Liquidity and Investment in General Equilibrium*

Nicolas Caramp — UC Davis

Julian Kozlowski — FRB St Louis

Keisuke Teeple — U Waterloo

September 15, 2022

Abstract

This paper studies the implications of trading frictions in financial markets for firms' investment and dividend choices and their aggregate consequences. When equity shares trade in frictional asset markets, the firm's problem is time-inconsistent, and it is as if it faces quasi-hyperbolic discounting. The transmission of trading frictions to the real economy crucially depends on the firms' ability to commit. In a calibrated economy without commitment, larger trading frictions imply lower capital and production. In contrast, if firms can commit, trading frictions affect asset prices but have no effect on capital and production. Our findings rationalize several empirical regularities on liquidity and investment.

JEL Classifications: E44, G32, G12.

Keywords: Liquidity, Investment, Present bias.

*The views do not necessarily reflect official positions of the Federal Reserve Bank of St. Louis, the Federal Reserve System, or the Board of Governors. This work employed resources from the BigTex High Performance Computing Group at the Federal Reserve Bank of Dallas.

1 Introduction

Financial assets trade in frictional markets. This has important consequences for both asset pricing (Amihud et al., 2005) and household behavior (Kaplan et al., 2014). However, little is known about the transmission of trading frictions to the firms' investment plans and the aggregate economy. This paper studies the implications of trading frictions in financial markets for aggregate investment and asset prices in general equilibrium. Our findings help rationalize several empirical regularities concerning the relationship between stock market liquidity and investment in the cross-section and over the business cycle (Amihud and Levi, 2022; Naes et al., 2011), the differential behavior of public and private firms (Asker et al., 2015), and the short-termism observed in stock market-listed firms (Graham et al., 2005).

We present a theory of investment in an incomplete markets production economy in which households trade firms' shares subject to transaction costs. Because of financial frictions, agents disagree on the valuation of the firm and, crucially, on the discount factor used to value cash flows. Our main result is that trading frictions generate a disagreement between owners' discount factors that leads to a problem for the firms that is *time-inconsistent*: the optimal investment plan at any point in time is suboptimal at future dates. Moreover, we show that the firms' problem is equivalent to one in which there are no financial frictions but there is a representative household with preferences that exhibit *quasi-hyperbolic discounting*. Notably, we obtain this result from the frictions in financial markets rather than behavioral assumptions. In a calibrated economy, we find that the transmission of trading frictions to the real economy is highly sensitive to the firms' ability to commit to an investment plan. If firms cannot commit, trading frictions can have significant adverse effects on investment and output due to present bias. In contrast, if firms can commit, trading frictions affect asset prices but not the aggregate level of capital.

We build a production economy with heterogeneous agents in which households trade assets to smooth consumption and there is no aggregate risk. Our baseline model is an Aiyagari (1994) economy with households that face income risk and have access to two financial markets: a market for the stock of the representative firm and a market for risk-free one-period bonds. In the spirit of Aiyagari and Gertler (1991), we assume that (i) stock trading is subject to transaction costs while (ii) the bond market is frictionless, but agents can borrow only up to a limit. Crucially, we assume that the firm makes the investment decisions. In the presence of transaction costs and borrowing constraints, households' (expected) intertemporal marginal rates of substitution (IMRS) do not equalize. As a consequence, owners of the firm disagree on the optimal investment plan.[1] There is a long tradition in microeconomics that studies ways to specify the firm's objective in this scenario (see Diamond, 1967; Dreze, 1974; Grossman and Hart, 1979). In this paper, we follow Grossman and Hart (1979) and assume that the firm maximizes the weighted valuation of owners, where the weights are given by the initial stock holdings.

More importantly, the absence of IMRS equalization leads to the paper's main result: the firm's problem is *time-inconsistent*. To understand why, consider the tradeoffs involved in the design of the firm's investment plan. Investment is an intertemporal decision: agents give up consumption in period t (through the distribution of dividends) for a higher level of production in $t+1$. If the IMRS of owners and buyers are not equal, the factor they apply to discount dividends will differ. This implies that owners and buyers will disagree on the optimal investment plan for the firm. In particular, if owners are more impatient than buyers (which they will typically be in our economy), owners will favor a higher level of dividends and a lower level of investment than buyers. In contrast, when planning investment multiple periods into the future, owners would like to commit to a high level of investment to satisfy the buyers' preferences and increase the firm's market value. However, if the decisions are revised in $t+1$ by the new set of owners, they will be tempted to again lower the investment level in t+1 and promise a higher level of investment starting in $t+2$. Thus, the problem of the firm is time-inconsistent. Notably, in the absence of transaction costs (e.g., as in Aiyagari, 1994), owners' and buyers' (expected) IMRS are equalized, and the problem of the firm is time-consistent, as all owners discount payoffs using the risk-free rate. Finally, we show that under plausible assumptions, the firm's problem is equivalent to one in which the firm exhibits quasi-hyperbolic discounting.

We then calibrate the model and study its quantitative properties in general equilibrium. We find that present bias is the empirically relevant case. When there are no trading frictions, the model is similar to the one-asset economy in Aiyagari (1994).

[1]If households' expected IMRS equalize, they all agree on the discount factor for the firms' cash flows and the optimal investment plan. In particular, it is optimal for the firm to choose the plan that maximizes its market valuation.

In this case, the steady-state level of capital is higher than in a complete markets economy due to precautionary savings. With trading frictions and commitment, we find large effects on asset prices but minor consequences for aggregate capital. On the one hand, trading frictions depress asset prices, implying a lower steady-state level of capital. On the other hand, there is a higher precautionary motive for saving, implying a higher steady-state level of capital. Quantitatively, these two forces are similar, and, as a result, capital does not change significantly. However, without commitment, there is a third force at play: present bias. Quantitatively, this force strongly favors more discounting and, as a result, we can obtain a lower level of capital than in the complete markets economy, contrary to the overaccumulation result in Aiyagari (1994). This result illustrates that the assumptions about trading frictions and the firm's problem are essential for understanding both aggregate quantities and asset prices.

Finally, we extend the main framework in several dimensions. First, we allow the firm to borrow in illiquid corporate bonds markets. We find that the problem of the firm is still time-inconsistent. Corporate bonds can affect the firm's financial structure but have no impact on investment decisions, as these are made by the stockholders. Even if corporate bonds are more illiquid than stocks, a firm without commitment may issue bonds due to present bias. This result provides a reason for corporate borrowing that does not rely on the tax advantage of debt. Second, we study how the demand and supply of liquid assets affect aggregate capital. On the demand side, we consider an increase in the idiosyncratic uncertainty, which raises the precautionary savings motive. When firms can commit, the increase in precautionary savings reduces the interest rate and increases aggregate capital. When firms cannot commit, more uncertainty also implies a more severe time-inconsistency problem, which generates a reduction of aggregate capital. On the supply side, we incorporate liquid government bonds into the model. Once again, we find that the aggregate effects depend on firms' commitment. When firms can commit, an increase in the supply of government bonds leads to a lower level of capital because the interest rate increases. When firms cannot commit, however, we get a higher level of capital because there is a less severe time-inconsistency problem.

Notably, our theory is consistent with recent empirical findings that connect stock market liquidity and firms' investment. First, in the cross-section, Amihud and Levi (2022) compare liquid and illiquid firms and conclude that illiquid firms invest less than liquid ones. We extend the model to have both liquid and illiquid firms and show that illiquid firms invest less, consistent with the empirical fact. Second, Naes et al. (2011) studies how the cross-section of firms evolves along the business cycle. During recessions, markets become less liquid, and there is a "*flight to liquidity*": investors shift their portfolios into liquid assets. We simulate a recession in the cross-sectional model and find that investors also shift their portfolios into liquid assets, as in the data. Third, Asker et al. (2015) find that comparable public firms invest substantially less than private firms. We incorporate private firms into the benchmark model and find that their problem is time-consistent and that they invest more than public ones. Fourth, the theory provides an alternative, rational explanation for *short-termism*, that is, the observation that public firms are willing to distort their investment plans to meet short-term profit targets (see Graham et al., 2005). Our theory rationalizes this as the outcome of conflicting objectives among different owners when markets are frictional.

Literature. The paper is related to several strands of the literature. First, it is related to the literature on the determinants of firms' investment decisions, pioneered by Brainard and Tobin (1968) and Tobin (1969). Tobin's q, in its modern version of *average* q (see Abel, 1983), is the cornerstone of the neoclassical investment theory. We contribute to this literature by studying the determinants of investment in a general equilibrium economy with trading frictions. A key result of our paper is that trading frictions lead to a discount factor that makes the firm's problem time-inconsistent.

Second, the paper is related to the literature on market liquidity and asset prices. Amihud and Mendelson (1986) is an early paper studying how liquidity affects the pricing of an exogenous stream of dividends (for a review of the literature, see Amihud et al., 2005). Aiyagari and Gertler (1991) incorporate trading frictions in a general equilibrium model. This literature considers endowment economies and focuses on the asset-pricing implications, abstracting from issues such as how trading frictions affect the firm's problem and the implications for the real economy. Recent papers in macroeconomics incorporate asset liquidity frictions into general equilibrium production economies (e.g., Kaplan and Violante, 2014; Kaplan et al., 2018; Cui and Radde, 2020; Jeenas and Lagos, 2022).[2]. However, this literature typically assumes

[2]Relatedly, see Jermann (1998); Cochrane (2008) for production-based asset pricing models.

that households directly own the capital of the economy (so that the problem of the firm is static) or that firms are owned by a financial intermediation sector, so households do not have direct holdings of firms' stocks. Under either of these assumptions, liquidity frictions do not directly affect the firms' problem, except for their impact on the equilibrium real interest rate. We show that allowing households to hold stocks directly has significant consequences for the firms' problem and the effect of liquidity on the macroeconomy.

The paper is also related to the literature that studies the problem of the firm in economies with incomplete markets (see Diamond, 1967; Dreze, 1974; Grossman and Hart, 1979; DeMarzo, 1993).[3] These papers focus on aggregate risk and typically study two-period models.Our paper considers an infinitely lived economy with idiosyncratic risk and trading frictions but no aggregate risk. In a steady-state equilibrium, we show that the firms' problem can be expressed as featuring quasi-hyperbolic discounting. Notably, we do not assume quasi-hyperbolic discounting as a behavioral phenomenon (as in Kang and Ye, 2019), but it arises endogenously from trading frictions in asset markets (see also Amador, 2012; Azzimonti, 2011). Recent papers have also encountered the problem of the appropriate discount factor of the firm in incomplete market economies. For example, Favilukis (2013) and Favilukis et al. (2017) similarly use a portfolio-weighted average of the agents' IMRS as the firms' discount factor. However, they do not consider the potential time-inconsistency problem. Relatedly, Espino et al. (2018) and Bisin et al. (2022) study the insurance properties of firm ownership. Espino et al. (2018) consider privately owned firms and focus on the solution of the centralized allocation with a finite number of agents and private information. Bisin et al. (2022) study how investors' hedging demand shapes the firms' capital structure. Instead, our paper focuses on how trading frictions affect firms' investment decisions.

Closest to our paper is Carceles-Poveda and Coen-Pirani (2010), who study the implications of household and firm ownership of capital in heterogeneous agents models. Their focus is on cases that lead to an equivalence result: whether the households or the firms own the capital is irrelevant for equilibrium under the appropriate choice of the firms' discount factor. We focus instead on cases where the distinction matters. Transaction costs are crucial for our analysis; absent transaction costs, we recover the

[3]See Magill and Quinzii (2002) for a review of the literature.

equivalence result.[4]

The paper is organized as follows. Section 2 presents the household and firm problems. Section 3 considers a simple three-period example to understand the sources of time-inconsistency. Section 4 defines the equilibrium, and Section 5 describes the solution to the quasi-hyperbolic firm problem. Section 6 presents the numerical evaluation. Section 7 connects the findings to empirical observations. Finally, Section 8 concludes.

2 The Model

We study an Aiyagari economy augmented to incorporate transaction costs on financial assets. Time is discrete and denoted by $t = 0, 1, 2, \ldots$ Households face labor income risk, and markets are incomplete. In particular, households have access to only two financial markets: a market for the stock of the representative firm and a market for a risk-free bond. The stock market is subject to transaction costs which we model as a wasteful use of resources.The market for bonds is frictionless but agents can borrow (i.e., negative bond holdings) up to a limit.[5] Moreover, there is a representative firm that combines labor and capital to produce the final consumption good. There is no aggregate risk.

Households. The economy is populated by a measure one of households, indexed by $j \in [0, 1]$. Households are subject to idiosyncratic labor shocks that determine the number of hours they can sell in the labor market, which we label as their *employment status* and denote by h. We assume that h is drawn from a finite set $\mathbb{H} = \{h_1, h_2, \ldots, h_S\}$ with associated transition density $dF(h_{s'}|h_s)$. Households' preferences can be represented by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t\right), \tag{1}$$

where $\mathbb{E}_t$ represents the expectation operator conditional on the information set at period t, β is the households' subjective discount factor, c_t denotes consumption in period t, and $u(\cdot)$ is a continuously differentiable and strictly concave function that

[4]See also Carceles-Poveda and Coen-Pirani (2009).

[5]We think of bonds as bank deposits and loans rather than assets that trade in frictional markets like corporate bonds. See Aiyagari and Gertler (1991).

satisfies Inada conditions.

Markets in this economy are incomplete. Households can trade two classes of financial assets: the stock of the representative firm and a one-period risk-free bond. Trade in the stock market entails a transaction cost. Let q_t denote the price of a share, and Δ_t^- and Δ_t^+ denote the shares sold and bought by a household. We assume that the transaction cost incurred in a sale is $\frac{\phi}{2}(\Delta_t^-)^2 q_t$. Quadratic transaction costs simplify the solution of the agents' problem, as it avoids the existence of inaction regions (see Heaton and Lucas, 1996).[6] Moreover, short-selling is not allowed in this market. In contrast, trading bonds is frictionless but households face a borrowing limit $\overline{b}$. Thus, the constraints faced by a household in period t are

$$c_t + q_t\Delta_t^+ + \frac{b_{t+1}}{1+r_t} \leq w_t h_t + d_t\theta_t + \left(\Delta_t^- - \frac{\phi}{2}(\Delta_t^-)^2\right) q_t + b_t, \tag{2}$$

$$\theta_{t+1} = \theta_t + \Delta_t^+ - \Delta_t^-, \tag{3}$$

$$\Delta_t^+ \geq 0, \quad \theta_t \geq \Delta_t^- \geq 0, \tag{4}$$

$$b_{t+1} \geq \overline{b}, \tag{5}$$

where b_t denotes the holdings of one-period risk-free bonds, r_t is the real interest rate, w_t is the real wage, θ_t denotes the household's holdings of stock at the beginning of period t, and d_t denotes the dividends distributed by the representative firm. Equation (2) is a standard budget constraint. The household receives labor income, $w_t h_t$, dividend income, $d_t\theta_t$, the proceeds from the sale of stock net of transaction costs, $\left(\Delta_t^- - \frac{\phi}{2}(\Delta_t^-)^2\right) q_t$, and the maturing bonds, b_t. They use their income to consume, c_t, buy stock, $q_t\Delta_t^+$, and buy bonds, $\frac{b_{t+1}}{1+r_t}$. Equation (3) represents the law of motion of stock holdings. Condition (4) imposes natural constraints on trades, ensuring that purchases and sales are non-negative and that the household does not sell more shares than it owns. Finally, equation (5) represents the borrowing constraint.

Thus, the problem of a household is to choose processes $\{c_t, b_{t+1}, \theta_{t+1}, \Delta_t^+, \Delta_t^-\}_{t=0}^{\infty}$ in order to maximize (1) subject to (2), (3), (4), and (5) for every $t \geq 0$, and given an initial portfolio (θ_0, b_0) and prices and dividends $\{w_t, q_t, r_t, d_t\}_{t=0}^{\infty}$.

[6] In Appendix B.1, we show that our results also hold in a 3-period model in which both sellers and buyers pay transaction costs. Assuming that buyers do not pay transaction costs simplifies the solution of the infinite-horizon model.

Firms. There is a representative firm that operates a non-increasing returns to scale technology that combines labor and capital to produce the final consumption good, given by

$$y_t = \left(l_t^{\gamma} k_t^{1-\gamma}\right)^{\psi},$$

where l_t denotes the amount of labor hired, k_t denotes the amount of capital operated, $\gamma \in (0,1)$ and $\psi \leq 1$.[7] Moreover, the firm operates the economy's investment technology, $k_{t+1} = (1-\delta)k_t + i_t$.

As is standard, we assume that the firm acts in the best interest of its shareholders. Because the choice of labor is an intratemporal decision, it is immediate that optimality implies

$$l_t = \psi\gamma \frac{y_t}{w_t}.$$

Let $\pi_t = y_t - w_t l_t$ denote the firm's per-period profits. Then, $\pi_t = (1-\gamma\psi)\, y_t$, or $\pi_t = z_t k_t^{\alpha}$, where $z_t = (1-\gamma\psi)\left(\frac{\gamma\psi}{w_t}\right)^{\frac{\gamma\psi}{1-\gamma\psi}}$ and $\alpha = \frac{(1-\gamma)\psi}{1-\gamma\psi}$. Dividends are then given by $d_t = \pi_t - i_t$, or

$$d_t = F_t(k_t, k_{t+1}) \equiv z_t k_t^{\alpha} + (1-\delta)k_t - k_{t+1}.$$

Investment is an intertemporal decision that requires knowing the shareholders' intertemporal preferences. Because of the assumed financial frictions, shareholders may have conflicting preferences about the firm's investment plans. In this paper, we assume that the firm maximizes a weighted average of the shareholders' valuations, where the weights are given by their stock holdings at the beginning of the period (see Grossman and Hart, 1979). We proceed in two steps. First, we compute the shareholders' valuations. Then, we specify the firm's objective.

Shareholders' valuations. Let $V_t(\theta, b, h)$ denote the value function in period t of a household with stock holdings θ, bond holdings b, and employment status h. The envelope condition of a shareholder (i.e., a household with $\theta > 0$) with respect to θ

[7] In the background, we are assuming that there is a measure one of identical firms with non-increasing returns to scale. Since all firms make the same choices in equilibrium, we have $k_i = k$ and $l_i = l$ for all i. Then, there exists a representative firm with the same technology that operates all the capital and labor, i.e., $\int_0^1 k_i di = k$ and $\int_0^1 l_i di = l$. Finally, we assume that the entry costs are such that there is no firm entry in equilibrium.

is given by

$$V_{\theta t}\left(\theta,b,h\right)\equiv\frac{\partial V_t(\theta,b,h)}{\partial\theta}=\lambda_t\left(\theta,b,h\right)d_t+\mu_t\left(\theta,b,h\right)+\overline{\eta}_t^-\left(\theta,b,h\right),$$

where $\lambda_t(\theta,b,h)$ denotes the marginal utility of wealth (measured as the Lagrange multiplier associated to the budget constraint (2)), $\mu_t(\theta,b,h)$ denotes the marginal utility of an extra unit of the stock (measured by the Lagrange multiplier of constraint (3)), and $\overline{\eta}_t^-(\theta,b,h)$ denotes the Lagrange multiplier associated to the short-selling constraint in (4). Let $\tilde{q}_t(\theta,b,h)$ denote the shareholder's valuation of the firm in units of the consumption good, which is given by

$$\tilde{q}_t(\theta,b,h)=\frac{V_{\theta t}\left(\theta,b,h\right)}{\lambda_t\left(\theta,b,h\right)}=d_t+\frac{\mu_t\left(\theta,b,h\right)+\overline{\eta}_t^-\left(\theta,b,h\right)}{\lambda_t\left(\theta,b,h\right)}.$$

The shareholder's valuation has two components. First, shareholders value the dividend they receive, d_t. Since the dividend is in units of the consumption good, all shareholders agree on its valuation. The second term denotes the *ex-dividend* value of the stock. Since the stock is a long-lived asset, agents value holding shares of the firm above and beyond the dividend they receive in the current period. This term may differ across agents.

There are two types of shareholders: those who keep all their holdings (and potentially buy more) and those who sell at least part of their portfolio. We call them buyers and sellers, respectively. Absent transaction costs, this distinction would be inconsequential: at the margin, the valuation of buyers and sellers would always coincide. However, transaction costs introduce a wedge in the agents' valuation, generating disagreement. The following lemma characterizes the shareholders' valuation. All proofs of the results below are in Appendix A.

Lemma 1. *The shareholders' valuation of the firm is given by*

$$\tilde{q}_t(\theta,b,h)=d_t+(1-\phi\Delta_t^-(\theta,b,h))q_t. \tag{6}$$

Lemma 1 characterizes the valuation of all agents that start the period with $\theta>0$. Note that since buyers choose $\Delta_t^-=0$, their valuation simplifies to $\tilde{q}_t(\theta,b,h)=d_t+q_t$, which is independent of agent-specific variables. In contrast, the seller's valuation is decreasing in the amount of stock they sell. The more the agent sells, the higher the transaction cost and the lower the benefits of holding the stock.

Next, we use Lemma 1 to specify the firm's objective.

The firm's problem. There is a vast literature studying the problem of the firm when shareholders disagree.[8] In this paper, we follow Grossman and Hart (1979) and assume that the firm maximizes an *ownership-weighted valuation*, which can be thought of as giving shareholders votes in proportion to their holdings.

Assumption 1. *The firm maximizes an ownership-weighted valuation given by:*

$$\int_{\theta,b,h} \theta \left[d_t + (1 - \phi\Delta_t^-(\theta, b, h))q_t\right] d\Gamma_t(\theta, b, h) = d_t + (1 - \overline{\Phi}_t)q_t,$$

where $\Gamma_t(\theta, b, h)$ denotes the cross-section distribution over the portfolio holdings and employment status, $d_t = F(k_t, k_{t+1})$, and

$$\overline{\Phi}_t \equiv \phi \int_{\theta,b,h} \theta\Delta_t^-(\theta, b, h)d\Gamma_t(\theta, b, h). \tag{7}$$

Maximizing the firm's value involves choosing the path of capital that trades off the effects on the dividend distributed in period t, d_t, and the continuation market value, q_t. For example, higher investment today implies a lower current dividend but potentially higher dividends in the future. Crucially, the continuation value q_t is discounted by $\overline{\Phi}_t$, which represents the weighted-average marginal cost faced by *current owners*. Consider the valuations of owners who sell some of their holdings relative to owners who keep everything. Since the firm can distribute dividends costlessly, those who sell tend to favor the distribution of dividends at the expense of a higher continuation value. The firm aggregates these differences by internalizing a *weighted-average* transaction cost in its problem.

A crucial determinant of the firm's problem is the sensitivity of the stock price, q_t, to the firm's investment plan. Naturally, the firm does not directly control q_t, which is determined in equilibrium. However, the firm understands that the equilibrium value of q_t depends on its investment decisions through their consequences on the dividend payout plan. In the case where $\phi = 0$, determining this sensitivity is relatively simple, as we show next.

[8] See, e.g, Diamond (1967), Dreze (1974), Grossman and Hart (1979).

The frictionless case. Consider the benchmark case without transaction costs, i.e. $\phi = 0$. In this case, the firm's objective simplifies to $d_t + q_t$, and the price is equal to

$$q_t = \sum_{s=1}^{\infty} \prod_{z=0}^{s-1} \left(\frac{1}{1 + r_{t+z}} \right) d_{t+s},$$

where the firm takes the sequence of interest rates $\{r_{t+z}\}_{z=0}^{\infty}$ as given. When $\phi = 0$, bonds and stocks are perfect substitutes, so their rates of return must equalize. Moreover, all shareholders agree that the appropriate discount of the firm's dividends is $\frac{1}{1+r_{t+z}}$. Thus, $\{r_{t+z}\}_{z=0}^{\infty}$ summarizes the shareholders' intertemporal preferences, and the problem of the firm is to maximize the net present value of all future dividends.

When $\phi > 0$, we get two important differences. First, shareholders disagree on the trade-off between the current dividends and the firm's continuation value. The firm internalizes this disagreement with Assumption 1. Second, because stocks are more *illiquid* than bonds, the rate the firm uses to discount future dividends is different from the interest rate on the liquid asset, r_t. To tackle this problem, we first solve a simplified 3-period version of the model in Section 3. We turn back to the infinite-horizon setting in Section 4.

3 Motivating Example: A 3-Period Model

Before considering the infinite-horizon economy described above, we study a simplified 3-period version that will allow us to identify the source of time-inconsistency in the firm's problem. Because there are only three time periods, $t = 0, 1, 2$, we have that $c_t = 0$ for $t \geq 3$ and (2) $-$ (5) only hold for $t = 0, 1, 2$. Note that $q_2 = 0$ because there is no need to hold assets beyond $t = 2$.[9]

Besides assuming a shorter time horizon, we make two additional simplifying assumptions relative to the model described in Section 2. First, we assume that households face no income risk. Instead, we follow Woodford (1990) and assume that households' employment status oscillates in a deterministic fashion between a low value h_{low} and a high value h_{high}. We assume that half of the households receive

[9] When we solve the infinite-horizon in Section 5, we will focus on a stationary equilibrium where aggregate variables are constant over time. Since the 3-period model has a final period where the stock price is zero, a stationary equilibrium does not exist. In Appendix B.2, we present a natural extension of this model that generates a stationary equilibrium. All our results go through in that setting.

the low labor endowment in period $t = 0$, and the other half receive the high labor endowment. We refer to these two groups as the low and high groups $j \in \{l, h\}$, respectively. Second, we assume that households cannot borrow, that is, the borrowing limit is $\bar{b} = 0$. Thus, there effectively is a single financial asset in the economy, the illiquid stock. All other aspects of the example are as described in the previous section.

Our main object of interest is the firm's intertemporal problem. We first characterize the solution for a firm that commits in period 0 to an investment path for capital in periods 1 and 2. Then, we show that if the firm is allowed to reoptimize in period 1, it chooses a different allocation for capital in period 2. Hence, the problem is not time-consistent.

The firm maximizes the value to its shareholders, which by Assumption 1 is given by:

$$\sum_{j\in\{l,h\}} \frac{\theta_t^j}{2} \left[d_t + (1-\phi\Delta_t^{j-})q_t\right], \quad t \in \{0,1\}, \tag{8}$$

with $d_t = F(k_t, k_{t+1})$. As discussed above, while q_t is determined in equilibrium, the firm understands that its decisions (investment, dividend payout policy) will impact its value. Here, we assume that the firm knows that the stock price satisfies the following households' Euler equations

$$\left(1-\phi\Delta_t^{j-}\right) q_t = \beta\frac{u'\left(c_{t+1}^j\right)}{u'\left(c_t^j\right)} d_{t+1} + \beta\frac{u'\left(c_{t+1}^j\right)}{u'\left(c_t^j\right)}\left(1-\phi\Delta_{t+1}^{j-}\right) q_{t+1}, \quad t \in \{0,1\}. \tag{9}$$

Note that we focus on equilibria where the short-selling constraint does not bind. In the numerical illustration below, we choose parameter values that satisfy this requirement. Then, introducing (9) into (8), we get that the firm's value in period 0 is given by

$$V_0^F(k_0) = \max_{k_1,k_2\geq 0} \sum_{j\in\{l,h\}} \frac{\theta_0^j}{2}\left[d_0 + \beta\frac{u'\left(c_1^j\right)}{u'\left(c_0^j\right)} d_1 + \beta^2\frac{u'\left(c_2^j\right)}{u'\left(c_0^j\right)} d_2\right], \tag{10}$$

where $d_t = F_t(k_t, k_{t+1})$ and the firm takes the households' consumption plans as given.[10] The optimal investment plan from period 0's perspective is the solution to

[10]This is an application of the standard *"big D little d"* argument. The firm takes aggregate consumption—which depends on dividends—as given, but optimizes on its own dividend payments.

problem (10).

Consider now the problem of the firm in period 1. The firm starts the period with capital stock k_1, which was chosen in period 0. Suppose we give the firm the option to reoptimize its investment plan, i.e., the choice of k_2. If the plan in period 0 is time-consistent, the firm would not have an incentive to change its choice. However, we show next that the firm may choose to change its plan; that is, its problem may be time-inconsistent. To see this, note that the firm's problem in period 1 along the equilibrium path is

$$\begin{aligned} V_1^F(k_1) &= \max_{k_2 \geq 0} \sum_{j \in \{l,h\}} \frac{\theta_1^j}{2} \left[d_1 + (1 - \phi \Delta_1^{j-}) q_1 \right] \\ &= \max_{k_2 \geq 0} \sum_{j \in \{l,h\}} \frac{\theta_1^j}{2} \left[d_1 + \beta \frac{u'\left(c_2^j\right)}{u'\left(c_1^j\right)} d_2 \right] \end{aligned} \tag{11}$$

where $\theta_1^j = \theta_0^j + \Delta_1^{j+} - \Delta_1^{j-}$. Then, the firm's problem is time-consistent if and only if the discounting between $t = 1$ and $t = 2$ in problem (10) and problem (11) coincide, or, equivalently, if

$$\frac{\sum_{j \in \{l,h\}} \frac{\theta_0^j}{2} \beta^2 \frac{u'(c_2^j)}{u'(c_0^j)}}{\sum_{j \in \{l,h\}} \frac{\theta_0^j}{2} \beta \frac{u'(c_1^j)}{u'(c_0^j)}} = \sum_{j \in \{l,h\}} \frac{\theta_1^j}{2} \beta \frac{u'(c_2^j)}{u'(c_1^j)}. \tag{12}$$

First consider the case with $\phi = 0$. The Euler equation (9) becomes

$$\frac{q_t}{d_{t+1} + q_{t+1}} = \beta \frac{u'\left(c_{t+1}^j\right)}{u'\left(c_t^j\right)}.$$

Crucially, in this case, the IMRS are equalized across agents. In this case, the firm's problem is time-consistent:

$$\frac{\sum_{j \in \{l,h\}} \frac{\theta_0^j}{2} \beta^2 \frac{u'(c_2^j)}{u'(c_0^j)}}{\sum_{j \in \{l,h\}} \frac{\theta_0^j}{2} \beta \frac{u'(c_1^j)}{u'(c_0^j)}} = \frac{\frac{q_0}{d_1+q_1} \frac{q_1}{d_2+q_2}}{\frac{q_0}{d_1+q_1}} = \frac{q_1}{d_2 + q_2} = \sum_{j \in \{l,h\}} \frac{\theta_1^j}{2} \beta \frac{u'(c_2^j)}{u'(c_1^j)}.$$

When financial markets are frictionless, agents' valuations of consumption across periods coincide. As a consequence, they will agree on the optimal investment plan for the firm across periods, and the firm's problem is time-consistent.

In contrast, when $\phi > 0$, the agents' IMRS might not equalize if there is positive

Table 1: Equilibrium in the 3-Period Model

Household type	c_0	c_1	c_2	Δ_0^-	Δ_1^-	θ_1	θ_2
low	2.6226	2.7290	2.6030	0.3358	0.0000	0.6642	1.6031
high	3.3667	3.2271	3.3970	0.0000	0.9389	1.3358	0.3969

trade (i.e., if $\Delta_t^{j-} > 0$ for some households). In that case, there is no guarantee that condition (12) will hold. In particular, as agents' valuation and ownership change, the optimal plan for the firm may also change. We illustrate this with a numerical example.

Suppose $w_t = 1$ and $d_t = 1$ for all t, $u = ln(c)$, $\phi = 0.1$, $\beta = 0.95$, $h_{low} = 1$, and $h_{high} = 3$.[11] Initial asset holdings are $\theta_0 = 1$. The household equilibrium is summarized in Table 1, and asset prices are given by $q_0 = 1.8856$ and $q_1 = 0.9960$.

The problem of the firm in period 0 becomes

$$V_0^F(k_0) = \max_{k_1,k_2 \geq 0} \; d_0 + 0.9520 d_1 + 0.9019 d_2,$$

and the problem of the firm in period 1 becomes

$$V_1^F(k_1) = \max_{k_2 \geq 0} \; d_1 + 0.9335 d_2.$$

Now let's compare the firm's problem in $t = 0$ and $t = 1$ along the equilibrium path. From its period 0 perspective, the firm's discount factor between $t = 1$ and $t = 2$ is 0.9474, which is the ratio between 0.9019 and 0.9520. When period 1 arrives, the firm's discount factor between $t = 1$ and $t = 2$ is 0.9335. Hence, the problem of the firm is time-inconsistent: the time preference between $t = 1$ and $t = 2$ is different in $t = 0$ than in $t = 1$. Furthermore, in this calibration, the direction of inconsistency is towards *present bias*. That is, when period 1 arrives, the firm discounts period 2 more than it did in period 0.

The purpose of this example was to show, in a simple and transparent model, that time-inconsistency is a natural outcome when the IMRS of the agents are not equalized. Intratemporal disagreement among current shareholders, as well as intertemporal disagreement among current and future shareholders, can make the optimal plan—from period 0's perspective—suboptimal in period 1.

[11] We can obtain $w_t = 1$ and $d_t = 1$ by choosing the parameters of the production function and the capital accumulation technology appropriately.

The key step that allowed us to characterize the solution was to replace the price of the stock, q_t, from the firm's objective by substituting the households' Euler equation:

$$
\begin{aligned}
\left(1-\phi\Delta_0^{j-}\right) q_0 &= \beta\frac{u'\left(c_1^j\right)}{u'\left(c_0^j\right)}d_1 + \beta\frac{u'\left(c_1^j\right)}{u'\left(c_0^j\right)}\left(1-\phi\Delta_1^{j-}\right) q_1, \\
&= \beta\frac{u'\left(c_1^j\right)}{u'\left(c_0^j\right)}d_1 + \beta^2\frac{u'\left(c_2^j\right)}{u'\left(c_0^j\right)}d_2,
\end{aligned}
$$

which holds true when $q_2 = 0$ and short-selling constraints are not binding. Note, however, that this substitution requires a great deal of information on the part of the firm. The firm needs to know not only the current preferences of its owners but also their future preferences. Crucially, the firm cannot recover these valuations from asset prices. While the firm's stock market price is observable, the *sensitivity* of the price to changes in the investment plan is unobservable.

Unfortunately, these problems get exacerbated in an infinite-horizon setting. The firm would need information on its shareholders' valuation from the present to the infinite future. Moreover, realistic settings will allow for the set of shareholders to change over time (rather than the same shareholders changing the amount they own), and some shareholders might be constrained by the short-selling constraint. These issues substantially complicate the firm's problem and the computation of equilibrium. To make progress, Section 4 discusses a set of assumptions that render the infinite-horizon problem tractable.

4 The Firm in Infinite-Horizon

In this section, we set up the problem of the representative firm in the infinite horizon setting. While simplifying assumptions will allow us to sidestep the aforementioned complications related to an infinite-horizon setting, two outcomes from the previous section remain: time-inconsistency and present bias. Moreover, under these assumptions, we show that the firm behaves as if it faced quasi-hyperbolic discounting.

4.1 The Stock Price Elasticity

A crucial element of the firm's problem is its understanding of how its stock price, q_t, changes in response to its future dividend choices, $\{d_{t+s}\}_{s\geq 1}$. Like in the 3-period model of Section 3, the firm understands that the answer to this question lies in the households' Euler equation. Let $\mathcal{C}_t(\theta, b, h)$, $\Theta_{t+1}(\theta, b, h)$, and $\mathcal{B}_{t+1}(\theta, b, h)$ denote the policy functions for consumption, stock holdings, and bonds, given (θ, b, h). To avoid excessive clutter, we simplify the notation by removing the dependence of $\mathcal{C}_t$, Θ_{t+1}, and $\mathcal{B}_{t+1}$ on (θ, b, h) whenever it does not lead to confusion.

The Euler equation of a household is given by

$$(1-\phi\Delta_t^-)q_t = \mathbb{E}_t\left[\beta\frac{u'\left(\mathcal{C}_{t+1}\right)}{u'\left(\mathcal{C}_t\right)}\right]d_{t+1} + \mathbb{E}_t\left[\beta\frac{u'\left(\mathcal{C}_{t+1}\right)}{u'\left(\mathcal{C}_t\right)}\left(1-\phi\Delta_{t+1}^-\right)\right]q_{t+1} + \eta_t,$$

where η_t denotes the Lagrange multiplier on the household's short-selling constraint for stocks in (4), and the expectation is taken with respect to h_{t+1}. Let

$$\Phi_t \equiv \mathbb{E}_t\left[\phi\Delta_{t+1}^-\right] + \phi\frac{\operatorname{cov}_t\left(u'(c_{t+1}), \Delta_{t+1}^-\right)}{\mathbb{E}_t\left[u'(c_{t+1})\right]}. \tag{13}$$

Then, we can rewrite the households' Euler as

$$(1-\phi\Delta_t^-)q_t = \mathbb{E}_t\left[\beta\frac{u'\left(\mathcal{C}_{t+1}\right)}{u'\left(\mathcal{C}_t\right)}\right]\left[d_{t+1} + \left(1-\Phi_t\right)q_{t+1}\right] + \eta_t. \tag{14}$$

The household uses its expected IMRS between t and $t+1$ to discount future cash flows. But the stock valuation has an extra component, Φ_t, which reflects the household's expected marginal transaction cost. A higher transaction cost (indexed by ϕ) reduces the household's valuation, as it lowers the stock return in a sale. This effect is amplified by the fact that households will sell (i.e. $\Delta_{t+1}^- > 0$) when their marginal utility of consumption, $u'(\mathcal{C}_{t+1})$, is relatively high. Hence, a positive covariance between marginal utility and quantity sold further depresses asset prices.

Equation (14) is at the core of the firm's investment decisions. In the 3-period model of Section 3, we used (14) to iteratively replace $(1-\phi\Delta_t^-)q_t$ from the firm's problem (8). This strategy is not helpful in the infinite-horizon version of the model for two reasons. First, recall that in the simple model, it was important that there were no agents against their short-selling constraints, so that $\eta_t = 0$ for all agents and periods. However, this will not be true in the infinite-horizon model with income risk. Households that suffer long spells of low employment will eventually hit the constraint. Thus, the Lagrange multiplier associated with the short-selling constraint will be positive for some agents. Second, after introducing the Euler equations into the firm's problem, we obtained a characterization of the firm's objective that depended on the *whole distribution of marginal utilities at all horizons*. This would require the firm to obtain information on agents' *individual* marginal utilities over time, information that is not readily available from aggregate variables or the price system.

Our strategy will use equation (14), but it will leverage the fact that if buyers are not borrowing constrained, it is possible to recover the sensitivity of the stock price to the firm's investment plan from two financial prices: the risk-free interest rate and a *liquidity premium*.

4.2 The Liquidity Premium

For buyers, condition (14) reduces to

$$q_t = \mathbb{E}_t \left[\beta \frac{u'(\mathcal{C}_{t+1})}{u'(\mathcal{C}_t)} \right] \left[d_{t+1} + (1 - \Phi_t(\theta, b, h)) q_{t+1} \right],$$

because $\Delta_t^- = \eta_t = 0$ for buyers. In principle, buyers may disagree on how much they value the dividend d_{t+1} relative to the *ex-dividend* market price q_{t+1}.[12] That is, $\mathbb{E}_t \left[\beta \frac{u'(\mathcal{C}_{t+1})}{u'(\mathcal{C}_t)} \right]$ and $\Phi_t(\theta, b, h)$ could vary depending on the buyer's initial portfolio and realization of the employment shock. However, their valuations coincide if their borrowing constraint does not bind. To see this, note that the agents' optimality condition for bonds is given by

$$1 = (1 + r_t) \mathbb{E}_t \left[\beta \frac{u'(\mathcal{C}_{t+1})}{u'(\mathcal{C}_t)} \right] + \gamma_t, \tag{15}$$

where γ_t is the Lagrange multiplier associated with the borrowing constraint (5). For unconstrained agents $\gamma_t = 0$, and hence

$$\mathbb{E}_t \left[\beta \frac{u'(\mathcal{C}_{t+1})}{u'(\mathcal{C}_t)} \right] = \frac{1}{1 + r_t} \implies q_t = \frac{d_{t+1}}{1 + r_t} + \frac{(1 - \Phi_t) q_{t+1}}{1 + r_t},$$

so Φ_t must be the same for all unconstrained buyers. In what follows, we focus on economies where the borrowing constraint does not bind for buyers. We verify their existence in the numerical exercise.

Let the yield of the stock be $r_t^\theta \equiv \frac{d_{t+1} + q_{t+1}}{q_t} - 1$. Define the liquidity premium as

[12] All buyers agree on the stock's valuation; if they did not, they would not be willing to buy at the common price q_t.

the excess return of the stock over the return of the bond, i.e. $r_t^\theta - r_t = \Phi_t \frac{q_{t+1}}{q_t}$, which reduces to Φ in steady-state. Then, the pricing equation of the firm's stock depends on two widely used financial prices: the risk-free rate, r_t, and the liquidity premium, Φ_t. With this, we are ready to set up the firm's intertemporal problem.

4.3 The Firm's Problem

Recall that Assumption 1 states that the objective of the firm is to maximize an owner-weighted valuation given by

$$d_t + (1 - \overline{\Phi}_t) q_t. \tag{16}$$

The discussion in Section 4.2 implies that the stock price can be expressed as

$$q_t = \frac{d_{t+1}}{1 + r_t} + \frac{(1 - \Phi_t) q_{t+1}}{1 + r_t}. \tag{17}$$

The following assumption provides the final piece necessary to solve the firm's problem.

Assumption 2. *The firm takes* $\{\overline{\Phi}_t, \Phi_t\}_{t \geq 0}$ *as given.*

Assumption 2 has two parts. First, it states that the firm takes the owners' average transaction cost, $\overline{\Phi}_t$, as given. This variable can be computed by a firm with access to data on how many stocks each owner bought or sold. Second, it states that the firm also takes the liquidity premium, Φ_t, as given. A common feature of these two assumptions is that the firm does not anticipate how its decisions impact households' trading patterns, summarized by the cross-sectional distribution of Δ_t^- (see the definitions of $\overline{\Phi}_t$ and Φ_t in equations (7) and (13), respectively). Note that Assumption 2 implies that the firm always has the correct expectation with respect to the *levels* of these variables. However, it ignores the potential *changes* that arise from the effect of its investment decisions on the households' trading strategies.[13]

The resulting pricing equation (17) is intuitive. Since dividends are distributed without frictions, and the economy features no aggregate risk, the firm's dividend is discounted by the same discount factor as the bonds, i.e., $\frac{1}{1+r_t}$. Note that if $\phi = 0$,

[13] Grossman and Hart (1979) make a similar assumption to set up the firm's problem in a model with heterogeneous agents and aggregate risk. They argue that since the assumption involves an elasticity, the agents' beliefs are neither verified nor falsified in equilibrium.

the pricing equation simplifies to $q_t = \frac{d_{t+1}+q_{t+1}}{1+r_t}$. In contrast, when $\phi > 0$, the resale price q_{t+1} is further discounted by the liquidity premium Φ_t. Thus, the firm's problem will be to maximize (16) subject to (17).

4.4 Equilibrium

Given Assumptions 1 and 2, we can define an equilibrium for the economy. An equilibrium for this economy consists of household allocations $\left\{\left(c_{j,t}, b_{j,t+1}, \theta_{j,t+1}, \Delta^+_{j,t}, \Delta^-_{j,t}\right)_{\forall j\in[0,1]}\right\}_{t=0}^{\infty}$, firm allocations $\{l_t, k_{t+1}, d_t\}_{t=0}^{\infty}$ and aggregates $\{w_t, r_t, q_t, \overline{\Phi}_t, \Phi_t, \Gamma_t\}_{t=0}^{\infty}$, such that, given $(\theta_{j,0}, b_{j,0})_{\forall j\in[0,1]}$ and k_0,

1. Given $\{d_t, w_t, r_t, q_t\}_{t=0}^{\infty}$, households optimize;
2. Given $\{w_t, r_t, \overline{\Phi}_t, \Phi_t\}_{t=0}^{\infty}$, firms optimize [14];
3. $\{\overline{\Phi}_t, \Phi_t\}_{t=0}^{\infty}$ are consistent with the cross-sectional distribution $\{\Gamma_t\}_{t=0}^{\infty}$ according to (7) and (13);
4. Markets clear

$$\int_{j\in[0,1]} h_{j,t}dj = l_t, \quad \int_{j\in[0,1]} \theta_{j,t}dj = 1, \quad \int_{j\in[0,1]} b_{j,t}dj = 0$$

   for all $t \geq 0$.

A steady-state equilibrium is an equilibrium in which firm allocations and aggregates are constant over time. Our analysis will focus on steady-state equilibria.

5 Equilibrium Characterization

In a steady-state equilibrium, $\overline{\Phi}_t = \overline{\Phi}$ and $\Phi_t = \Phi$ for all t. Iterating the price (17) forward and replacing it in the objective function, the problem of the firm simplifies to

$$V^F(k_t) = \max_{\{k_{t+s}\}_{s\geq 1}} F(k_t, k_{t+1}) + \frac{1-\overline{\Phi}}{1-\Phi}\sum_{s=1}^{\infty}\left(\frac{1-\Phi}{1+r}\right)^s F(k_{t+s}, k_{t+s+1}). \tag{FP}$$

[14] The optimization of the firm will depend on whether we consider the solution with or without commitment. This will be made explicit in the next section.

5.1 Quasi-Hyperbolic discounting

The program (FP) features quasi-hyperbolic discounting. To see this, let $\tilde{\delta} = \frac{1-\Phi}{1+r}$ and $\tilde{\beta} = \frac{1-\bar{\Phi}}{1-\Phi}$. Then, the value of the firm can be written as

$$V^F(k_t) = \max_{\{k_{t+s}\}_{s\geq 1}} F(k_t, k_{t+1}) + \tilde{\beta} \sum_{s=1}^{\infty} \tilde{\delta}^s F(k_{t+s}, k_{t+s+1}).$$

On the one hand, $\tilde{\delta}$ represents the "natural" discount factor obtained from the pricing equation (17): it combines the risk-free rate, r, and the liquidity premium, Φ. That is, $\tilde{\delta}$ reflects the discounting implied in asset prices. On the other hand, $\tilde{\beta}$ summarizes the disagreement between current and future owners. If $\bar{\Phi} = \Phi$, then current and future owners agree on how to value the firm, so $\tilde{\beta} = 1$ and the problem simplifies to a standard one with exponential discounting. However, if $\bar{\Phi} > \Phi$, then current owners are more impatient than the market, which implies that $\tilde{\beta} < 1$ and the firm exhibits present bias. Similarly, if $\bar{\Phi} < \Phi$, then current owners are more patient than the market, so $\tilde{\beta} > 1$ and the firm exhibits future bias. In the next proposition, we decompose the difference $\Phi - \overline{\Phi}$ to identify the factors contributing to time-inconsistency.

Proposition 1. *The difference $\Phi - \overline{\Phi}$ is equal to the sum of a* persistence effect *and a* risk premium*:*

$$\Phi - \bar{\Phi} = \underbrace{\phi \left[\widetilde{\mathbb{E}}_t \left[\mathbb{E}_t \left[\Delta^-_{t+1} \right] \middle| \, buyer \, \right] - \widetilde{\mathbb{E}}_t \left[\mathbb{E}_t \left[\Delta^-_{t+1} \right] \right] \right]}_{persistence\ effect} + \underbrace{\phi \widetilde{\mathbb{E}}_t \left[\frac{\mathrm{cov}_t \left(u'\left(\mathcal{C}_{t+1}\right), \Delta^-_{t+1} \right)}{\mathbb{E}_t \left[u'\left(\mathcal{C}_{t+1}\right) \right]} \middle| \, buyer \right]}_{risk\ premium}$$

where tilde moments are taken with respect to the cross-sectional portfolio-weighted density $\Theta_{t+1} d\Gamma(\theta, b, h)$, and the non-tilde moments are taken with respect to the density $dF(h'|h)$.

Proposition 1 presents several results. First, it shows that time-inconsistency can only emerge in an economy with transaction costs, i.e., $\phi > 0$. When $\phi = 0, \Phi = \bar{\Phi}$ and $\tilde{\beta} = 1$, recovering the standard exponential discounting problem. Second, it shows that the degree and direction of time-inconsistency (i.e., present or future bias) depends on the interaction of two economic forces.

The persistence effect captures the difference in transaction costs for different agents. On the one hand, we have the expected transaction costs next period for those that are buyers today. On the other hand, we have the average transaction cost for owners—which includes both buyers and sellers. Note that, by stationarity, this term is the same in periods t and $t+1$. If there is *persistence* in trades, so that buyers in t expect to also be buyers in $t+1$, then the expected transaction cost of buyers is smaller than the average transaction cost in the economy, i.e. $\widetilde{\mathbb{E}}_t\left[\mathbb{E}_t\left[\Delta^-_{t+1}|buyer\right]\right] < \widetilde{\mathbb{E}}_t\left[\mathbb{E}_t\left[\Delta^-_{t+1}\right]\right]$, which represents a force towards present bias. This is the case we expect to obtain in our model, as purchasing the illiquid assets is profitable only if there is a relatively high chance that the agent will *not* sell the asset immediately (the bond is a better asset for this strategy). We confirm that this is true in our numerical exercise.

The second term in the expression is the risk premium effect, which captures the fact that it is precisely when the agents need the resources the most, i.e., high marginal utility states, that households sell the most. This positive covariance makes the equity a risky investment, which is priced in by the buyers and represents a force towards future bias. The net effect of these two competing forces (persistence versus risk) depends on parameter values. In our numerical simulations of Section 6, we find that the persistence effect significantly outweighs the risk premium, leading to present bias.

This result reflects, once again, the differences in IMRS across agents. In particular, the result in Proposition 1 can be understood as comparing the rate of impatience of buyers and owners. While agents' IMRS does not appear explicitly in Proposition 1, their impatience is reflected in the quantity sold, Δ^-. Since selling stock is costly while selling bonds (i.e., borrowing) is not, sellers of the stocks are likely borrowing constrained.[15] Due to the presence of constrained sellers, we have that

$$\widetilde{\mathbb{E}}_t\left[\mathbb{E}_t\left[\frac{\beta u'(c_{t+1})}{u'(c_t)}\right]\middle| seller\right] < \frac{1}{1+r_t}$$

In contrast, buyers are not borrowing constrained today, which implies that their IMRS is $\frac{1}{1+r_t}$. Moreover, as we argued above, buyers today are likely to be buyers tomorrow (the persistence effect), so their expected IMRS is close to $\frac{1}{1+r_t}$. Thus, in

[15] Since transaction costs are quadratic, some agents will start selling stocks before reaching the borrowing constraint if they anticipate that they will have liquidity needs in the near future. However, they will exhaust their borrowing capacity before selling all their stock holdings.

equilibrium, we expect to have

$$\widetilde{\mathbb{E}}_t \left[\mathbb{E}_t \left[\frac{\beta u'(\mathcal{C}_{t+1})}{u'(\mathcal{C}_t)} \right] \right] < \widetilde{\mathbb{E}}_t \left[\mathbb{E}_t \left[\frac{\beta u'(\mathcal{C}_{t+2})}{u'(\mathcal{C}_{t+1})} \right] \middle| buyer \right]$$

where the LHS is the (weighted) average IMRS across owners (i.e., both buyers and sellers) in period t, while the RHS is the (weighted) average IMRS across buyers in $t+1$. This implies that owners are more impatient than buyers. Note that the firm's discount factor between $t+1$ and $t+2$ in period t is determined by the stock price q_t, and so is the IMRS of the buyers. However, the discount between $t+1$ and $t+2$ *in period* $t+1$ is determined by the owners. Using our result that owners are likely to be more impatient than buyers, we conclude that the firm's problem is time-inconsistent and, in particular, it suffers from present bias.

Next, we solve the problem of the firm analytically. First, we consider the problem of a firm that can commit to future policies. This firm chooses an investment plan in the initial period and never reoptimizes. Then, we study the problem of a firm that cannot commit to an investment plan but fully anticipates its future incentives and understands how present decisions can affect its future self.

5.2 Firm with commitment

A firm that can commit to a future investment policy chooses a sequence of capital $\{k_{t+s}\}_{s=1}^{\infty}$ in period t to maximize its value:

$$V^F(k_t) = \max_{\{k_{t+s}\}_{s\geq 1}} F(k_t, k_{t+1}) + \tilde{\beta} \sum_{s=1}^{\infty} \tilde{\delta}^s F(k_{t+s}, k_{t+s+1}).$$

The first-order condition with respect to k_{t+1} is

$$F_2(k_t, k_{t+1}) + \tilde{\beta}\tilde{\delta} F_1(k_{t+1}, k_{t+2}) = 0$$

and the first-order condition with respect to k_{t+s+1} for $s \geq 1$ is

$$F_2(k_{t+s}, k_{t+s+1}) + \tilde{\delta} F_1(k_{t+s+1}, k_{t+s+2}) = 0 \tag{18}$$

where $F_1(k_t, k_{t+1}) \equiv z\alpha k_t^{\alpha-1} + (1-\delta)$ and $F_2(k_t, k_{t+1}) \equiv -1$. Note that when $\tilde{\beta} \neq 1$, the choice of initial investment involves a different trade-off than future investment.

Focusing on steady-state equilibria, and using that $z = (1-\gamma\psi)\left(\frac{\gamma\psi}{w}\right)^{\frac{\gamma\psi}{1-\gamma\psi}}$, $\alpha = \frac{(1-\gamma)\psi}{1-\gamma\psi}$, and $w = \gamma\psi k^{(1-\gamma)\psi} H^{\gamma\psi-1}$ (where H is the unconditional expectation of the employment process), we get that the level of capital when the firm has commitment is

$$k^C = \left(\frac{(1-\gamma)\,\psi\tilde{\delta}}{1-\tilde{\delta}\,(1-\delta)} H^{\gamma\psi}\right)^{\frac{1}{1-(1-\gamma)\psi}}. \tag{19}$$

5.3 No commitment

We now turn to the problem of a firm that cannot commit to an entire sequence of investments but optimizes period by period. We assume that the firm fully understands its future-self incentives and takes them into account when making its choice. This is known as a *sophisticated solution* in the literature on time-inconsistent preferences.

We solve for a Markov perfect equilibrium (MPE). Given an initial level of capital k, the firm takes the future policy function, $k' = g(k)$, as given. Thus, we can write the firm's problem as

$$V^F(k) = \max_{k'} F(k,k') + \tilde{\beta}\tilde{\delta}W(k')$$

subject to

$$W(k') = F(k', g(k')) + \tilde{\delta}W(g(k')).$$

Let $k' = \zeta(k)$ denote the solution of this equation for a given level of initial capital k. A MPE requires $\zeta(k) = g(k)$. The function $W(\cdot)$ takes into account the preference disagreement between the current and the future firm. If $\tilde{\beta} = 1$, the problem simplifies to a standard Bellman equation, with $V^F(k) = W(k)$. However, when $\tilde{\beta} \neq 1$, the current firm discounts the future at a different rate than the future firm. Note that, because $\zeta(k) = g(k)$, the future firm also intends to discount the immediate future at the rate $\tilde{\beta}\tilde{\delta}$.

The following proposition shows that there is a unique differentiable solution that is the limit of the finite-horizon version of this problem. Moreover, it shows that this solution features a lower level of capital than with commitment if and only if the firm suffers from present bias.

Proposition 2. *The firm's problem with no commitment has two differentiable,* *Markovian solutions, but only one is the limit of the unique finite-horizon equilibrium. This solution is given by*

$$k^N = \left(\frac{(1-\gamma)\,\psi\tilde{\beta}\tilde{\delta}}{1-\tilde{\beta}\tilde{\delta}\,(1-\delta)} H^{\gamma\psi} \right)^{\frac{1}{1-(1-\gamma)\psi}}. \tag{20}$$

Finally, $k^N < k^C$ if and only if $\tilde{\beta} < 1$.

Notice that (20) is directly comparable to (19), where $\tilde{\beta}\tilde{\delta}$ replaces $\tilde{\delta}$. When $\tilde{\beta} < 1$, Proposition 2 shows that firms with no commitment under-invest compared to those with commitment. It is as if the firm without commitment is more impatient, which then delivers a lower level of capital. Intuitively, the incentive for firms is to increase dividend payouts today and leave little capital for future selves. This is a best response, considering that future selves have the same incentive to over-distribute dividends.

5.4 The Role of Corporate Bonds

Up to now, we assumed that the firm funds itself from retained earnings. We now explore the consequences of allowing the firm to also issue illiquid corporate bonds. We find that corporate bonds might alter the financing decisions of the firm, depending on the liquidity of bonds. However, the presence of corporate bonds does not alter the investment decision and the time-inconsistency problem. Intuitively, the firm uses the stockholders' discount factor to make investment decisions independently of the interest rate of corporate bonds.

Suppose the firm can borrow up to an exogenous limit at the gross interest rate $1+r^{cb} = \frac{1+r}{1-\tilde{\phi}}$. The parameter $\tilde{\phi}$ captures a premium that corporate bonds pay relative to the risk-free rate in reduced form. This is motivated by the fact that corporate bond markets tend to be more illiquid than stock markets. We assume that, in contrast, the rate of return from savings is equal to r. It should be clear that since $\frac{1}{1+r} > \tilde{\beta}\tilde{\delta}$ and $\frac{1}{1+r} > \tilde{\delta}$, the firm never finds it optimal to save.

The firm's strategy depends on how r^{cb} compares to $\tilde{\beta}\tilde{\delta}$ and $\tilde{\delta}$. Intuitively, the firm borrows only if the interest rate on corporate bonds is lower than its discount rate. But note that the discount rate the firm uses for the comparison depends on its ability to commit to an investment plan. It might seem natural to conclude that if $\tilde{\phi} > \Phi$, that is, if the premium on corporate bonds is higher than the premium on stocks (recall that $\Phi = r^{\theta} - r$), then the firm does not issue corporate debt. However, this logic is correct only if the firm has commitment. If it does not, recall that the firm uses $\overline{\Phi}$ to discount future cash flows. Thus, the firm will borrow even if the corporate rate is high relative to the stock market discount. This result provides a rationalization for corporate bond issuance that does not rely on the canonical tax advantage of debt.

Important for our results is that incorporating corporate bonds does not affect our conclusions: the firm still discounts futures cash flows using $\tilde{\beta}\tilde{\delta}^{s}$.[16] The following proposition formalizes this intuition.

Proposition 3. *Suppose the firm suffers from present bias and it has access to the bond market with an interest rate of* $1 + r^{cb} = \frac{1+r}{1-\tilde{\phi}}$. *If* $\tilde{\phi} < \Phi$ *the firm always borrows to the limit independently of its degree of commitment. If* $\Phi < \tilde{\phi} < \overline{\Phi}$ *the firm without commitment borrows up to the limit, while the firm with commitment does not borrow. Furthermore, optimal levels of capital are determined according to* (19) *and* (20) *with and without commitment, respectively.*

6 Numerical Solution

We now solve the model numerically to study its properties in general equilibrium. We solve the model globally using the equations derived in the previous sections.

6.1 Calibration

We have three sets of parameters: i) standard parameters, ii) the income process, and iii) transaction costs. Table 2 summarizes the calibration.

Standard parameters. There are standard parameters that we borrow from the literature. We set the discount factor equal to 0.95, which is standard for annual calibrations, and the coefficient of relative risk aversion equal to 2. For the production function, we assume $\gamma = 0.80$ and $\psi = 0.95$ (e.g., Gavazza et al., 2018). We set the depreciation rate at 5%.

[16] Naturally, whether the firm can borrow or not can affect the general equilibrium determination of $\tilde{\beta}$ and $\tilde{\delta}$. However, its qualitative properties do not change.

Table 2: Parameter Values

Parameter	**Value**
Discount factor β	0.95
Risk aversion σ	2.00
Production weight on labor γ	0.80
Returns to scale ψ	0.95
Depreciation δ	0.05
Borrowing limit $\bar{b}$	-1.00
Labor persistence ρ_h	0.50
Labor st dev σ_h	0.30

Income process. Both the income process and the borrowing constraint are important as they determine the degree of disagreement on IMRS across agents. Labor h can take two values, interpreted as high and low income states. We approximate its distribution with the Rouwenhorst method with persistence $\rho_h = 0.5$ and a standard deviation of innovations $\sigma_h = 0.3$.[17] Hence, the standard deviation of log income in the model is 0.3. The comparison with the data is not straightforward because we should only consider households participating in the stock market, which are less than 50% (Morelli, 2021). To get a rough comparison, the standard deviation of residual log income for the U.S. population is 0.9 (Guvenen et al., 2022). Our calibration is, then, on the conservative side, as it generates a lower level of disagreement.Below we show that as we increase idiosyncratic uncertainty, the results in the model are amplified. Furthermore, we set the borrowing limit equal to negative one, which corresponds to a household borrowing capacity equal to the average annual income.[18]

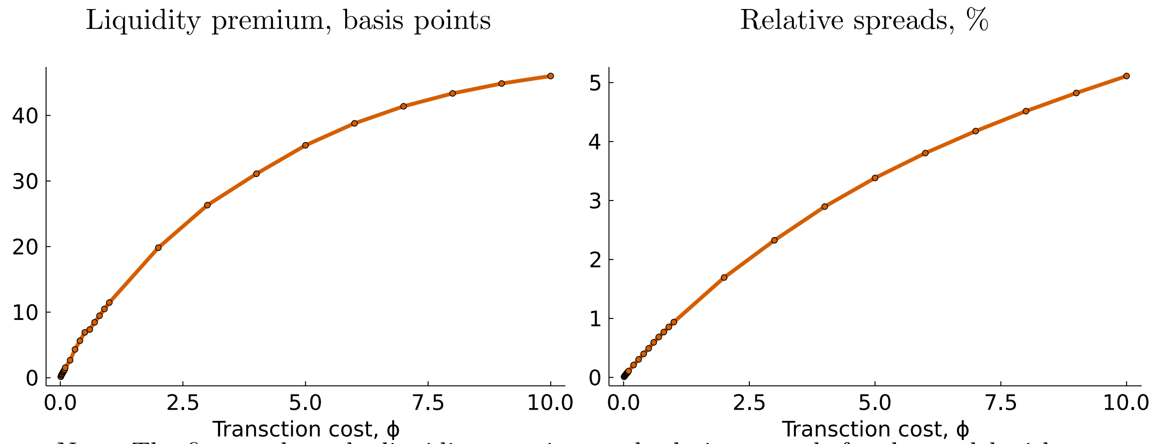
Transaction costs. To discipline the range for the transaction cost parameter ϕ, we consider two moments that connect ϕ with measures of market liquidity.

First, we choose ϕ to target empirically plausible values of the liquidity premium (given by Φ in the model). The left panel of Figure 1 shows that the liquidity premium is always below 40 basis points for $\phi \leq 10$. We believe that this is a conservative value for the liquidity premium, although direct empirical counterparts are limited

[17] For the transition matrix, we define p as the probability of staying in the same income state. The Rouwenhorst method implies that $p = (1+\rho_h)/2$. For the values of $log(h)$ define $\Sigma_h = \sigma_h/\sqrt{1-\rho_h^2}$. Then $log(h)$ can take on values Σ_h or $-\Sigma_h$.

[18] Kaplan et al. (2018) also consider a borrowing constraint of 1 period of labor income but their model is quarterly, so effectively we are allowing more borrowing, which tends to dampen our results.

Figure 1: Transaction Costs



Note: The figures show the liquidity premium and relative spreads for the model without commitment for different transaction costs, ϕ.

(see, e.g., Amihud et al., 2005). For comparison, the liquidity premium is much lower than the value in Kaplan et al. (2018), which is equal to 370 basis points.[19] Thus, we will consider $\phi \in [0, 10]$ for our numerical explorations.

We also consider the implications of ϕ on relative spreads. Naes et al. (2011) measure the relative spreads as the difference between the ask and bid quotes as a fraction of the price. In the model, this is equivalent to $\mathbb{E}\left[\frac{\phi}{2}\Delta^- \middle| \Delta^- > 0\right]$, where the expectation is over the cross-sectional distribution of households.

The right panel of Figure 1 shows the model's relative spreads. The relative spread increases with the value of ϕ and is always below 5% for the range of ϕ considered in these exercises. Thus, the frictions associated with transaction costs in these exercises are in the range of the empirical estimates. Our benchmark of $\phi = 4$ has a relative spread of 2.8%, which is close to the estimate of Naes et al. (2011) of 2.1% on average.[20]

Non-targeted moments. Although the model is stylized and does not have many *quantitative add-ons*, the composition of liquid and illiquid assets—which is not targeted in the calibration—is consistent with the data. For example, the average share

[19]The asset structure is very similar as they also consider liquid and illiquid assets and map illiquid assets to the firm's capital and profits.

[20]In the cross-section there is significant heterogeneity. For example, Livingston (2019) finds that the smallest companies in the Russell Microcap Index have a relative spread of almost 8%, while the average value of 2% corresponds to a firm in Russell 2000.

Table 3: Asset Composition

	Model	**Data**
Mean illiquid assets	3.7	2.9
Mean liquid assets	0.2	0.6
Frac. with $b < 0$	0.5	0.2

Note: The benchmark model has $\phi = 4$. Means are expressed as ratios to annual output. Data from SCF 2004 (see Kaplan et al., 2018).

of illiquid assets to GDP is 3.7 in the model and 2.9 in the data, while the average share of liquid assets is 0.2 in the model and 0.6 in the data. Moreover, the model generates a fraction of borrowers of 0.5 in the model while it is 0.2 in the data. This large difference is likely driven by our assumption that the borrowing rate is equal to the risk-free rate, r. The literature typically assumes a wedge between borrowing and saving rates, which reduces the fraction of borrowers in the economy.[21] However, incorporating a wedge would substantially complicate the solution of our model, as it would create an inaction region for bond holdings.

6.2 Liquidity and Investment in General Equilibrium

Relative to a complete markets economy, our model features three forces impacting the firm's discount factor: (i) precautionary savings, (ii) transaction costs, and (iii) lack of commitment. We now analyze the importance of each channel numerically.

Figure 2 shows the equilibrium allocations for economies with different values of the transaction cost, ϕ. First, the dotted blue line shows the solution under complete markets, when the firm discounts at rate $r = 1/\beta - 1$ (independent of ϕ). Second, the yellow point highlights the case with no transaction costs, i.e., $\phi = 0$, but incomplete markets, so the model is analogous to the economy studied in Aiyagari (1994). The absence of trading frictions implies that the firm's problem is time-consistent. In this economy, the interest rate is lower, and capital is higher, than with complete markets due to precautionary savings. In our parametrization, capital is 9% higher than under complete markets.

Third, the red line shows the solution when markets are incomplete, $\phi > 0$, and the firm cannot commit to future policies. In this case, capital is decreasing in ϕ. For example, when $\phi = 4$, the liquidity premium is about 30 basis points, and capital is

[21] For example, Kaplan et al. (2018), assume a wedge of 600 basis points.

27% lower than with complete markets. Hence, the combination of transaction costs and lack of commitment generates significant changes in aggregate capital relative to the standard Aiyagari (1994) economy. Lack of commitment is critical for this result. To see this, the green line shows the solution when the firm can commit. In this case, capital is roughly constant in ϕ and only marginally higher than in the Aiyagari case.

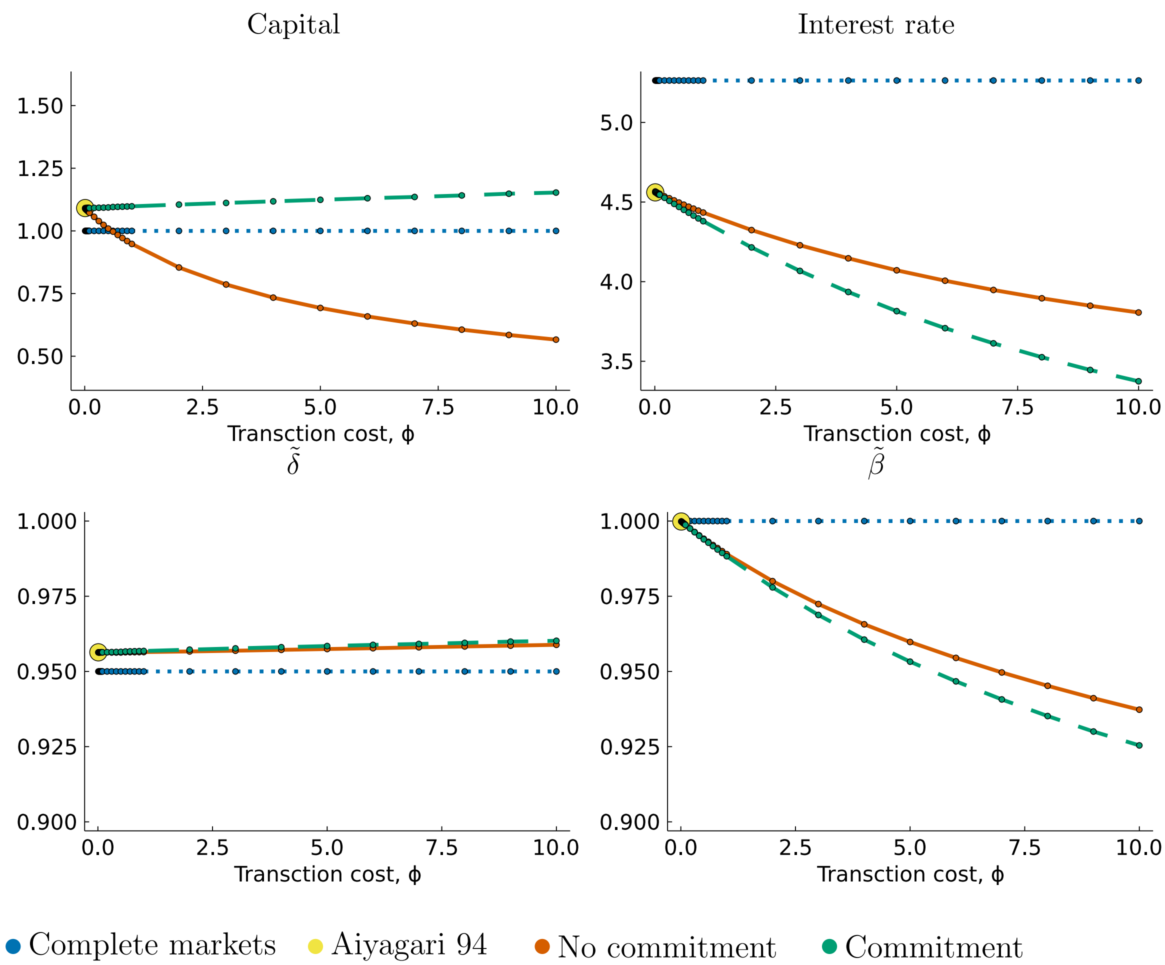
To understand the discrepancy between firms with and without commitment, consider again Proposition 1. When the firm has commitment, the discount factor in the stationary equilibrium, $\tilde{\delta}$, is equal to $(1 - \Phi)/(1 + r)$, which in our numerical exercise is roughly constant in ϕ. The reason is that, as ϕ increases, bonds become more desirable assets than stocks, and, as a result, the liquidity premium increases and the interest rate decreases. In equilibrium, both forces approximately offset each other, with a small net effect on the steady-state level of capital. When the firm lacks commitment, the disagreement among owners is captured by the difference between Φ and $\bar{\Phi}$, which is increasing in the trading friction ϕ, as captured by the last panel of Figure 2 ($\tilde{\beta}$). For example, when $\phi = 4$, $\tilde{\beta}$ is about 0.96. Recall that k^N is increasing in $\tilde{\beta}$. Hence, without commitment, increases in ϕ decrease $\tilde{\beta}$, which decreases the steady-state level of capital.

These results imply that the transmission of trading frictions to the real economy, and capital accumulation in particular, crucially depends on the ability of firms to commit to future policies. If firms cannot commit, trading frictions can have large and negative effects on the real economy because of present bias. Interestingly, the economy can end up with less capital than with complete markets. That is, the over-accumulation of capital present in Aiyagari (1994) is overturned once we introduce trading frictions and lack of commitment. In contrast, if firms can commit, trading frictions affect asset pricing with almost no consequences for aggregate capital. These results illustrate that the assumptions about trading frictions and the firm's problem are important for understanding both aggregate quantities and asset prices.

6.3 Demand and Supply of Liquidity

We end this section with comparative statics with respect to changes in the demand and supply of assets. In particular, we consider the effects of changes in the level of idiosyncratic risk and the supply of government bonds.

Figure 2: Liquidity and Investment in General Equilibrium

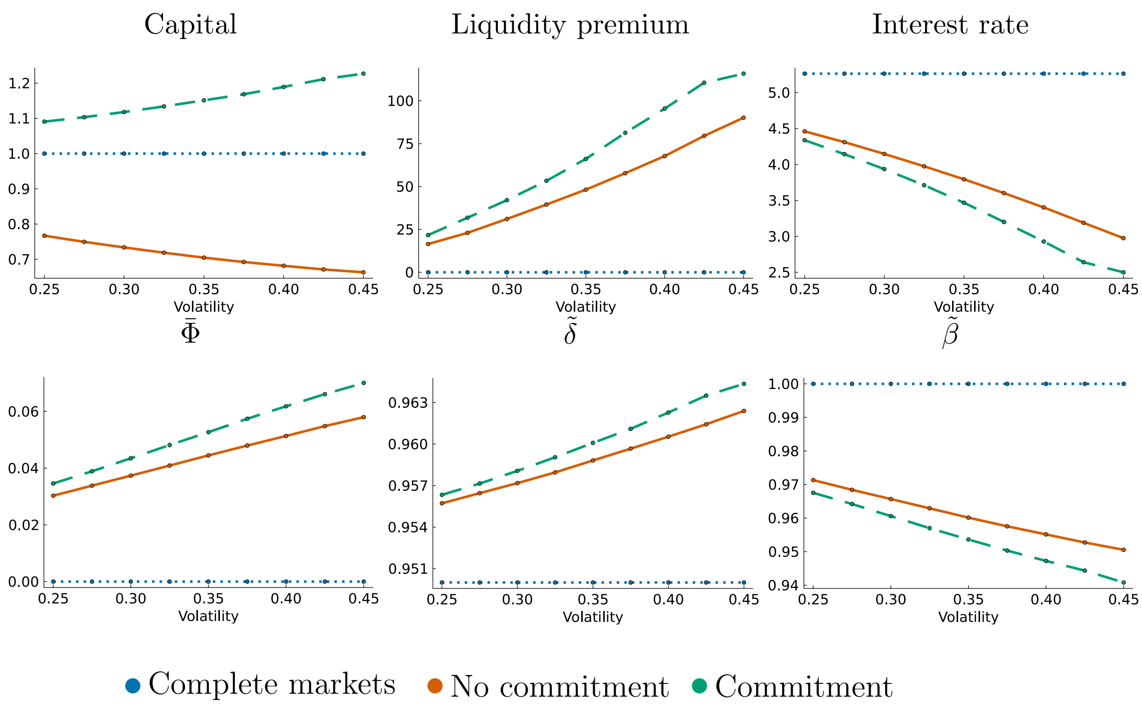


Note: The figures show the allocations under different transaction costs, ϕ.

Demand of Liquidity. In this section, we evaluate the role of increasing uncertainty, which is akin to increasing the demand for liquidity. In particular, we consider increases in σ_h, which increase the uncertainty of the income process while keeping the average income and the persistence of the process constant (i.e., a mean preserving spread).

Figure 3 shows how the economy responds to changes in σ_h. On the one hand, a higher level of uncertainty increases the agents' precautionary savings motive and—as a result—represents a force towards a higher level of capital. This effect dominates when firms can commit to an investment plan. However, when firms cannot commit, more uncertainty implies more time-consistency problems, and—as a result—

Figure 3: Uncertainty



Note: The figures show the allocations under different volatility, σ_h.

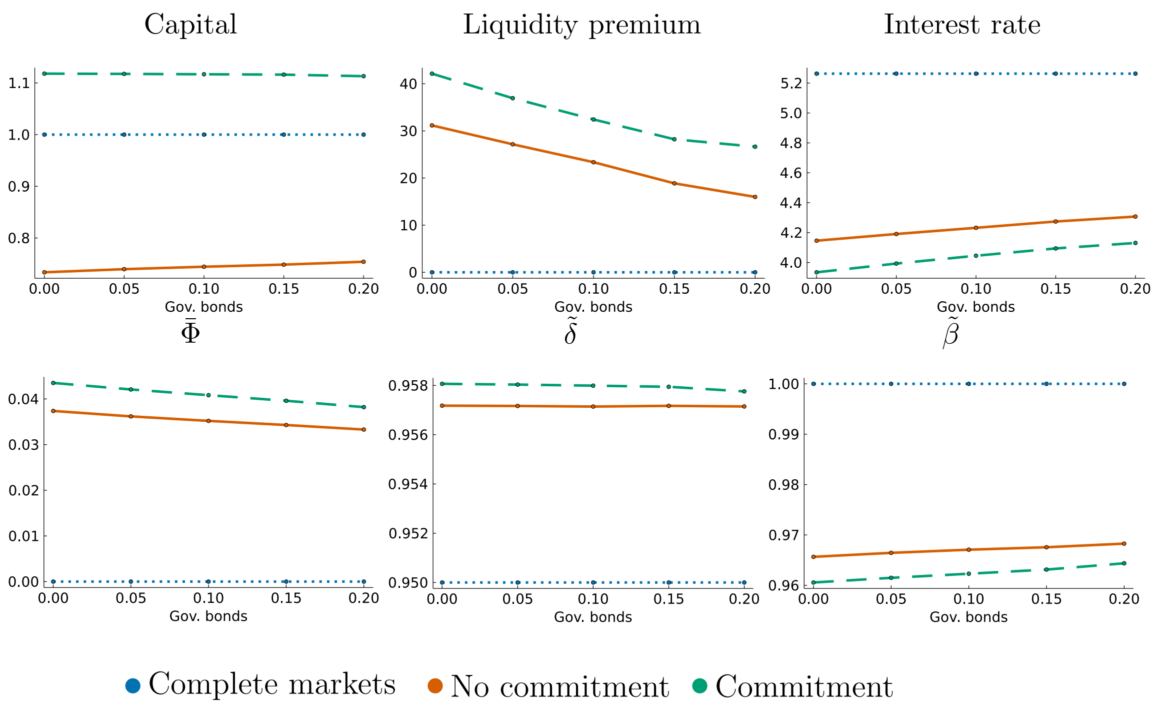
represents a force towards a lower level of capital. The reason is that higher liquidity needs exacerbate the disagreement between current and future owners. Current owners increase their preference for immediate dividend distribution, leading to lower levels of investment.

Supply of Liquidity. Now suppose the government can issue one-period bonds, denoted by B^G, and it uses lump-sum taxes to pay for the debt services. Note that as B^G increases, the economy has a larger supply of liquid assets.[22]

Figure 4 shows how the economy responds to changes in B^G. As the supply of bonds increases, bond prices decrease and the interest rate increases. Not surprisingly, then, the liquidity premium decreases. The effect on aggregate capital crucially depends on the firm's ability to commit. On the one hand, when firms can commit, a higher supply of liquid assets implies a lower residual demand for private assets

[22]We assume that the return on government bonds is the risk-free rate r. While government bonds trade in over-the-counter markets, short-term U.S. bonds are among the most liquid securities.

Figure 4: Shortage of Liquid Assets



Note: The figures show the allocations under different government bonds, B^G.

and—as a result—less capital. On the other hand, when firms cannot commit, a higher supply of liquid assets implies a less severe time-consistency problem and—as a result—a higher level of capital.

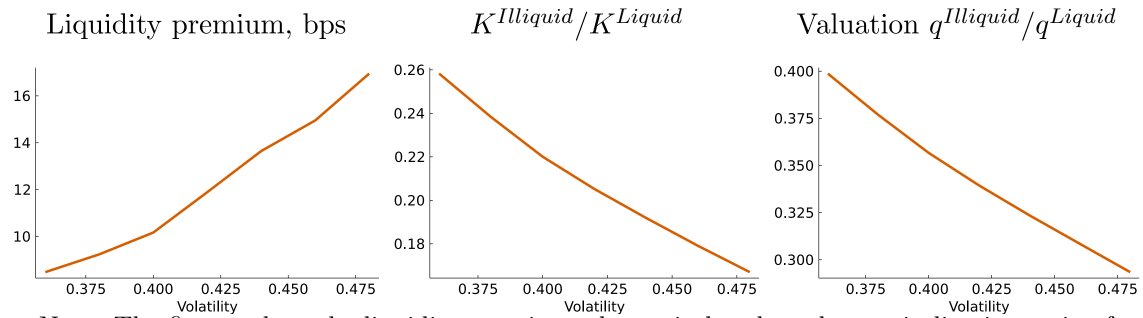
7 Empirical Observations

This section shows that the theory in this paper helps us rationalize several empirical findings relating to liquidity and investment both in the cross-section and in the aggregate. We evaluate the model's prediction with and without commitment and conclude that lack of commitment is crucial to replicate the empirical findings.

7.1 Liquidity and Investment in the Cross-section

Amihud and Levi (2022) use cross-sectional data for US public firms to compare liquid and illiquid firms and conclude that illiquid firms invest less than liquid ones. Our model is consistent with this fact.

Figure 5: Liquid vs Illiquid Firms



Note: The figures show the liquidity premium, the capital and market capitalization ratio of illiquid to liquid firms for different values of idiosyncratic uncertainty, σ_h.

We extend the model to have liquid and illiquid firms. A fraction ω of firms are illiquid as in the benchmark model, and a fraction $(1-\omega)$ of firms are liquid.[23] By no-arbitrage, the liquid stock has the same return as the bond. Hence, the liquid firm—not surprisingly—discounts at rate $\frac{1}{1+r}$, with standard exponential discounting.

The discount factor of liquid firms, $\frac{1}{1+r}$, is larger than the discount factor of illiquid firms without commitment, given by $\tilde{\beta}\tilde{\delta} = \frac{1-\overline{\Phi}}{1+r}$. Using arguments analogous to those in Proposition 2, liquid firms invest more than illiquid ones, consistent with the empirical evidence in Amihud and Levi (2022).

7.2 Liquidity and Investment over the Business Cycle

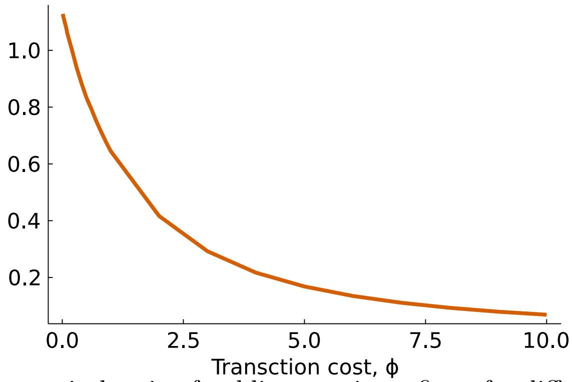
The model is also consistent with how the cross-section evolves along the business cycle. During recessions, markets become less liquid, and there is a "*flight to liquidity*": investors shift their portfolios into liquid assets (Naes et al., 2011).

To rationalize this fact, we study the comparative statics with respect to idiosyncratic uncertainty, σ_h. An increase in σ_h generates an increase in the liquidity premium and a decrease in the discount factor of the illiquid firm. As a result, the capital of illiquid firms decreases; see middle panel of Figure 5.[24] Hence, an increase in σ_h is consistent with a reduction in liquidity and flight to liquidity as the market capitalization of liquid firms relative to illiquid firms increases when σ_h increases; see the right panel of Figure 5.

[23] Appendix B.3 contains the detail of the extension. Note that the difference between this exercise and Figure 2 is that here we consider the cross-section of firms while Figure 2 considers changes in aggregate liquidity.

[24] To generate enough demand of liquidity we vary σ_h between 0.36 and 0.48 and we set $\omega = 0.9$.

Figure 6: Private Firms $K^{Public}/K^{Private}$



Note: The figure shows the capital ratio of public vs. private firms for different transaction costs, ϕ.

7.3 Public vs. Private Firms

Asker et al. (2015) compare similar public and private firms and conclude that public firms invest substantially less than private firms. We add private firms to the benchmark equilibrium in Appendix B.4. Private firms are owned by one household and are not traded in financial markets.

Figure 6 shows how these differences affect the relative equilibrium of capital of public (without commitment) and private firms as a function of ϕ. Note that, conditional on consumption levels and the real interest rate (which is common to both public and private firms), the investment decisions of private firms are independent of ϕ, while we have already shown that investment in public firms decreases with transaction costs. In particular, an increase in ϕ reduces the discount factor used by public firms while it does not change the one used by private firms. Hence, as ϕ increases, the ratio of average capital in public versus private firms decreases. Interestingly, for most values of ϕ, the ratio is below 1, implying that private firms invest more (on average) than public firms, consistent with the empirical evidence.

7.4 Short-Termism

There is an extensive literature on short-termism showing that public firms are concerned about meeting short-term targets. For example, a survey by Graham et al. (2005) shows that almost half of U.S. executives would prefer to reject a positive net present value project over missing their target. Terry (2017) finds that firms barely meeting Wall Street forecasts have lower R&D growth. Laurence Fink, the

CEO of BlackRock, one of the largest money managers, wrote that "the effects of the short-termist phenomenon are troubling ... more and more corporate leaders have responded with actions that can deliver immediate returns to shareholders, such as buybacks or dividend increases, while underinvesting in innovation, killed workforces or essential capital expenditures necessary to sustain long-term growth."

Short-termism is consistent with firms that cannot commit to future policies and have quasi-hyperbolic discounting with present bias. Our theory shows that this short-termism can be attributed to trading frictions in financial markets and asset liquidity.

8 Conclusion

This paper studies the implications of trading frictions in financial markets for firm investment and dividend choices. The main result is that when equity shares trade in frictional asset markets, the firm's problem is time-inconsistent, and the empirically relevant direction of inconsistency is present bias. Financial transaction costs cause buyers to be patient and value future firm investment and sellers to be impatient and prefer immediate dividend distribution. Hence, future owners (buyers) make investment plans, yet current owners (including sellers) readjust plans for immediate dividend payout, resulting in a problem of the firm featuring present bias. Furthermore, under a reasonable set of assumptions, the firm problem is equivalent to one where a representative agent features quasi-hyperbolic discounting.

When firms are able to commit to future investments, present bias has large effects on asset prices but small consequences on real variables. However, without commitment, firms over-issue dividends and under-invest in capital. The no-commitment case is consistent with empirical findings pertaining to liquidity and investment in the cross-section, over the business cycle, and with the short-termism narrative for publicly traded firms.

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A Proofs

Proof of Lemma 1. In what follows, the dependence of each variable on (θ, b, h) is suppressed for brevity. The first-order condition for Δ_t^+ when household are buyers $(\overline{\eta}_t^- = 0)$ is given by:

$$\mu_t = \lambda_t q_t$$

The first-order condition for Δ_t^- when households are sellers is given by:

$$\mu_t = \lambda_t q_t (1 - \phi \Delta_t^-) - \overline{\eta}_t^-$$

Expression (6) follows from plugging these conditions into the envelope condition:

$$\tilde{q}_t = d_t + \frac{\mu_t + \overline{\eta}_t^-}{\lambda_t}$$

□

Proof of Proposition 1. Begin with the definitions of Φ and $\overline{\Phi}$.

$$\begin{aligned}\Phi =& \phi \left[\mathbb{E}_t[\Delta_{t+1}^-] + \frac{\text{cov}_t(u'(\mathcal{C}_{t+1}), \Delta_{t+1}^-)}{\mathbb{E}_t[u'(\mathcal{C}_{t+1})]} \right] \\ \overline{\Phi} =& \phi \sum_h \int_\theta \int_b \theta \Delta_t^-(\theta, b, h) d\Gamma(\theta, b, h)\end{aligned}$$

By stationarity $\overline{\Phi}$ also equals:

$$\begin{aligned}\overline{\Phi} =& \phi \sum_{h'} \int_{\theta_{t+1}} \int_{b_{t+1}} \theta_{t+1} \Delta_{t+1}^-(\theta_{t+1}, b_{t+1}, h') d\Gamma(\theta_{t+1}, b_{t+1}, h') \\ =& \phi \sum_h \int_\theta \int_b \Theta_{t+1} \sum_{h'} \Delta_{t+1}^-(\Theta_{t+1}, \mathcal{B}_{t+1}, h') dF(h'|h) d\Gamma(\theta, b, h)\end{aligned}$$

where the second equality follows from plugging in policy functions Θ_{t+1} and $\mathcal{B}_{t+1}$. Because we focus on equilibria where buyers are unconstrained, Φ does not depend on (θ, b, h) for buyers. Hence we make use of the following tautology:

$$\Phi = \frac{\sum_h \int_\theta \int_b \Theta_{t+1} \Phi 1_{\{\Delta_t^+ > 0\}} d\Gamma(\theta, b, h)}{\sum_h \int_\theta \int_b \Theta_{t+1} 1_{\{\Delta_t^+ > 0\}} d\Gamma(\theta, b, h)}$$

We call the denominator B. Now take the difference between Φ and $\overline{\Phi}$:

$$\begin{aligned}\Phi - \overline{\Phi} &= \frac{1}{B}\sum_h \int_\theta \int_b \Theta_{t+1}\Phi 1_{\{\Delta_t^+>0\}} d\Gamma(\theta,b,h) - \overline{\Phi} \\ &= \frac{\phi}{B}\sum_h \int_\theta \int_b \Theta_{t+1}\mathbb{E}_t[\Delta_{t+1}^-] 1_{\{\Delta_t^+>0\}} d\Gamma(\theta,b,h) \\ &+ \frac{\phi}{B}\sum_h \int_\theta \int_b \Theta_{t+1}\frac{cov_t(u'(\mathcal{C}_{t+1}),\Delta_{t+1}^-)}{\mathbb{E}_t[u'(\mathcal{C}_{t+1})]} 1_{\{\Delta_t^+>0\}} d\Gamma(\theta,b,h) \\ &- \phi\sum_h \int_\theta \int_b \Theta_{t+1}\mathbb{E}_t[\Delta_{t+1}^-] d\Gamma(\theta,b,h)\end{aligned} \tag{21}$$

where we have plugged in the definitions of Φ and $\overline{\Phi}$. Notice that the second term in condition (21) is the risk premium. Focusing on the first and third term in (21):

$$\begin{aligned}\text{Persistence effect} =& \phi\sum_h \int_\theta \int_b \Theta_{t+1}\mathbb{E}_t[\Delta_{t+1}^-]\frac{1_{\{\Delta_t^+>0\}}}{B} d\Gamma(\theta,b,h) \\ &- \phi\sum_h \int_\theta \int_b \Theta_{t+1}\mathbb{E}_t[\Delta_{t+1}^-] d\Gamma(\theta,b,h) \\ =& \phi\left[\widetilde{\mathbb{E}}_t[\mathbb{E}_t[\Delta_{t+1}^-] \mid \text{buyer}] - \widetilde{\mathbb{E}}_t[\mathbb{E}_t[\Delta_{t+1}^-]]\right]\end{aligned}$$

which completes the proof. □

Proof of Proposition 2. We prove the proposition for a more general firm problem with any twice-differentiable production function $y_t = f(l_t, k_t)$ that is weakly concave in both variables (jointly), and strictly concave in k_t. Profits are given by

$$\pi_t = y_t - w_t l_t$$

so the intratemporal decision satisfies

$$f_1(l_t, k_t) = w_t \tag{22}$$

and the subscript denotes the partial derivative. Condition (22) implicitly defines the labor function, which we denote $l(k_t)$. Next we solve the firm's intertemporal problem with no commitment. The first-order condition of the firm's problem is given by

$$F_2(k, k') + \tilde{\beta}\tilde{\delta}W'(k') = 0$$

where

$$W'(k') \equiv F_1(k', g(k')) + g'(k') \left(F_2(k', g(k')) + \tilde{\delta} W'(g(k')) \right)$$

Plugging in the function $F(k, k') = f(l(k), k) - wl(k) + (1 - \delta)k - k'$

$$\frac{1}{\tilde{\beta}\tilde{\delta}} = \pi'(k') + 1 - \delta + g'(k') \left(-1 + \frac{1}{\tilde{\beta}} \right) \tag{23}$$

where

$$\begin{aligned} \pi'(k) &= f_1(H, k) l_1(k) + f_2(H, k) - wl_1(k) \\ &= f_2(H, k) \end{aligned}$$

and the second equality follows from the intratemporal condition (22), and H is the unconditional expectation of the labor process. Because f is strictly concave in k, $\pi'(k)$ is a strictly decreasing function. First we guess that $g'(k') = 0$, yielding the steady-state level of capital:

$$k^N = (\pi')^{-1} \left(\frac{1}{\tilde{\beta}\tilde{\delta}} - 1 + \delta \right)$$

where π' has an inverse because π' was shown to be strictly decreasing. By the same reasoning, $(\pi')^{-1}$ must be strictly decreasing. Similar logic for the commitment case yields

$$k^C = (\pi')^{-1} \left(\frac{1}{\tilde{\delta}} - 1 + \delta \right)$$

and so we have shown that $k^N < k^C$ if and only if the firm has present bias, $\tilde{\beta} < 1$. Now say $g'(k') \neq 0$, and one of two scenarios can occur. First, (23) could pin down an optimal value of k' (notice that this value does not depend on the value of k). But since $g'(k') \neq 0$, $g(k')$ is not a constant function. And by symmetry, $g(k)$ is also not a constant function, which contradicts the optimal choice k' (which was irrespective of the value of k). Second, (23) could be satisfied for any value of k'. The $g(k')$ required for this indeterminacy can be found by solving (23) for $g'(k')$:

$$g'(k') = \frac{1 + \tilde{\beta}\tilde{\delta}[\delta - 1 - \pi'(k')]}{\tilde{\delta}(1 - \tilde{\beta})}$$

Integrating this last expression yields:

$$g(k') = \frac{k' + \tilde{\beta}\tilde{\delta}[k'(\delta - 1) - \pi(k')]}{\tilde{\delta}(1 - \tilde{\beta})} + c$$

which leads to indeterminacy of the steady-state, based on this choice of integration constant c. Now consider a resource-exhausting finite-horizon version of the same problem.[25] Because there is zero investment in the final period, the firm objective in the preceding period would be

$$F(k, k') + \tilde{\beta}\tilde{\delta}F(k', 0)$$

and first-order conditions yield k^N. Using backward induction, the firm in each preceding period would also choose k^N and hence our claim is proved. $\square$

Proof of Proposition 3. The problem of the firm that can borrow up to a limit is given by

$$V^F(k_t, b_t^-, b_t^+) = \max_{\{k_{t+s}, b_{t+s}^-, b_{t+s}^+\}_{s \geq 1}} d_t + \tilde{\beta} \sum_{s=1}^{\infty} \tilde{\delta}^s d_{t+s}$$

subject to

$$d_t = F(k_t, k_{t+1}) + \frac{b_{t+1}^-(1 - \tilde{\phi})}{1 + r} - b_t^- - \frac{b_{t+1}^+}{1 + r} + b_t^+, \quad 0 \leq b_{t+1}^- \leq \bar{b}, \quad 0 \leq b_{t+1}^+$$

The firm can borrow the amount b_{t+1}^-, up to a limit $\bar{b}$. The parameter $\tilde{\phi} \geq 0$ captures the illiquidity of corporate bonds. Firms can save the amount b_{t+1}^+ at the interest rate r. First-order conditions for capital do not change from Proposition 2. First-order conditions for bonds are

$$\begin{aligned}
(b_{t+1}^-) &: \frac{1 - \tilde{\phi}}{1 + r} - \tilde{\beta}\tilde{\delta} + \mu_t^1 - \mu_t^2 = 0 \\
(b_{t+s}^-) &: \frac{1 - \tilde{\phi}}{1 + r} - \tilde{\delta} + \mu_{t+s-1}^1 - \mu_{t+s-1}^2 = 0 \\
(b_{t+1}^+) &: -\frac{1}{1 + r} + \tilde{\beta}\tilde{\delta} + \mu_t^3 = 0 \\
(b_{t+s}^+) &: -\frac{1}{1 + r} + \tilde{\delta} + \mu_{t+s-1}^3 = 0
\end{aligned}$$

[25] Laibson (1997) uses the same equilibrium refinement.

where μ_t^1, μ_t^2, and μ_t^3 are the Lagrange multipliers on the three inequality constraints, respectively. Because $\tilde{\beta}\tilde{\delta} = \frac{1-\overline{\Phi}}{1+r} < \frac{1}{1+r}$ and $\tilde{\delta} = \frac{1-\Phi}{1+r} < \frac{1}{1+r}$, we have that $b_{t+s}^{+} = 0$ for all $s \geq 1$. In other words, no savings occurs. The firm's borrowing decision depends on the illiquidity parameter $\tilde{\phi}$. We consider the empirically relevant case of present bias, $\overline{\Phi} > \Phi$.

<u>Case 1</u>: $\tilde{\phi} > \overline{\Phi}$ and, hence, corporate bonds are extremely illiquid. Then $\tilde{\beta}\tilde{\delta} = \frac{1-\overline{\Phi}}{1+r} > \frac{1-\tilde{\phi}}{1+r}$ and $\tilde{\delta} = \frac{1-\Phi}{1+r} > \frac{1-\tilde{\phi}}{1+r}$ so $b_{t+s}^{-} = 0$ for all $s \geq 1$.

<u>Case 2</u>: $\tilde{\phi} < \Phi$ and, hence, corporate bonds are extremely liquid. Then $\tilde{\beta}\tilde{\delta} = \frac{1-\overline{\Phi}}{1+r} < \frac{1-\tilde{\phi}}{1+r}$ and $\tilde{\delta} = \frac{1-\Phi}{1+r} < \frac{1-\tilde{\phi}}{1+r}$ so $b_{t+s}^{-} = \overline{b}$ for all $s \geq 1$.

<u>Case 3a</u>: $\Phi = \tilde{\phi} < \overline{\Phi}$ and, hence, corporate bonds are characterized by intermediate liquidity. Then $\tilde{\beta}\tilde{\delta} = \frac{1-\overline{\Phi}}{1+r} < \frac{1-\tilde{\phi}}{1+r}$ yet $\tilde{\delta} = \frac{1-\Phi}{1+r} = \frac{1-\tilde{\phi}}{1+r}$, so $b_{t+1}^{-} = \overline{b}$ yet $0 \leq b_{t+s}^{-} \leq \overline{b}$ for $s > 1$. This problem is time-inconsistent.

<u>Case 3b</u>: $\Phi < \tilde{\phi} < \overline{\Phi}$ and, again, corporate bonds are characterized by intermediate liquidity. Then $\tilde{\beta}\tilde{\delta} = \frac{1-\overline{\Phi}}{1+r} < \frac{1-\tilde{\phi}}{1+r}$ yet $\tilde{\delta} = \frac{1-\Phi}{1+r} > \frac{1-\tilde{\phi}}{1+r}$, so $b_{t+1}^{-} = \overline{b}$ yet $b_{t+s}^{-} = 0$ for $s > 1$. This problem is time-inconsistent.

<u>Case 3c</u>: $\Phi < \tilde{\phi} = \overline{\Phi}$ and, again, corporate bonds are characterized by intermediate liquidity. Then $\tilde{\beta}\tilde{\delta} = \frac{1-\overline{\Phi}}{1+r} = \frac{1-\tilde{\phi}}{1+r}$ yet $\tilde{\delta} = \frac{1-\Phi}{1+r} > \frac{1-\tilde{\phi}}{1+r}$, so $0 \leq b_{t+1}^{-} \leq \overline{b}$ yet $b_{t+s}^{-} = 0$ for $s > 1$. This problem is time-inconsistent.

In the time-inconsistent parameter range, we must consider a MPE when the firm cannot commit. Letting $g(b)$ denote the future policy function, the first-order condition with respect to b_{t+1}^{-} is

$$\frac{1-\tilde{\phi}}{1+r} + \mu^1 - \mu^2 + \tilde{\beta}\tilde{\delta}W'(b') = 0$$

where

$$W'(b') \equiv -1 + g'(b')\left[\frac{1-\tilde{\phi}}{1+r} + \mu^{1'} - \mu^{2'} + \tilde{\delta}W'(g(b'))\right]$$

and one of the solutions is found by setting $g'(b') = 0$. This collapses the first-order

Table 4: Equilibrium with Symmetric Transaction Costs

Household type	c_0	c_1	c_2	Δ_0^-	Δ_1^-	θ_1	θ_2
low	2.5939	2.7925	2.5635	0.3253	0.0000	0.6747	1.5635
high	3.3865	3.1324	3.4365	0.0000	0.8888	1.3253	0.4365

condition into those of Case 3 above. One can apply similar arguments to those in Proposition 2 to show that this is, in fact, the only equilibrium that is the limit of a finite-horizon version of the same problem. □

B Alternative Models

B.1 Symmetric Transaction Costs

Consider a 3-period economy, as in Section 3, but with symmetric transaction costs. That is, the budget constraint (2) now has one additional term $\frac{\phi}{2}(\Delta_t^+)^2 q_t$, which denotes the transaction cost incurred for buying the stock. The new budget constraint is

$$c_t + q_t\Delta_t^+ + \frac{b_{t+1}}{1+r_t} \leq w_t h_t + d_t\theta_t + \left(\Delta_t^- - \frac{\phi}{2}(\Delta_t^-)^2 - \frac{\phi}{2}(\Delta_t^+)^2\right) q_t + b_t$$

and the Euler equation of the household becomes

$$(1+\phi\Delta_t^j)q_t = \beta\frac{u'(c_{t+1}^j)}{u'(c_t^j)}d_{t+1} + \beta\frac{u'(c_{t+1}^j)}{u'(c_t^j)}(1+\phi\Delta_{t+1}^j)q_{t+1}, \quad t \in \{0,1\}$$

where $\Delta_t^j = (\Delta_t^{j+} - \Delta_t^{j-})$. Intuitively, buyers now have an even higher valuation of the stock than in the asymmetric setting because they must incur a transaction cost to buy. Calculations similar to those of Lemma 1 yield a firm objective equal to

$$\sum_{j\in\{l,h\}} \frac{\theta_t^j}{2}[d_t + (1+\phi\Delta_t^j)q_t]$$

With this new setup, we solve a numerical example using the same parameter values as in Section 3. The household equilibrium is summarized in Table 4, and asset prices are given by $q_0 = 1.8560$ and $q_1 = 0.9504$.

The problem of the firm in period 0 becomes

$$V_0^F(k_0) = \max_{k_1,k_2 \geq 0} d_0 + 0.9547 d_1 + 0.9013 d_2$$

and the problem of the firm in period 1 becomes

$$V_1^F(k_1) = \max_{k_2 \geq 0} d_1 + 0.9229 d_2$$

so that, from its period 0 perspective, the firm discounts between time $t = 1$ and $t = 2$ at the rate 0.9441, which is the ratio between 0.9013 and 0.9547. When period 1 arrives, the firm discounts between time $t = 1$ and $t = 2$ at the rate 0.9229. Hence the problem of the firm remains time-inconsistent, with a direction of inconsistency towards present bias. Transaction costs on buyers further restrict asset trading (beyond the case of asymmetric transaction costs), which creates larger differences in IMRS across agents; this exacerbates problems of time-inconsistency.

B.2 Stationary 3-Period Model

Consider a stationary version of the 3-period model laid out in Section 3. Like before, there are only three time periods $t = 0, 1, 2$ and $c_t = 0$ for $t \geq 3$; however $q_2 \neq 0$. The reason households hold stock beyond period 2 is due to an exogenous "continuation value" we append to the household's objective. This trick allows us to induce a stationary 3-period equilibrium with constant asset prices. The household objective becomes

$$\sum_{t=0}^{2} \beta^t u(c_t^j) + \beta^2 \mu^j \theta_3^j$$

The Euler equations (9) remain unchanged; however, the household Euler equation for $t = 2$ becomes

$$q_2 u'(c_2^j)(1 - \phi \Delta_2^{j-}) = \mu^j$$

If μ^j and initial assets θ_0^j are selected appropriately, then the resulting equilibrium will be stationary. In this stationary equilibrium, consumption will simply fluctuate (i.e. c^l, c^h, c^l), asset sales will also fluctuate (i.e. $\Delta^-, 0, \Delta^-$), and asset prices will be a

Table 5: Stationary Equilibrium

Household type	c_0	c_1	c_2	Δ_0^-	Δ_1^-	Δ_2^-	θ_0
low	2.0385	2.1389	2.0385	0.4810	0.0000	0.4810	1.2405
high	2.1389	2.0385	2.1389	0.0000	0.4810	0.0000	0.7595

constant q. Using this notation, the required μ^j are given by

$$\mu^l = \beta u'(c^h)(d+q), \text{ and } \mu^h = \beta u'(c^l)(d + (1-\phi\Delta^-)q)$$

We now re-solve the numerical example in Section 3 with all the same parameter values but one. Moving to this stationary setup, asset prices are larger in magnitude (because we mimic the first-order conditions of an infinite-horizon problem). With larger asset prices, asset trades are smaller and time-inconsistency issues become more difficult to discern. For no other reason than to reduce asset prices, we let $d_t = 0.1$ for all t. The household equilibrium is summarized in Table 5, and the stationary asset price is given by $q = 1.9480$.

The problem of the firm is defined as before, which, in period 0, becomes

$$V_0^F(k_0) = \max_{k_1,k_2\geq 0} d_0 + 0.9401 d_1 + 0.9025 d_2$$

and, in period 1, becomes

$$V_1^F(k_1) = \max_{k_2\geq 0} d_1 + 0.9401 d_2$$

Due to the stationary nature of our new equilibrium, the one-period-ahead discount rate is always the same, 0.9401, and the two-period-ahead discount rate is simply β^2 (that is, consumption at time t and time $t+2$ agree). From its period 0 perspective, the firm discounts between time $t = 1$ and $t = 2$ at the rate 0.9600, which is the ratio between 0.9025 and 0.9401. When period 1 arrives, the firm discounts between time $t = 1$ and $t = 2$ at the rate 0.9401. Hence the problem of the firm is time-inconsistent and, like in Section 3, the direction of inconsistency is towards present bias.

B.3 Liquid Firms

Consider an economy with both liquid and illiquid stocks. The proportion of liquid stocks is $(1-\omega)$, and the proportion of illiquid stocks is ω so that the total mass of stocks is still normalized to one. The special case of $\omega = 1$ corresponds to our main model. With two types of firms in the economy (with potentially differing discount rates), we must restrict our analysis to decreasing returns technologies to retain an interior solution to both firms' problems. The household budget constraint becomes

$$c_t + q_t\Delta_t^+ + \frac{b_{t+1}}{1+r_t} + \hat{q}_t\hat{\theta}_{t+1} \leq w_t h_t + d_t\theta_t + (\hat{d}_t + \hat{q}_t)\hat{\theta}_t + \left(\Delta_t^- - \frac{\phi}{2}(\Delta_t^-)^2\right) q_t + b_t$$

where $\hat{q}_t$ denotes the price of the liquid stock, $\hat{\theta}_{t+1}$ denotes the amount of liquid stock purchased, and $\hat{d}_t$ denotes dividends paid by the liquid firm. Beyond the standard constraints on illiquid stocks, we have the short-selling constraint on the liquid stock and the borrowing constraint

$$\hat{\theta}_{t+1} \geq 0, \quad b_{t+1} \geq \bar{b} \tag{24}$$

Using calculations analogous to those of Lemma 1, the valuation of the liquid firm is given by

$$\hat{d}_t + \hat{q}_t \tag{25}$$

and the Euler equation for the liquid stock is

$$\hat{q}_t = \mathbb{E}_t\left[\beta\frac{u'(c_{t+1})}{u'(c_t)}\right][\hat{d}_{t+1} + \hat{q}_{t+1}] + \hat{\eta}_t$$

where $\hat{\eta}_t$ is the Lagrange multiplier on the liquid short-selling constraint. If there is a household that is neither borrowing constrained nor short-selling constrained, their Euler equation reduces to the no arbitrage condition

$$\hat{q}_t = \frac{\hat{d}_{t+1} + \hat{q}_{t+1}}{1+r_t} \tag{26}$$

To simplify this problem, we note that (26) implies liquid stocks and bonds are redundant, and hence we can combine them into one state variable

$$W_{t+1} \equiv (\hat{d}_{t+1} + \hat{q}_{t+1})\hat{\theta}_{t+1} + b_{t+1}$$

so that the budget constraint can be rewritten

$$c_t + q_t \Delta_t^+ + \frac{W_{t+1}}{1+r_t} \leq w_t h_t + d_t \theta_t + W_t + \left(\Delta_t^- - \frac{\phi}{2}(\Delta_t^-)^2\right) q_t$$

The constraints (24) can be combined

$$W_{t+1} \geq \bar{b}$$

Market clearing conditions for bonds, illiquid stocks, and labor become

$$\begin{aligned}
&\int_{j\in[0,1]} W_{j,t} dj = (1-\omega)(\hat{d}_t + \hat{q}_t) = (1-\omega)\frac{1+r}{r}\hat{d} \\
&\int_{j\in[0,1]} \theta_{j,t} dj = \omega \\
&\int_{j\in[0,1]} h_{j,t} dj = \omega l_t + (1-\omega)\hat{l}_t
\end{aligned}$$

On the first line above, the first equality follows from the fact that the proportion of liquid stocks is $(1-\omega)$, the second equality follows from (26) evaluated at steady-state, and $\hat{d}$ denotes the steady-state value of dividends. On the third line above, $\hat{l}_t$ denotes labor demanded by the liquid firm, which is defined by the intratemporal first-order condition

$$\hat{l}_t = \psi\gamma \frac{\hat{y}_t}{w_t}, \text{ where } \hat{y}_t = [(\hat{l}_t)^\gamma (\hat{k}_t)^{1-\gamma}]^\psi$$

and because the liquid firm maximizes (25) subject to (26), assuming a steady-state interest rate $r_t = r$ for all t, this leads to a steady-state level of capital

$$\hat{k} = \left(\frac{z\alpha}{r+\delta}\right)^{\frac{1}{1-\alpha}}$$

where

$$z_t = (1-\gamma\psi)\left(\frac{\gamma\psi}{w_t}\right)^{\frac{\gamma\psi}{1-\gamma\psi}} \text{ and } \alpha = \frac{(1-\gamma)\psi}{1-\gamma\psi}$$

Labor demanded by the illiquid firm, l_t, is defined by similar conditions

$$l_t = \psi\gamma\frac{y_t}{w_t}, \text{ where } y_t = [(l_t)^{\gamma}(k_t)^{1-\gamma}]^{\psi}$$

and the steady-state level of capital for the illiquid firm, k, depends on commitment versus no commitment. Note that, in either case, $\hat{k} > k^C$ and $\hat{k} > k^{NC}$. This follows from a comparison of their discount rates: the liquid firm discounts at $\frac{1}{1+r}$, the firm with commitment discounts at $\tilde{\delta} = \frac{1-\Phi}{1+r}$, and the firm with no commitment discounts at $\tilde{\beta}\tilde{\delta} = \frac{1-\overline{\Phi}}{1+r}$.

B.4 Private Firms

We add a countable number of households who each own an entire zero-mass firm. Note that the joint mass of these *entrepreneurs* is small enough so as to not affect aggregate variables. Private firms are not traded actively in financial markets, however their valuation can be calculated.[26] The entrepreneur faces budget constraint

$$c_t + \frac{b_{t+1}}{1+r_t} + q_t\Delta_t^+ \leq w_t h_t + d_t\theta + q_t\left(\Delta_t^- - \frac{\phi}{2}(\Delta_t^-)^2\right) + b_t + \hat{d}_t\hat{\theta}_t \tag{27}$$

where the new term $\hat{d}_t\hat{\theta}_t$ denotes dividends paid out by the private firm. Because it is not traded, the law of motion on private firm ownership is trivial

$$\hat{\theta}_{t+1} = \hat{\theta}_t$$

The entrepreneur valuation is calculated

$$\frac{V_{\theta t}}{\lambda_t} = \sum_{s=0}^{\infty}\frac{\mathbb{E}_t[\lambda_{t+s}\hat{d}_{t+s}]}{\lambda_t}$$

where V_t denotes the value function and λ_t denotes the Lagrange multiplier on (27). Plugging in the first-order condition for c_{t+s} and multiplying by λ_t, the objective of

[26] For the same reasons as in Appendix B.3, we restrict our analysis to decreasing returns technologies.

the private firm becomes

$$\sum_{s=0}^{\infty} \beta^s \mathbb{E}_t[u'(c_{t+s})\hat{d}_{t+s}] = \sum_{s=0}^{\infty} \beta^s \mathbb{E}_t[u'(c_{t+s})F(k_{t+s}, k_{t+s+1})]$$

Note that, in period $(t+s)$, the firm chooses k_{t+s+1} and hence the first-order condition is

$$F_2(k_{t+s}, k_{t+s+1}) + \beta \frac{\mathbb{E}_{t+s}[u'(c_{t+s+1})]F_1(k_{t+s+1}, k_{t+s+2})}{u'(c_{t+s})} = 0$$

where the expectation is only applied to marginal utility because $F_1(k_{t+s+1}, k_{t+s+2})$ does not depend on k_{t+s+2}. We call the discount factor

$$\hat{\beta}_t(\theta, b, h) = \beta \frac{\mathbb{E}_t[u'(\mathcal{C}_{t+1}(\theta, b, h))]}{u'(\mathcal{C}_t(\theta, b, h))}$$

We assume that entrepreneurs are drawn independently from the population $\Gamma(\theta, b, h)$. Then by the strong law of large numbers, the average amount of investment by private firms approaches

$$\hat{k} = \int_{\theta,b,h} k(\hat{\beta}_t(\theta, b, h)) d\Gamma(\theta, b, h), \text{ where } k(x) = \left(\frac{z\alpha x}{1 - x(1-\delta)} \right)^{\frac{1}{1-\alpha}}$$

almost surely, and the fact that $\hat{k}$ does not depend on t relies on the stationarity of the household equilibrium.