divided by nominal GDP (NIPA Table 1.1.5 line 1). The stock of debt is the market value of government debt held by the private sector from Hall, Payne and Sargent (2018) plus reserves of depository institutions (Fred TOTRESNS). Finally, the nominal rate is the federal funds effective rate (Fred DFF).

Since we look at primary surplus over GDP in the data, we nee to adjust our fiscal shock to include movements in GDP. To account for this, we combine the growth component of GDP into the purely fiscal component. Thus, our fiscal shock is a composite of an exogenous transfer shock and the contribution of growth to the debt-to-GDP ratio.

Formally, we have that the government's budget constraint is given by

$$\dot{B}_t = (i_t - \pi_t)\, B_t - (\rho + \gamma)\, B_t + \Psi_t.$$

Dividing by quarterly output, we get

$$\frac{\dot{B}_t}{X_t} - \frac{B_t}{X_t}\frac{\dot{X}_t}{X_t} + \frac{B_t}{X_t}\frac{\dot{X}_t}{X_t} = (i_t - \pi_t)\,\frac{B_t}{X_t} - (\rho + \gamma)\,\frac{B_t}{X_t} + \frac{\Psi_t}{X_t}.$$

Let $b_t \equiv \frac{B_t}{X_t}$ and $g_t \equiv \frac{\dot{X}_t}{X_t}$, to get

$$\dot{b}_t = (i_t - \pi_t)\, b_t - (\rho + \gamma)\, b_t + \underbrace{\left(\frac{\Psi_t}{X_t} - b_t g_t\right)}_{\equiv \psi_t}.$$

Let $\hat{b}_t \equiv b_t - b_n$ and $\hat{\psi}_t \equiv \psi_t - \overline{\psi}$. We assume that γ is such that $\gamma = \frac{\overline{\psi}}{b_n}$. Then

$$\dot{\hat{b}}_t = \underbrace{(i_t - \pi_t - \rho)\, \hat{b}_t}_{=\mathcal{O}(||\psi_t||^2)} + (i_t - \pi_t - \rho)\, b_n - \gamma \hat{b}_t + \hat{\psi}_t,$$

or

$$\dot{\hat{b}}_t \approx (i_t - \pi_t - \rho)\, b_n - \gamma \hat{b}_t + \hat{\psi}_t.$$

E.3 Connection to the Textbook Model

The system of equations characterizing the equilibrium of our economy is isomorphic to the system in the textbook model, with the difference that the expectations include the possibility of a monetary-fiscal reform. To see this, assume that the economy starts in the fiscal-expansion phase. The system of equations characterizing the equilibrium is given by

$$\begin{aligned} x_t^I &= E_t^h[x_{t+\Delta t}] - (i_t - E_t^h[\pi_{t+\Delta t}] - \rho)\Delta t \\ \pi_t^I &= \beta E_t^f[\pi_{t+\Delta t}] + \kappa x_t^I \Delta t \\ i_t &= \rho + \phi_\pi \pi_t^I + u_t \\ b_t &= b_{t-\Delta t} + (i_{t-\Delta t} - \pi_t^I - \rho) b^n - \gamma b_{t-\Delta t}\Delta t + \psi_t, \end{aligned}$$

where $\{x_t^I, \pi_t^I\}$ denote the output gap and inflation in the fiscal-expansion phase, respectively, $\{E_t^h, E_t^f\}$ denote the households' and firms' expectation operator, respectively, $\{x_{t+\Delta t}, \pi_{t+\Delta t}\}$ are random variables representing the output gap and inflation in period $t + \Delta t$, and the time period is of length Δt.

If $\lambda_h = \lambda_f = 0$, then $x_{t+\Delta t} = x_{t+\Delta t}^I$ and $\pi_{t+\Delta t} = \pi_{t+\Delta t}^I$. Assuming that $\Delta t = 1$, the system above becomes the textbook system of difference equations. In contrast, with $\lambda_h, \lambda_f > 0$ we have

$$\begin{aligned} E_t^h[x_{t+\Delta t}] &= (1 - \lambda_h \Delta t) x_{t+\Delta t}^I + \lambda_h \Delta t x_{t+\Delta t}^{II}, \\ E_t^j[\pi_{t+\Delta t}] &= (1 - \lambda_j \Delta t) \pi_{t+\Delta t}^I + \lambda_j \Delta t \pi_{t+\Delta t}^{II}, \quad \text{for } j \in \{h, f\}, \end{aligned}$$

where $\{x^{II}_{t+\Delta t}, \pi^{II}_{t+\Delta t}\}$ denote the output gap and inflation in Phase II, respectively.[31] Removing the superscript I, and using that $x^{II}_{t+\Delta t} = b_{t+\Delta t} - b^n$ and $\pi^{II}_{t+\Delta t} = \kappa\Phi(b_{t+\Delta t} - b^n)$, the system becomes

$$
\begin{aligned}
x_t &= (1-\lambda_h\Delta t)x_{t+\Delta t} - (i_t - ((1-\lambda_h\Delta t)\pi_{t+\Delta t} + \lambda_h\Delta t\kappa\Phi(b_{t+\Delta t} - b^n)) - \rho)\Delta t + \lambda_h\Delta t(b_{t+\Delta t} - b^n) \\
\pi_t &= \beta(1-\lambda_f\Delta t)\pi_{t+\Delta t} + \left[\kappa x_t + \beta\lambda_f\kappa\Phi(b_{t+\Delta t} - b^n)\right]\Delta t \\
i_t &= \rho + \phi_\pi\pi_t + u_t \\
b_t &= b_{t-\Delta t} + \left[(i_{t-\Delta t} - \pi_t - \rho)b^n - \gamma b_{t-\Delta t} + \psi_t\right]\Delta t,
\end{aligned}
$$

In the limit as $\Delta t \to 0$, it simplifies to

$$
\begin{aligned}
\dot{x}_t &= i_t - \pi_t - r^n_t + \lambda_h x_t \\
\dot{\pi}_t &= (\rho + \lambda_f)\pi_t - \kappa x_t - \mu_t \\
i_t &= \rho + \phi_\pi\pi_t + u_t \\
\dot{b}_t &= (i_t - \pi_t - \rho)b^n - \gamma b_t + \psi_t,
\end{aligned}
$$

where $r^n_t \equiv \rho + \lambda_h(b_t - b^n)$ and $\mu_t \equiv \beta\lambda_f\kappa\Phi(b_t - b^n)$, and we used that $\beta = \frac{1}{1+\rho\Delta t}$. These expressions make it clear that, through the expectation of a reform, the system of equations characterizing equilibrium changes relative to the textbook version in the following ways: $i)$ the Euler equation features "discounting," $ii)$ the natural rate is endogenous and depends on the level of debt, $iii)$ the NKPC also features "discounting," $iv)$ the NKPC features a cost-push shock that depends on the stock of debt.

[31]Note that we are assuming, for simplicity, perfect foresight conditional on the regime.