Liquidity and Investment in General Equilibrium*

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Abstract

This paper studies the implications of trading frictions in financial markets for firms’ investment and dividend choices, and their aggregate consequences. When equity shares trade in frictional asset markets, the firm’s problem is time-inconsistent, and it is as if it faces quasi-hyperbolic discounting. The transmission of trading frictions to the real economy crucially depends on the firms’ ability to commit. In a calibrated economy without commitment, larger trading frictions imply lower capital and production. In contrast, if firms can commit, trading frictions affect asset prices but have no aggregate effect on capital and production. Our findings rationalize several empirical regularities on liquidity and investment.

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1 Introduction

Asset liquidity matters for a wide range of macroeconomic outcomes such as asset prices (e.g., Aiyagari and Gertler, 1991; Amihud et al., 2005), consumption risk-sharing (e.g., Kaplan et al., 2014, 2018), and firm investment, the capital structure, and the transmission of monetary policy (e.g., Amihud and Levi, 2022; Cui and Radde, 2020; Jeenas and Lagos, 2023). Importantly, liquidity frictions generate heterogeneous valuations: agents disagree on how to discount future cash flows. In this paper, we study the consequences of this disagreement on aggregate investment and asset prices in general equilibrium. Because firm owners disagree, the aggregation of preferences leads to a discount factor that is time-inconsistent. Our findings help rationalize several empirical regularities concerning the relationship between stock market liquidity and investment in the cross-section and in the aggregate (Amihud and Levi, 2022; Naes et al., 2011), and the short-termism observed in stock market-listed firms (Graham et al., 2005; Terry, 2023).

We present a theory of investment in an incomplete markets production economy in which households trade firms’ shares subject to transaction costs. Because of financial frictions, agents disagree on the valuation of the firm and, crucially, on the discount factor used to value cash flows. Our main result is that trading frictions generate a disagreement between owners’ discount factors that leads to a problem for the firms that is time-inconsistent: the optimal investment plan at any point in time is suboptimal at future dates. Moreover, we show that the firms’ problem is equivalent to one in which there are no financial frictions but the firm uses a stochastic discount factor with quasi-hyperbolic discounting. Notably, we obtain this result from the frictions in financial markets rather than behavioral assumptions. In a calibrated economy, we find that the transmission of trading frictions to the real economy is highly sensitive to the firms’ ability to commit to an investment plan. If firms cannot commit, trading frictions can have significant adverse effects on investment and output due to present bias. In contrast, if firms can commit, trading frictions affect asset prices but have very small effects on the aggregate level of capital.

We build a model with two standard blocks: incomplete markets as in Aiyagari (1994), and neoclassical firms. Households face income risk and have access to two financial markets: a market for the stock of the representative firm and a market for risk-free one-period bonds. In the spirit of Aiyagari and Gertler (1991) and Kaplan
et al. (2014), we assume that stock trading is subject to transaction costs while the bond market is frictionless but agents can borrow only up to a limit. Unlike Aiyagari and Gertler (1991) and Kaplan et al. (2014), we consider a production economy, which allows us to study the implications of financial market frictions on aggregate investment. Crucially, we assume that firms make the investment decisions and, therefore, they have a dynamic problem. This requires to find an appropriate discount factor for the firms. In the presence of transaction costs, households’ expected rates of return for holding the stock do not equalize. As a consequence, owners of the firm disagree on the discount factor the firm should use and the implied investment plan.\footnote{Absent aggregate risk and transaction costs, households’ expected returns coincide, so they all agree on the discount factor for the firms’ cash flows and the investment plan. In particular, it is optimal for the firm to choose the plan that maximizes its market valuation.} There is a long tradition in microeconomics that studies ways to specify the firm’s objective in this scenario (see DeMarzo, 1993; Dreze, 1974; Grossman and Hart, 1979; Makowski, 1983). In this paper, we assume that the firm maximizes the weighted valuation of owners, where the weights are given by the initial stock holdings.\footnote{We show that our results are similar under alternative weights such as Dreze (1974), or majority voting (DeMarzo, 1993).}

Disagreement about the discount factor leads to the paper’s main result: the firm’s problem is \textit{time-inconsistent}. To understand why, consider the tradeoffs involved in the design of the firm’s investment plan. Investment is an intertemporal decision: agents give up consumption in period $t$ (through the distribution of dividends) for a higher level of production in $t + 1$. If the discount factor of owners and buyers are not equal, they will disagree. Hence, owners and buyers disagree on the optimal investment plan for the firm. In particular, if owners are more impatient than buyers (which they typically are in this economy), owners will favor a higher level of dividends and a lower level of investment than buyers. In contrast, when planning investment multiple periods into the future, owners would like to commit to a high level of investment to satisfy the buyers’ preferences and increase the firm’s market value. However, if the decisions are revised in $t + 1$ by the new set of owners, they will be tempted to again lower the investment level in $t + 1$ and promise a higher level of investment starting in $t + 2$. Thus, the problem of the firm is time-inconsistent.

We show that the firm’s problem is equivalent to one in which the firm exhibits quasi-hyperbolic discounting. Notably, transaction costs are crucial for these results. In the absence of transaction costs (e.g., as in Aiyagari, 1994), owners’ and buyers’
discount factors are equalized, and the problem of the firm is time-consistent, as all owners discount payoffs using the risk-free rate.

We then calibrate the model and study its quantitative properties in general equilibrium. Most of the parameters are standard and we take them from existing literature. The main parameter to calibrate is the transaction cost. We consider stock data from the Center for Research in Security Prices (CRSP) between 2000 and 2022. The median relative spread is 2.8%, consistent with previous studies (e.g., Naes et al., 2011; Corwin and Schultz, 2012; Goyenko et al., 2009; Abdi and Ranaldo, 2017). We discipline the transaction cost to match the data on relative spreads, but we also analyze the economy for alternative values of trading frictions. Finally, as non-targeted moments, the model is consistent with the composition of liquid and illiquid assets.

We find that present bias is the empirically relevant case. When there are no trading frictions, the model is isomorphic to the one-asset economy in Aiyagari (1994). In this case, the steady-state level of capital is higher than in a complete markets economy due to precautionary savings. With trading frictions and commitment, we find that trading frictions affect asset prices but have minor consequences for aggregate capital. On the one hand, trading frictions depress asset prices, implying a lower steady-state level of capital. On the other hand, there is a higher precautionary motive for saving, implying a higher steady-state level of capital. Quantitatively, these two forces are of similar magnitude, and, as a result, capital does not change significantly. However, without commitment, there is a third force at play: present bias. Quantitatively, this force strongly favors more discounting and, as a result, we obtain a lower level of capital than in the complete markets economy, contrary to the overaccumulation result in Aiyagari (1994). This result illustrates that the assumptions about trading frictions and the firm’s problem are essential for understanding both aggregate quantities and asset prices.

Finally, we extend the main framework in several dimensions. First, we consider a setting with heterogeneous firms which differ in the illiquidity of their stock. Consistent with the cross-sectional empirical evidence in Amihud and Levi (2022), we find that firms that face higher transaction costs invest less than those with lower transaction costs. The cross-sectional evidence, however, is not enough to understand the aggregate effects of liquidity on investment. Instead, we use the model with heterogeneous firms to evaluate the aggregate implications. We find that as the transaction cost increases, aggregate capital decreases if firms cannot commit, while it increases if
firms can commit. Hence, the aggregate effects are qualitatively different depending on the firms’ ability to commit. These results highlight that: (i) the model is consistent with the cross-sectional evidence, and (ii) the cross-sectional evidence alone is insufficient to make aggregate predictions.

In a second extension, we allow the firm to borrow in illiquid corporate bonds markets. We find that the problem of the firm is still time-inconsistent. Corporate bonds can affect the firm’s financial structure but have no impact on investment decisions, as these are made by the stockholders. Even if corporate bonds are more illiquid than stocks, a firm without commitment may issue bonds due to present bias. This result provides a reason for corporate borrowing that does not rely on the tax advantage of debt.

In a third extension, we study how the demand and supply of liquid assets affect aggregate capital. On the demand side, we consider an increase in the idiosyncratic uncertainty, which raises the precautionary savings motive. When firms can commit, the increase in precautionary savings reduces the interest rate and increases aggregate capital. When firms cannot commit, more uncertainty also implies a more severe time-inconsistency problem, which generates a reduction of aggregate capital. On the supply side, we incorporate liquid government bonds into the model. Once again, we find that the aggregate effects depend on firms’ commitment. When firms can commit, an increase in the supply of government bonds leads to a lower level of capital because the interest rate increases. When firms cannot commit, however, we get a higher level of capital because there is a less severe time-inconsistency problem. Thus, under no commitment, a higher supply of government debt crowds-in capital investment.

**Related Literature.** The paper is related to several strands of the literature.

First, there is an ample empirical, theoretical, and quantitative literature arguing that liquidity matters for asset pricing (e.g., Amihud et al., 2005; Duffie et al., 2005; Lagos and Rocheteau, 2009; Lagos, 2010). More specifically, our results relate to the literature that studies the effects of transaction costs on financial markets (e.g., Constantinides, 1986; Aiyagari and Gertler, 1991; Heaton and Lucas, 1996; Vayanos, 1998; Gârleanu and Pedersen, 2013; Abel et al., 2013). This literature considers either exogenous dividend streams or endowment economies and focuses on the asset-

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3 See Vayanos et al. (2012) for a survey of this literature.
pricing implications, abstracting from issues such as how trading frictions affect the firm’s problem and the implications for investment.

Second, recent papers in macroeconomics incorporate asset liquidity frictions. For example, Kaplan and Violante (2014) and Kaplan et al. (2018) study the importance of asset liquidity for consumption risk-sharing. Cui and Radde (2020) and Jeenas and Lagos (2023) study the effects of liquidity on firms’ investment decisions. However, this literature does not study the implications of owner disagreement. For example, Kaplan et al. (2018) assumes that firms are owned by a financial intermediation sector, so households do not have direct holdings of firms’ stocks. Cui and Radde (2020) assumes that households directly own the capital of the economy, so that the problem of the firm is static. Jeenas and Lagos (2023) assumes that investment decisions are made by a single entrepreneur. As a consequence, liquidity frictions do not directly affect the firms’ problem but only through their impact on asset prices. In our paper, liquidity frictions affect asset prices but also generate disagreement among owners, which leads to a problem for the firms that is time-inconsistent. Closest to our paper is Carceles-Poveda and Coen-Pirani (2010), who study the implications of household and firm ownership of capital in heterogeneous agents models. Their focus is on cases that lead to an equivalence result: whether the households or the firms own the capital is irrelevant for equilibrium under the appropriate choice of the firms’ discount factor. We focus instead on cases where the distinction matters. Transaction costs are crucial for our analysis; absent transaction costs, we recover the equivalence result.\footnote{Relatively, see Jermann (1998) and Cochrane (2008) for production-based asset pricing models.}

Third, the paper is also related to the literature that studies the problem of the firm in economics with incomplete markets (see Diamond, 1967; Dreze, 1974; Grossman and Hart, 1979; DeMarzo, 1993; Makowski, 1983).\footnote{See also Carceles-Poveda and Coen-Pirani (2009).} These papers focus on aggregate risk and study models with a finite number of time periods. Our paper considers an infinitely lived economy with idiosyncratic risk and trading frictions but no aggregate risk. In a steady-state equilibrium, we show that the firms’ problem can be expressed as featuring quasi-hyperbolic discounting. Notably, we do not assume quasi-hyperbolic discounting as a behavioral phenomenon (as in Kang and Ye, 2019), but it arises endogenously from trading frictions in asset markets (see also Amador,\footnote{See Magill and Quinzii (2002) for a review of the literature.}}
Recent papers have also encountered the problem of the appropriate discount factor of the firm in incomplete market economies. For example, Favlukis (2013) and Favlukis et al. (2017) similarly use a portfolio-weighted average of the agents’ intertemporal marginal rates of substitution (IMRS) as the firms’ discount factor. However, they do not consider the potential time-inconsistency problem. Jackson and Yariv (2015) show, in a very different setting than ours, that preference aggregation naturally leads to time inconsistency. Present bias obtains because, in the distant future, weighted discount rates are determined by the most patient individual. Espino et al. (2018) and Bisin et al. (2022) study the insurance properties of firm ownership. Espino et al. (2018) consider privately owned firms and focus on the solution of the centralized allocation with a finite number of agents and private information. Bisin et al. (2022) study how investors’ hedging demand shapes the firms’ capital structure. Instead, our paper focuses on how trading frictions affect firms’ investment decisions.

Finally, there is an extensive literature on short-termism showing that public firms are concerned about meeting short-term targets. For example, a survey by Graham et al. (2005) shows that almost half of U.S. executives would prefer to reject a positive net present value project over missing their target. Terry (2023) finds that firms barely meeting Wall Street forecasts have lower R&D growth. Our theory provides an alternative, rational explanation for short-termism, namely, as the outcome of conflicting objectives among different owners when markets are frictional.

The paper is organized as follows. Section 2 presents the household and firm problems. Section 3 considers a simple three-period example to understand the sources of time inconsistency. Section 4 defines the equilibrium in the infinite horizon economy, and Section 5 describes the solution to the quasi-hyperbolic firm problem. Section 6 presents the quantitative evaluation. Finally, Section 7 concludes.

2 The Model

We study an Aiyagari economy augmented to incorporate transaction costs on financial assets. Time is discrete and denoted by $t = 0, 1, 2, \ldots$ Households face labor income risk, and markets are incomplete. In particular, households have access to only two financial markets: a market for the stock of the representative firm and a market for a risk-free bond. The stock market is subject to transaction costs which
we model as a wasteful use of resources.\textsuperscript{7} The market for bonds is frictionless but agents can borrow (i.e., negative bond holdings) up to a limit.\textsuperscript{8} Moreover, there is a representative firm that combines labor and capital to produce the final consumption good. There is no aggregate risk.

**Households.** The economy is populated by a measure one of households, indexed by $j \in [0, 1]$. Households are subject to idiosyncratic labor shocks that determine the number of hours they can sell in the labor market, which we label as their employment status and denote by $h$. We assume that $h$ is drawn from a finite set $\mathbb{H} = \{h_1, h_2, \ldots, h_S\}$, where $h_1 < h_2 < \cdots < h_S$, with associated transition density $dF(h_s|h_s)$. Households’ preferences can be represented by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t),$$

where $\mathbb{E}_t$ represents the expectation operator conditional on the information set at period $t$, $\beta$ is the households’ subjective discount factor, $c_t$ denotes consumption in period $t$, and $u(\cdot)$ is a continuously differentiable and strictly concave function that satisfies Inada conditions.

Markets in this economy are incomplete. Households can trade two classes of financial assets: the stock of the representative firm and a one-period risk-free bond. Trade in the stock market entails a transaction cost. Let $q_t$ denote the price of a share, and $\Delta_t^{-}$ and $\Delta_t^{+}$ denote the shares sold and bought by a household. We assume that the transaction cost incurred in a sale is $\frac{1}{2} (\Delta_t^{-})^2 q_t$. Quadratic transaction costs simplify the solution of the agents problem, as it avoids the existence of inaction regions.\textsuperscript{9} Short-selling is not allowed in this market. In contrast, trading bonds is frictionless but households face a borrowing limit $\bar{b}$. Thus, the constraints faced by a

\textsuperscript{7}The result would not change if we assumed that transaction costs are paid to a financial intermediary.

\textsuperscript{8}We think of bonds as bank deposits and loans rather than assets that trade in frictional markets like corporate bonds. See Aiyagari and Gertler (1991).

\textsuperscript{9}See Heaton and Lucas (1996).
household in period $t$ are

$$c_t + q_t \Delta_t^+ + \frac{b_{t+1}}{1 + r_t} \leq w_t h_t + d_t \theta_t + \left( \Delta_t^- - \frac{\phi}{2} (\Delta_t^-)^2 \right) q_t + b_t, \quad (2)$$

$$\theta_{t+1} = \theta_t + \Delta_t^+ - \Delta_t^-, \quad (3)$$

$$\Delta_t^+ \geq 0, \quad \theta_t \geq \Delta_t^- \geq 0, \quad (4)$$

$$b_{t+1} \geq b, \quad (5)$$

where $b_t$ denotes the holdings of one-period risk-free bonds, $r_t$ is the real interest rate, $w_t$ is the real wage, $\theta_t$ denotes the household’s holdings of stock at the beginning of period $t$, and $d_t$ denotes the dividends distributed by the representative firm. Equation (2) is a standard budget constraint. The household receives labor income, $w_t h_t$, dividend income, $d_t \theta_t$, the proceeds from the sale of stock net of transaction costs, $(\Delta_t^- - \frac{\phi}{2} (\Delta_t^-)^2) q_t$, and the maturing bonds, $b_t$. They use their income to consume, $c_t$, buy stock, $q_t \Delta_t^+$, and buy bonds, $\frac{b_{t+1}}{1 + r_t}$. Equation (3) represents the law of motion of stock holdings. Condition (4) imposes natural constraints on trades, ensuring that purchases and sales are non-negative and that the household does not sell more shares than it owns. Finally, equation (5) represents the borrowing constraint. We assume that $\bar{b} = -\sum_{t=1}^{\infty} \prod_{s=1}^{t-1} w_t h_t \frac{k_{t+1}^{1-\gamma}}{1+r_s}$. Thus, the problem of a household is to choose processes $\{c_t, b_{t+1}, \theta_{t+1}, \Delta_t^+, \Delta_t^-\}_{t=0}^{\infty}$ in order to maximize (1) subject to (2), (3), (4), and (5) for every $t \geq 0$, and given an initial portfolio $(\theta_0, b_0)$ and prices and dividends $\{w_t, q_t, r_t, d_t\}_{t=0}^{\infty}$.

**Firms.** There is a representative firm that operates a non-increasing returns to scale technology that combines labor and capital to produce the final consumption good, given by

$$y_t = \left( l_t^{\gamma} k_t^{1-\gamma} \right)^\psi,$$

where $l_t$ denotes the amount of labor hired, $k_t$ denotes the amount of capital operated, $\gamma \in (0, 1)$ and $\psi \leq 1$.\(^{10}\) Moreover, the firm operates the economy’s investment technology, $k_{t+1} = (1 - \delta)k_t + i_t$, where $i_t$ denotes the level of investment in period $t$

\(^{10}\)In the background, we are assuming that there is a measure one of identical firms with non-increasing returns to scale. Since all firms make the same choices in equilibrium, we have $k_i = k$ and $l_i = l$ for all $i$. Then, there exists a representative firm with the same technology that operates all the capital and labor, i.e., $\int_0^1 k_i di = k$ and $\int_0^1 l_i di = l$. Finally, we assume that the entry costs are such that there is no firm entry in equilibrium.
and \( \delta \in (0, 1) \) is the depreciation rate.

As is standard, we assume that the firm acts in the best interest of its shareholders. Because the choice of labor is an intratemporal decision, it is immediate that optimality implies \( l_t = \gamma \psi w_t \). Let \( \pi_t = y_t - w_t l_t \) denote the firm’s per-period profits. Then, \( \pi_t = (1 - \gamma \psi) y_t \), or \( \pi_t = z_t k_t^\alpha \), where \( z_t = (1 - \gamma \psi) \left( \frac{\gamma \psi}{w_t} \right)^{1-\gamma \psi} \) and \( \alpha = \frac{(1-\gamma)\psi}{1-\gamma \psi} \). Dividends are then given by \( d_t = \pi_t - i_t \), or

\[
d_t = F_t(k_t, k_{t+1}) = z_t k_t^\alpha + (1 - \delta) k_t - k_{t+1}.
\]

Investment is an intertemporal decision that requires knowledge of shareholders’ intertemporal preferences. Because of the assumed financial frictions, shareholders may have conflicting preferences about the firm’s investment plans. In this paper, we assume that the firm maximizes a weighted average of the shareholders’ valuations, where the weights are given by their stock holdings at the beginning of the period (see Grossman and Hart, 1979; Favilukis, 2013). We proceed in two steps. First, we compute the shareholder’s valuations. Then, we specify the firm’s objective.

**Shareholders’ valuations.** Let \( \tilde{q}_t(\theta, b, h) \) denote the shareholder’s valuation of the firm in units of the consumption good, which is given by

\[
\tilde{q}_t(\theta, b, h) = \frac{V_{\theta t}(\theta, b, h)}{\lambda_t(\theta, b, h)} = d_t + \frac{\mu_t(\theta, b, h) + \eta_t^{-}(\theta, b, h)}{\lambda_t(\theta, b, h)},
\]

where \( V_t(\theta, b, h) \) denotes the value function in period \( t \) of a household with stock holdings \( \theta \), bond holdings \( b \), and employment status \( h \), and \( V_{\theta t}(\theta, b, h) \) denotes the envelope condition of a shareholder (i.e., a household with \( \theta > 0 \)) with respect to \( \theta \), \( \lambda_t(\theta, b, h) \) denotes the marginal utility of wealth (measured as the Lagrange multiplier associated to the budget constraint (2)), \( \mu_t(\theta, b, h) \) denotes the marginal utility of an extra unit of the stock (measured by the Lagrange multiplier of constraint (3)), and \( \eta_t^{-}(\theta, b, h) \) denotes the Lagrange multiplier associated to the short-selling constraint in (4).

The shareholder’s valuation has two components. First, shareholders value the dividend they receive, \( d_t \). Since the dividend is in units of the consumption good, all shareholders agree on its valuation. The second term denotes the *ex-dividend* value of the stock. Since the stock is a long-lived asset, agents value holding shares of the
firm above and beyond the dividend they receive in the current period. This term may differ across agents.

There are two types of shareholders: those who keep all their holdings (and potentially buy more) and those who sell at least part of their portfolio. We call them buyers and sellers, respectively. Absent transaction costs, this distinction would be inconsequential: at the margin, the valuation of buyers and sellers would always coincide. However, transaction costs introduce a wedge in the agents’ valuation, generating disagreement. The following lemma characterizes the shareholders’ valuation. All proofs are in Appendix A.

**Lemma 1.** The shareholders’ valuation of the firm is given by

\[ \tilde{q}_t(\theta, b, h) = d_t + (1 - \phi \Delta^-_t(\theta, b, h))q_t. \]  

(6)

Lemma 1 characterizes the valuation of all agents that start the period with \( \theta > 0 \). Note that since buyers choose \( \Delta^-_t = 0 \), their valuation simplifies to \( \tilde{q}_t(\theta, b, h) = d_t + q_t \), which is independent of agent-specific variables. In contrast, the seller’s valuation is decreasing in the amount of stock they sell. The more the agent sells, the higher the transaction cost and the lower the benefits of holding the stock.

Next, we use Lemma 1 to specify the firm’s objective.

**The firm’s problem.** There is a vast literature studying the problem of the firm when shareholders disagree.\(^{11}\) In this paper, we assume that the firm maximizes an ownership-weighted valuation, which can be thought of as giving shareholders votes in proportion to their holdings, in the spirit of Grossman and Hart (1979) (see also Favilukis, 2013).\(^{12}\)

**Assumption 1.** The firm maximizes an ownership-weighted valuation given by:

\[ \int_{\theta, b, h} \theta \left[ d_t + (1 - \phi \Delta^-_t(\theta, b, h))q_t \right] d\Gamma_t(\theta, b, h) = d_t + (1 - \Phi_t)q_t, \]  

(7)

where \( \Gamma_t(\theta, b, h) \) denotes the cross-section distribution over the portfolio holdings and

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\(^{12}\)In Appendix B, we generalize the weights used in the firm’s problem to encompass weighting by future ownership as in Dreze (1974) and majority voting as in DeMarzo (1993). We show that our results are consistent with these alternative specifications.
employment status, \( d_t = F_t(k_t, k_{t+1}) \), and
\[
\Phi_t \equiv \phi \int_{\theta, b, h} \theta \Delta_t^{-1}(\theta, b, h) d\Gamma_t(\theta, b, h).
\] (8)

Maximizing the firms value involves choosing the path of capital that trades off the effects on the dividend distributed in period \( t \), \( d_t \), and the continuation market value, \( q_t \). For example, higher investment today implies a lower current dividend but potentially higher dividends in the future. Crucially, the continuation value \( q_t \) is discounted by \( \Phi_t \), which represents the weighted-average marginal cost faced by current owners. Consider the valuations of owners who sell some of their holdings relative to owners who keep everything. Since the firm can distribute dividends costlessly, those who sell tend to favor the distribution of dividends at the expense of a higher continuation value. The firm aggregates these differences by internalizing a weighted-average transaction cost in its problem.

A crucial determinant of the firm’s problem is the sensitivity of the stock price, \( q_t \), to the firm’s investment plan. Naturally, the firm does not directly control \( q_t \), which is determined in equilibrium. However, the firm understands that the equilibrium value of \( q_t \) depends on its investment decisions through their consequences on the dividend payout plan. In the case where \( \phi = 0 \), determining this sensitivity is relatively simple, as we show next.

**The frictionless case.** Consider the benchmark case without transaction costs, i.e. \( \phi = 0 \). In this case, the firm’s objective simplifies to \( d_t + q_t \), and the price is equal to
\[
q_t = \sum_{s=1}^{\infty} \prod_{z=0}^{s-1} \left( \frac{1}{1 + r_{t+z}} \right) d_{t+s},
\]
where the firm takes the sequence of interest rates \( \{r_{t+z}\}_{z=0}^\infty \) as given. When \( \phi = 0 \), bonds and stocks are perfect substitutes, so their rates of return must equalize. Moreover, all shareholders agree that the appropriate discount of the firm’s dividends is \( \frac{1}{1 + r_{t+z}} \). Thus, \( \{r_{t+z}\}_{z=0}^\infty \) summarizes the shareholders’ intertemporal preferences, and the problem of the firm is to maximize the net present value of all future dividends.

This result is similar to Makowski (1983), which shows that in a setting with incomplete markets and trading of firms’ stock market value maximization is an objective for the firm that is unanimously favored by its shareholders. Absence of
transaction costs is crucial for their result: in Makowski (1983), households’ IMRS do not equalize but shareholders nonetheless agree on the appropriate discount factor for the firm. In contrast, the presence of transaction costs generates dispersion in the agents’ discount factors, which leads to the intertemporal disagreement that we will discuss next.

When $\phi > 0$, we get two important differences. First, shareholders disagree on the trade-off between the current dividends and the firm’s continuation value. The firm internalizes this disagreement with Assumption 1. Second, because stocks are more illiquid than bonds, the rate the firm uses to discount future dividends is different from the interest rate on the liquid asset, $r_t$. To tackle this problem, we first solve a simplified 3-period version of the model in Section 3. We turn back to the infinite-horizon setting in Section 4.

3 Motivating Example: A 3-Period Model

Before considering the infinite-horizon economy described above, we study a simplified 3-period version that will allow us to identify the source of time-inconsistency in the firm’s problem. Because there are only three time periods, $t = 0, 1, 2$, we have that $c_t = 0$ for $t \geq 3$ and (2) – (5) only hold for $t = 0, 1, 2$. Note that $q_2 = 0$ because there is no need to hold assets beyond $t = 2$.

Besides assuming a shorter time horizon, we make two additional simplifying assumptions relative to the model described in Section 2. First, we assume that households face no income risk. Instead, we follow Woodford (1990) and assume that households’ employment status oscillates in a deterministic fashion between a low value $h_{\text{low}}$ and a high value $h_{\text{high}}$. We assume that half of the households receive the low labor endowment in period $t = 0$, and the other half receive the high labor endowment. We refer to these two groups as the low and high groups $j \in \{l, h\}$, respectively. Second, we assume that households cannot borrow, that is, the borrowing limit is $b = 0$. Thus, there effectively is a single financial asset in the economy, the illiquid stock. All other aspects of the example are as described in the previous

\[13\]When we solve the infinite-horizon in Section 5, we will focus on a stationary equilibrium where aggregate variables are constant over time. Since the 3-period model has a final period where the stock price is zero, a stationary equilibrium does not exist. In Appendix C.1, we present a natural extension of this model that generates a stationary equilibrium. All our results go through in that setting.
Our main object of interest is the firm’s intertemporal problem. We first characterize the solution for a firm that commits in period 0 to an investment path for capital in periods 1 and 2. Then, we show that if the firm is allowed to reoptimize in period 1, it chooses a different allocation for capital in period 2. Hence, the problem is not time-consistent.

The firm maximizes the value to its shareholders, which by Assumption 1 is given by:

$$\sum_{j \in \{t, k\}} \frac{\theta_t^j}{2} [d_t + (1 - \phi \Delta_t^j) q_t], \quad t \in \{0, 1\},$$

with $d_t = F_t(k_t, k_{t+1})$. As discussed above, while $q_t$ is determined in equilibrium, the firm understands that its decisions (investment, dividend payout policy) will impact its value. Here, we assume that the firm knows that the stock price satisfies the following households’ Euler equations

$$(1 - \phi \Delta_t^j) q_t = \beta \frac{u'(c_{t+1}^j)}{u'(c_t^j)} d_{t+1} + \beta \frac{u'(c_{t+1}^j)}{u'(c_t^j)} (1 - \phi \Delta_{t+1}^j) q_{t+1}, \quad t \in \{0, 1\}.$$  

Note that we focus on equilibria where the short-selling constraint does not bind. In the numerical illustration below, we choose parameter values that satisfy this requirement. Then, introducing (10) into (9), we get that the firm’s value in period 0 is given by

$$V_0^F(k_0) = \max_{k_1, k_2 \geq 0} \sum_{j \in \{t, k\}} \frac{\theta_0^j}{2} \left[ d_0 + \beta \frac{u'(c_1^j)}{u'(c_0^j)} d_1 + \beta^2 \frac{u'(c_2^j)}{u'(c_0^j)} d_2 \right],$$

where $d_t = F_t(k_t, k_{t+1})$ and the firm takes the households’ consumption plans as given. The optimal investment plan from period 0’s perspective is the solution to problem (11).

The stochastic discount factor of the firm in problem (11) corresponds to a weighted average of the shareholders’ IMRS implied by the household’s Euler equations (10), where the weights correspond to the shareholder’s proportional ownership in the firm.

---

14 In Appendix C.2, we also consider a version of this 3-period model with symmetric transaction costs for buyers and sellers. All our results go through in that setting.

15 This is an application of the standard “big D little d” argument. The firm takes aggregate consumption—which depends on dividends—as given, but optimizes on its own dividend payments.
Note that this discount factor corresponds to the assumption in other papers such as Favilukis (2013) and Favilukis et al. (2017). The key difference is that we analyze the potential time-inconsistency problem arising from this discounting.

**Time Inconsistency**  Consider now the problem of the firm in period 1. The firm starts the period with capital stock \( k_1 \), which was chosen in period 0. Suppose we give the firm the option to reoptimize its investment plan, i.e., the choice of \( k_2 \). If the plan in period 0 is time-consistent, the firm would not have an incentive to change its choice. However, we show next that the firm may choose to change its plan; that is, its problem may be time-inconsistent. To see this, note that the firm’s problem in period 1 along the equilibrium path is

\[
V_1^F(k_1) = \max_{k_2 \geq 0} \sum_{j \in \{l, h\}} \frac{\theta_j^1}{2} \left[ d_1 + (1 - \phi \Delta_1^j) q_1 \right]
\]

\[
= \max_{k_2 \geq 0} \sum_{j \in \{l, h\}} \frac{\theta_j^1}{2} \left[ d_1 + \beta \frac{u'(c_j^2)}{u'(c_j^1)} d_2 \right]
\]

where \( \theta_1^j = \theta_0^j + \Delta_1^j + \Delta_1^{j-} \). Then, the firm’s problem is time-consistent if and only if the discounting between \( t = 1 \) and \( t = 2 \) in problem (11) and problem (12) coincide, or, equivalently, if

\[
\sum_{j \in \{l, h\}} \frac{\theta_0^j}{2} \beta^2 \frac{u'(c_j^2)}{u'(c_j^0)} = \sum_{j \in \{l, h\}} \frac{\theta_1^j}{2} \beta \frac{u'(c_j^2)}{u'(c_j^1)}.
\]

First consider the case with \( \phi = 0 \). The Euler equation (10) becomes

\[
\frac{q_t}{d_{t+1} + q_{t+1}} = \beta \frac{u'(c_{t+1}^j)}{u'(c_t^j)}.
\]

Crucially, in this case, the IMRS are equalized across agents. In this case, the firm’s problem is time-consistent:

\[
\sum_{j \in \{l, h\}} \frac{\theta_0^j}{2} \beta^2 \frac{u'(c_j^2)}{u'(c_j^0)} = \frac{q_0}{d_1 + q_1} = \frac{q_1}{d_2 + q_2} = \sum_{j \in \{l, h\}} \frac{\theta_1^j}{2} \beta \frac{u'(c_j^2)}{u'(c_j^1)}.
\]
Table 1: Equilibrium in the 3-Period Model

<table>
<thead>
<tr>
<th>Household type</th>
<th>$c_0$</th>
<th>$c_1$</th>
<th>$c_2$</th>
<th>$\Delta_0$</th>
<th>$\Delta_1$</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>low</td>
<td>2.6226</td>
<td>2.7290</td>
<td>2.6030</td>
<td>0.3358</td>
<td>0.0000</td>
<td>0.6642</td>
<td>1.6031</td>
</tr>
<tr>
<td>high</td>
<td>3.3667</td>
<td>3.2271</td>
<td>3.3970</td>
<td>0.0000</td>
<td>0.9389</td>
<td>1.3358</td>
<td>0.3969</td>
</tr>
</tbody>
</table>

When financial markets are frictionless, agents’ valuations of consumption across periods coincide. As a consequence, they will agree on the optimal investment plan for the firm across periods, and the firm’s problem is time-consistent.

In contrast, when $\phi > 0$, the agents’ IMRS might not equalize if there is positive trade (i.e., if $\Delta_t^j > 0$ for some households). In that case, there is no guarantee that condition (13) will hold. In particular, as agents’ valuation and ownership change, the optimal plan for the firm may also change. We illustrate this with a numerical example.

Suppose $w_t = 1$ and $d_t = 1$ for all $t$, $u = ln(c)$, $\phi = 0.1$, $\beta = 0.95$, $h_{low} = 1$, and $h_{high} = 3$.\(^{16}\) Initial asset holdings are $\theta_0 = 1$. The household equilibrium is summarized in Table 1, and asset prices are given by $q_0 = 1.8856$ and $q_1 = 0.9960$.

The problem of the firm in period 0 becomes

$$V_0^F(k_0) = \max_{k_1, k_2 \geq 0} d_0 + 0.9520d_1 + 0.9019d_2,$$

and the problem of the firm in period 1 becomes

$$V_1^F(k_1) = \max_{k_2 \geq 0} d_1 + 0.9335d_2.$$  

Now let’s compare the firm’s problem in $t = 0$ and $t = 1$ along the equilibrium path. From its period 0 perspective, the firm’s discount factor between $t = 1$ and $t = 2$ is 0.9474, which is the ratio between 0.9019 and 0.9520. When period 1 arrives, the firm’s discount factor between $t = 1$ and $t = 2$ is 0.9335. Hence, the problem of the firm is time-inconsistent: the time preference between $t = 1$ and $t = 2$ is different in $t = 0$ than in $t = 1$. Furthermore, in this calibration, the direction of inconsistency is towards present bias. That is, when period 1 arrives, the firm discounts period 2 more than it did in period 0.

The purpose of this example was to show, in a simple and transparent model,\(^{16}\) We can obtain $w_t = 1$ and $d_t = 1$ by choosing the parameters of the production function and the capital accumulation technology appropriately.
that time-inconsistency is a natural outcome in the absence of IMRS equalization across agents. Intratemporal disagreement among current shareholders, as well as intertemporal disagreement among current and future shareholders, can make the optimal plan—from period 0’s perspective—suboptimal in period 1.

The key step that allowed us to characterize the solution was to replace the price of the stock, $q_t$, from the firm’s objective by substituting the households’ Euler equation:

$$
(1 - \phi \Delta_{t-1}^i) q_0 = \beta \frac{u' (c_{t}^j)}{u'(c_0^j)} d_1 + \beta \frac{u' (c_{t}^i)}{u'(c_0^i)} (1 - \phi \Delta_{t-1}^i) q_1,
$$

$$
= \beta \frac{u' (c_{t}^j)}{u'(c_0^j)} d_1 + \beta^2 \frac{u' (c_{t}^j)}{u'(c_0^j)} d_2,
$$

which holds true when $q_2 = 0$ and short-selling constraints are not binding. Note, however, that this substitution requires a great deal of information on the part of the firm. The firm needs to know not only the current preferences of its owners but also their future preferences. Crucially, the firm cannot recover these valuations from asset prices. While the firm’s stock market price is observable, the sensitivity of the price to changes in the investment plan is unobservable.

Unfortunately, these problems get exacerbated in an infinite-horizon setting. The firm would need information on its shareholders’ valuation from the present to the infinite future. Moreover, realistic settings will allow for the set of shareholders to change over time (rather than the same shareholders changing the amount they own), and some shareholders might be constrained by the short-selling constraint. These issues substantially complicate the firm’s problem and the computation of equilibrium.\footnote{See Moll (2023) on why this (i) makes it very hard/impossible to solve, and (ii) it is an unrealistic assumption.} To make progress, Section 4 discusses an additional assumption that renders the infinite-horizon problem tractable.

## 4 The Firm in Infinite-Horizon

In this section, we set up the problem of the representative firm in the infinite horizon setting. While a simplifying assumption will allow us to sidestep the aforementioned complications, two outcomes from the previous section remain: time-inconsistency and present bias. Moreover, under this additional assumption, we show that the firm
behaves as if it faced quasi-hyperbolic discounting.

4.1 The Stock Price Elasticity

A crucial element of the firm’s problem is its understanding of how its stock price, \( q_t \), changes in response to its future dividend choices, \( \{d_{t+s}\}_{s \geq 1} \). Like in the 3-period model of Section 3, the firm understands that the answer to this question lies in the households’ Euler equation. Let \( C_t(\theta, b, h) \), \( \Theta_{t+1}(\theta, b, h) \), and \( B_{t+1}(\theta, b, h) \) denote the policy functions for consumption, stock holdings, and bonds, given \((\theta, b, h)\). To avoid excessive clutter, we simplify the notation by removing the dependence of \( C_t \), \( \Theta_{t+1} \), and \( B_{t+1} \) on \((\theta, b, h)\) whenever it does not lead to confusion.

The Euler equation of a household is given by

\[
(1 - \phi \Delta^-) q_t = \mathbb{E}_t \left[ \beta \frac{u'(C_{t+1})}{u'(C_t)} (d_{t+1} + (1 - \phi \Delta^{-}_{t+1} q_{t+1})) \right] + \eta_t,
\]

where \( \eta_t \) is the Lagrange multiplier on the households’ short-selling constraint for stocks in (4), and the expectation is taken with respect to \( h_{t+1} \). Let

\[
\Phi_t \equiv \mathbb{E}_t \left[ \phi \Delta^{-}_{t+1} \right] + \phi \frac{\text{cov}_t \left( u'(C_{t+1}), \Delta^{-}_{t+1} \right)}{\mathbb{E}_t \left[ u'(C_{t+1}) \right]}.
\]  

(14)

Then, we can rewrite the households’ Euler equation as

\[
(1 - \phi \Delta^-) q_t = \mathbb{E}_t \left[ \beta \frac{u'(C_{t+1})}{u'(C_t)} \right] \left[ d_{t+1} + (1 - \Phi_t) q_{t+1} \right] + \eta_t.
\]  

(15)

The household uses its expected IMRS between \( t \) and \( t+1 \) to discount future cash flows. But the stock valuation has an extra component, \( \Phi_t \), which reflects the household’s expected marginal transaction cost. A higher transaction cost (indexed by \( \phi \)) reduces the household’s valuation, as it lowers the stock return in a sale. This effect is amplified by the fact that households will sell (i.e. \( \Delta^{-}_{t+1} > 0 \)) when their marginal utility of consumption, \( u'(C_{t+1}) \), is relatively high. Hence, a positive covariance between marginal utility and quantity sold further depresses asset prices.

Equation (15) is at the core of the firm’s investment decisions. In the 3-period model of Section 3, we used (15) to iteratively replace \((1 - \phi \Delta^{-}_{t} q_{t})\) from the firm’s problem (9). This strategy is not helpful in the infinite-horizon version of the model
for two reasons. First, recall that in the simple model, it was important that there were no agents against their short-selling constraints, so that \( \eta_t = 0 \) for all agents and periods. However, this will not be true in the infinite-horizon model with income risk. Households that suffer long spells of low employment will eventually hit the constraint. Thus, the Lagrange multiplier associated with the short-selling constraint will be positive for some agents. Second, after introducing the Euler equations into the firm’s problem, we obtained a characterization of the firm’s objective that depended on the whole distribution of marginal utilities at all horizons. This would require the firm to obtain information on agents’ individual marginal utilities over time, information that is not readily available from aggregate variables or the price system.

Our strategy will use equation (15), but it will leverage the fact that if buyers are not borrowing constrained, it is possible to recover the sensitivity of the stock price to the firm’s investment plan from two financial prices: the risk-free interest rate and a liquidity premium.

### 4.2 The Liquidity Premium

For buyers, condition (15) reduces to

\[
q_t = \mathbb{E}_t \left[ \beta \frac{u'(C_{t+1})}{u'(C_t)} \right] \left[ d_{t+1} + (1 - \Phi_t(\theta, b, h))q_{t+1} \right],
\]

because \( \Delta_t^{-} = \eta_t = 0 \) for them. In principle, buyers may disagree on how much they value the dividend \( d_{t+1} \) relative to the ex-dividend market price \( q_{t+1} \).\(^{18}\) That is, \( \mathbb{E}_t \left[ \beta \frac{u'(C_{t+1})}{u'(C_t)} \right] \) and \( \Phi_t(\theta, b, h) \) could vary depending on the buyer’s initial portfolio and realization of the employment shock. However, their valuations coincide if their borrowing constraint does not bind. To see this, note that the agents’ optimality condition for bonds is given by

\[
\frac{1}{1 + r_t} = \mathbb{E}_t \left[ \beta \frac{u'(C_{t+1})}{u'(C_t)} \right] + \gamma_t,
\]

\(^{18}\)All buyers agree on the stock’s valuation; if they did not, they would not be willing to buy at the common price \( q_t \).
where $\gamma_t$ is the Lagrange multiplier associated with the borrowing constraint (5). For unconstrained agents $\gamma_t = 0$, and hence

$$
E_t \left[ \beta \frac{u'(C_{t+1})}{u'(C_t)} \right] = \frac{1}{1 + r_t} \implies q_t = \frac{d_{t+1}}{1 + r_t} + \frac{(1 - \Phi_t)q_{t+1}}{1 + r_t},
$$

so $\Phi_t$ must be the same for all unconstrained buyers. In what follows, we focus on economies where the borrowing constraint does not bind for buyers. We verify their existence in the numerical exercise.

Let the yield of the stock be $\theta_t = \frac{d_{t+1} + q_{t+1}}{q_t} - 1$. Define the liquidity premium as the excess return of the stock over the return of the bond, i.e. $\theta_t - r_t = \Phi_t \frac{q_{t+1}}{q_t}$, which reduces to $\Phi$ in steady-state. Then, the pricing equation of the firm’s stock depends on two widely used financial prices: the risk-free rate, $r_t$, and the liquidity premium, $\Phi_t$. With this, we are ready to set up the firm’s intertemporal problem.

### 4.3 The Firm’s Problem

Recall that Lemma 1 shows that the objective of the firm is to maximize a weighted valuation given by

$$
d_t + (1 - \Phi_t)q_t.
$$

The discussion in Section 4.2 implies that the stock price can be expressed as

$$
q_t = \frac{d_{t+1}}{1 + r_t} + \frac{(1 - \Phi_t)q_{t+1}}{1 + r_t}.
$$

The following assumption provides the final piece necessary to solve the firm’s problem.

**Assumption 2.** The firm takes $\{\Phi_t, \Phi_t\}_{t \geq 0}$ as given.

Assumption 2 has two parts. First, it states that the firm takes the owner’s average transaction cost, $\Phi_t$, as given. This variable can be computed by a firm with access to data on how many stocks were bought or sold. Second, it states that the firm also takes the liquidity premium, $\Phi_t$, as given. This implies that the firm does not anticipate how its decisions impact households’ trading patterns, summarized by the cross-sectional distribution of $\Delta_t'$ (see the definitions of $\Phi_t$ and $\Phi_t$ in equations (8) and (14), respectively). The firm always has the correct expectation with respect
to the *levels* of these variables but ignores the potential *changes* that arise from the effect of its investment decisions on the households’ trading strategies.\(^\text{19}\)

The resulting pricing equation equation (17) is intuitive. Since dividends are distributed without frictions, and the economy features no aggregate risk, the firm’s dividend is discounted by the same discount factor as the bonds, i.e., \(\frac{1}{1+r_t}\). Note that if \(\phi = 0\), the pricing equation simplifies to \(q_t = \frac{d_{t+1} + q_{t+1}}{1+r_t}\). In contrast, when \(\phi > 0\), the resale price \(q_{t+1}\) is further discounted by the liquidity premium \(\Phi_t\). Thus, the firm’s problem will be to maximize (16) subject to (17).

### 4.4 Equilibrium

An equilibrium for this economy consists of household allocations

\[
\left\{ (c_{jt}, b_{j,t+1}, \theta_{j,t+1}, \Delta_{j,t}^+, \Delta_{j,t}^-)_{\forall j \in [0,1]} \right\}_{t=0}^{\infty}, \text{ firm allocations } \left\{ l_t, k_{t+1}, d_t \right\}_{t=0}^{\infty} \text{ and aggregates }
\]

\[
\left\{ w_t, r_t, q_t, \Phi_t, \Gamma_t \right\}_{t=0}^{\infty}, \text{ such that, given } (\theta_{j,0}, b_{j,0})_{\forall j \in [0,1]} \text{ and } k_0,
\]

1. Given \(\{d_t, w_t, r_t, q_t\}_{t=0}^{\infty}\), households optimize;
2. Given \(\{w_t, r_t, \Phi_t, \Gamma_t\}_{t=0}^{\infty}\), firms optimize\(^\text{20}\);
3. \(\{\Phi_t, \Phi_t\}_{t=0}^{\infty}\) are consistent with the cross-sectional distribution \(\{\Gamma_t\}_{t=0}^{\infty}\) according to (8) and (14);
4. Markets clear:

\[
\int_{j \in [0,1]} h_{j,t} dj = l_t, \quad \int_{j \in [0,1]} \theta_{j,t} dj = 1, \quad \int_{j \in [0,1]} b_{j,t} dj = 0,
\]

for all \(t \geq 0\). By Walras Law, the goods market also clears.

A steady-state equilibrium is an equilibrium in which firm allocations and aggregates are constant over time. Our analysis will focus on steady-state equilibria.

\(^{19}\)Grossman and Hart (1979) argue that since the assumption involves an elasticity, the firm’s beliefs are neither verified nor falsified in equilibrium.

\(^{20}\)The optimization of the firm will depend on whether we consider the solution with or without commitment. This will be made explicit in the next section.
5 Equilibrium Characterization

In a steady-state equilibrium, $\Phi_t = \Phi$ and $\Phi_t = \Phi$ for all $t$. Iterating the price (17) forward and replacing it in the objective function, the problem of the firm simplifies to

$$V^F(k_t) = \max_{\{k_{t+s}\}_{s \geq 1}} F(k_t, k_{t+1}) + \frac{1 - \Phi}{1 - \Phi} \sum_{s=1}^{\infty} \left( \frac{1 - \Phi}{1 + r} \right)^s F(k_{t+s}, k_{t+s+1}).$$  (FP)

5.1 Quasi-Hyperbolic discounting

The program (FP) features quasi-hyperbolic discounting. To see this, let $\tilde{\delta} = \frac{1 - \Phi}{1 + r}$ and $\tilde{\beta} = \frac{1 - \Phi}{1 - \Phi}$. Then, the value of the firm can be written as

$$V^F(k_t) = \max_{\{k_{t+s}\}_{s \geq 1}} F(k_t, k_{t+1}) + \tilde{\beta} \sum_{s=1}^{\infty} \tilde{\delta}^s F(k_{t+s}, k_{t+s+1}).$$

On the one hand, $\tilde{\delta}$ represents the “natural” discount factor obtained from the pricing equation (17): it combines the risk-free rate, $r$, and the liquidity premium, $\Phi$. That is, $\tilde{\delta}$ reflects the discounting implied in asset prices. On the other hand, $\tilde{\beta}$ summarizes the disagreement between current and future owners. If $\Phi = \Phi$, then current and future owners agree on how to value the firm, so $\tilde{\beta} = 1$ and the problem simplifies to a standard one with exponential discounting. However, if $\Phi > \Phi$, then current owners are more impatient than the market, which implies that $\tilde{\beta} < 1$ and the firm exhibits present bias. Similarly, if $\Phi < \Phi$, then current owners are more patient than the market, so $\tilde{\beta} > 1$ and the firm exhibits future bias. In the next proposition, we decompose the difference $\Phi - \Phi$ to identify the factors contributing to time-inconsistency.

**Proposition 1.** The difference $\Phi - \Phi$ is equal to the sum of a persistence effect and
a risk premium:

\[
\Phi - \tilde{\Phi} = \phi \left[ \tilde{\mathbb{E}}_t \left[ \mathbb{E}_t[\Delta_{t+1}] \mid \text{buyer} \right] - \mathbb{E}_t \left[ \mathbb{E}_t[\Delta_{t+1}] \right] \right] + \frac{\phi \epsilon_t}{\mathbb{E}_t \left[ u'(C_{t+1}) \mid \text{buyer} \right]}
\]

where tilde moments are taken with respect to the cross-sectional weighted density \( \Theta_{t+1}d\Gamma(\theta, b, h) \), and the non-tilde moments are taken with respect to the density \( dF(h'\mid h) \).

Proposition 1 presents several results. First, it shows that time-inconsistency can only emerge in an economy with transaction costs, i.e., \( \phi > 0 \). When \( \phi = 0, \Phi = \tilde{\Phi} \) and \( \tilde{\beta} = 1 \), recovering the standard exponential discounting problem. Second, it shows that the degree and direction of time-inconsistency (i.e., present or future bias) depends on the interaction of two economic forces.

The persistence effect captures the difference in transaction costs for different agents. On the one hand, we have the expected transaction costs next period for those that are buyers today. On the other hand, we have the average transaction cost for owners—which includes both buyers and sellers. Note that, by stationarity, this term is the same in periods \( t \) and \( t+1 \). If there is persistence in trades, so that buyers in \( t \) expect to also be buyers in \( t+1 \), then the expected transaction cost of buyers is smaller than the average transaction cost in the economy, i.e. \( \tilde{\mathbb{E}}_t \left[ \mathbb{E}_t[\Delta_{t+1}] \mid \text{buyer} \right] < \mathbb{E}_t \left[ \mathbb{E}_t[\Delta_{t+1}] \right] \), which represents a force towards present bias. This is the case we expect to obtain in our model, as purchasing the illiquid asset is profitable only if there is a relatively high chance that the agent will not sell the asset immediately (the bond is a better asset for that case). We confirm that this is true in our quantitative exercise.

The second term in the expression is the risk premium effect, which captures the fact that it is precisely when the agents need the resources the most, i.e., high marginal utility states, that households sell the most. This positive covariance makes the equity a risky investment, which is priced in by the buyers and represents a force toward future bias. The net effect of these two competing forces (persistence versus
risk) depends on parameter values. In our calibrated economy of Section 6, we find that the persistence effect significantly outweighs the risk premium, leading to present bias.

Our results reflect, once again, the differences in rates of return across agents, closely linked to differences in IMRS. In particular, the main force in Proposition 1, the persistence effect, can be understood as comparing the rate of impatience of buyers and owners. While agents’ IMRS does not appear explicitly in Proposition 1, their impatience is reflected in the quantity sold, $\Delta^-$. Since selling stock is costly while selling bonds (i.e., borrowing) is not, sellers of the stocks are likely borrowing constrained. Due to the presence of constrained sellers, we have that

$$\tilde{E}_t \left[ \mathbb{E}_t \left[ \frac{\beta u'(c_{t+1})}{u'(c_t)} \right] \bigg| \text{seller} \right] \leq \frac{1}{1 + r_t}$$

In contrast, buyers are not borrowing constrained today, which implies that their IMRS is $\frac{1}{1 + r_t}$. Moreover, as we argued above, buyers today are likely to be buyers tomorrow (the persistence effect), so their expected IMRS is close to $(1 + r_t)^{-1}$. Thus, in equilibrium, we expect to have

$$\tilde{E}_t \left[ \mathbb{E}_t \left[ \frac{\beta u'(c_{t+1})}{u'(c_t)} \right] \bigg| \text{buyer} \right]$$

where the LHS is the (weighted) average IMRS across owners (i.e., both buyers and sellers) in period $t$, while the RHS is the (weighted) average IMRS across buyers in $t+1$. This implies that owners are more impatient than buyers. Note that the firm’s discount factor between $t + 1$ and $t + 2$ in period $t$ is determined by the stock price $q_t$, and so is the IMRS of the buyers. However, the discount between $t + 1$ and $t + 2$ in period $t + 1$ is determined by the owners. Using that owners are likely to be more impatient than buyers, we conclude that the firm’s problem is time-inconsistent and, in particular, it suffers from present bias.

Next, we solve the problem of the firm analytically. First, we consider the problem of a firm that can commit to future policies. This firm chooses an investment plan in the initial period and never reoptimizes. Then, we study the problem of a firm that cannot commit to an investment plan but fully anticipates its future incentives and understands how present decisions can affect its future self.

23
5.2 Firm with commitment

A firm that can commit to a future investment policy chooses a sequence of capital 
\( \{k_{t+s}\}_{s=1}^{\infty} \) in period \( t \) to maximize its value:

\[
V^F(k_t) = \max_{\{k_{t+s}\}_{s=1}^{\infty}} F(k_t, k_{t+1}) + \bar{\beta} \sum_{s=1}^{\infty} \delta^s F(k_{t+s}, k_{t+s+1}).
\]

The first-order condition with respect to \( k_{t+1} \) is

\[
F_2(k_t, k_{t+1}) + \tilde{\beta} \tilde{\delta} F_1(k_{t+1}, k_{t+2}) = 0
\]

and the first-order condition with respect to \( k_{t+s+1} \) for \( s \geq 1 \) is

\[
F_2(k_{t+s}, k_{t+s+1}) + \tilde{\delta} F_1(k_{t+s+1}, k_{t+s+2}) = 0
\]

where \( F_1(k_t, k_{t+1}) \equiv z\alpha k_t^{\gamma-1} + (1 - \delta) \) and \( F_2(k_t, k_{t+1}) \equiv -1 \). Note that when \( \tilde{\beta} \neq 1 \), the choice of initial investment involves a different trade-off than future investment.

Focusing on steady-state equilibria, and using that \( z = (1 - \gamma\psi)\left(\frac{\gamma\psi}{\psi w}\right)^{\psi/(\gamma\psi)} \), \( \alpha = \frac{(1 - \gamma)\psi}{1 - \gamma \psi} \), and \( w = \gamma \psi k_t(1 - \gamma) \psi H^{\gamma}\psi - 1 \) (where \( H \) is the unconditional expectation of the employment process), we get that the level of capital when the firm has commitment is

\[
k^C = \left( \frac{(1 - \gamma)\psi \tilde{\delta}}{1 - \tilde{\delta}(1 - \delta)} H^{\gamma}\psi \right)^{\frac{1}{1 - (1 - \gamma)\psi}}.
\]  

(18)

5.3 No commitment

We now turn to the problem of a firm that cannot commit to an entire sequence of investments but optimizes period by period. We assume that the firm fully understands its future-self incentives and takes them into account when making its choice. This is known as a sophisticated solution in the literature on time-inconsistent preferences.

We solve for a Markov perfect equilibrium (MPE). Given an initial level of capital \( k \), the firm takes the future policy function, \( k' = g(k) \), as given. Thus, we can write the firm’s problem as

\[
V^F(k) = \max_{k'} F(k, k') + \tilde{\beta} \tilde{\delta} W(k')
\]

24
subject to
\[ W(k') = F(k', g(k')) + \tilde{\delta}W(g(k')). \]

Let \( k' = \zeta(k) \) denote the solution of this equation for a given level of initial capital \( k \). A MPE requires \( \zeta(k) = g(k) \). The function \( W(\cdot) \) takes into account the preference disagreement between the current and the future firm. If \( \tilde{\beta} = 1 \), the problem simplifies to a standard Bellman equation, with \( V^F(k) = W(k) \). However, when \( \tilde{\beta} \neq 1 \), the current firm discounts the future at a different rate than the future firm. Note that, because \( \zeta(k) = g(k) \), the future firm also intends to discount the immediate future at the rate \( \tilde{\beta}\tilde{\delta} \).

The following proposition shows that there is a unique differentiable solution that is the limit of the finite-horizon version of this problem. Moreover, it shows that this solution features a lower level of capital than with commitment if and only if the firm suffers from present bias.

**Proposition 2.** The firm’s problem with no commitment has two differentiable, Markovian solutions, but only one is the limit of the unique finite-horizon equilibrium. This solution is given by

\[
k_N = \left( \frac{(1 - \gamma) \psi \tilde{\beta}\tilde{\delta}}{1 - \tilde{\beta}\tilde{\delta} (1 - \delta) H^\psi} \right)^{\frac{1}{1 - (1 - \gamma)\psi}}. \tag{19}
\]

Finally, \( k^N < k^C \) if and only if \( \tilde{\beta} < 1 \).

Notice that (19) is directly comparable to (18), where \( \tilde{\beta}\tilde{\delta} \) replaces \( \tilde{\delta} \). When \( \tilde{\beta} < 1 \), Proposition 2 shows that firms with no commitment under-invest compared to those with commitment. It is as if the firm without commitment is more impatient, which then delivers a lower level of capital. Intuitively, the incentive for firms is to increase dividend payouts today and leave little capital for future selves. This is a best response, considering that future selves have the same incentive to over-distribute dividends.

### 5.4 Extensions

Before presenting the numerical exercises, we consider two extensions to our baseline framework. First, we study a simple model of heterogenous firms to analyze the
cross-sectional implications of liquidity on firms’ investment decisions. Second, we examine the implications of allowing the firms to issue (illiquid) corporate bonds.

**Liquidity and Investment in the Cross-Section.** Consider an extension in which there is a fraction $\omega$ of firms that are illiquid as in the baseline model, and a fraction $(1 - \omega)$ of firms that are fully liquid, i.e., their stock trades in a frictionless market (subject to the no short-selling constraint). By no-arbitrage, the liquid stock has the same return as the bond, so it discounts the dividends it generates at the risk-free rate $\frac{1}{1+r}$, with standard exponential discounting. In contrast, the discount factor of illiquid firms is $\frac{1-\phi}{1+r} < \frac{1}{1+r}$ with commitment, and $\frac{1-\phi}{1+r} < \frac{1}{1+r}$ without commitment. Thus, firms with illiquid stock always have less capital than firms with liquid stock. This result is consistent with Amihud and Levi (2022), who use cross-sectional data for US public firms to compare liquid and illiquid firms and conclude that illiquid firms invest less than liquid ones.

In Section 6.3 we quantitatively analyze the aggregate implications. First, we show that the model is consistent with the cross-sectional evidence. Second, we argue that the cross-sectional evidence alone is not enough to make aggregate predictions. We show that the aggregate implications are qualitatively and quantitatively different if the firm can or cannot commit to future investment plans.

**The Role of Corporate Bonds.** Up to now, we assumed that the firm funds itself from retained earnings. We now explore the consequences of allowing the firm to also issue illiquid corporate bonds. We find that corporate bonds might alter the financing decisions of the firm, depending on their liquidity. However, the presence of corporate bonds does not alter the investment decision and the time-inconsistency problem. Intuitively, the firm uses the stockholders’ discount factor to make investment decisions independently of the interest rate of corporate bonds.

Suppose the firm can borrow up to an exogenous limit at the gross interest rate $1 + r^{cb} = \frac{1+r}{1-\tilde{\phi}}$. The parameter $\tilde{\phi}$ captures a premium that corporate bonds pay relative to the risk-free rate in reduced form. This is motivated by the fact that corporate bond markets tend to be more illiquid than stock markets. We assume that, in contrast, the rate of return from savings is equal to $r$. It should be clear that since $\frac{1}{1+r} > \tilde{\beta} \delta$ and $\frac{1}{1+r} > \delta$, the firm never finds it optimal to save.

---

21 Appendix D contains the details of the extension.
The firm’s strategy depends on how \( \frac{1}{1+r_{cb}} \) compares to \( \tilde{\beta}\tilde{\delta} \) and \( \tilde{\delta} \). Intuitively, the firm borrows only if the interest rate on corporate bonds is lower than its discount rate. But note that the discount rate the firm uses for the comparison depends on its ability to commit to an investment plan. It might seem natural to conclude that if \( \tilde{\phi} > \Phi \), that is, if the premium on corporate bonds is higher than the premium on stocks (recall that \( \Phi = r^g - r \)), then the firm does not issue corporate debt. However, this logic is correct only if the firm has commitment. If it does not, recall that the firm uses \( \Phi \) to discount future cash flows. Thus, the firm will borrow even if the corporate rate is high relative to the stock market discount. This result provides a rationalization for corporate bond issuance that does not rely on the canonical tax advantage of debt.

Important for our results is that incorporating corporate bonds does not affect our conclusions: the firm still discounts futures cash flows using \( \tilde{\beta}\tilde{\delta} \).\(^{22}\) The following proposition formalizes this intuition.

**Proposition 3.** Suppose the firm suffers from present bias and it has access to the bond market with an interest rate of \( 1 + r_{cb} = \frac{1+r}{1-\phi} \). If \( \tilde{\phi} < \Phi \) the firm always borrows to the limit independently of its degree of commitment. If \( \Phi < \tilde{\phi} < \Phi \) the firm without commitment borrows up to the limit, while the firm with commitment does not borrow. Furthermore, optimal levels of capital are determined according to (18) and (19) with and without commitment, respectively.

## 6 Quantitative Evaluation

We now calibrate and solve the model numerically to study how liquidity affects aggregate capital in general equilibrium.

### 6.1 Calibration

We have three sets of parameters. First, there are standard parameters that we take from the literature. Second, there are parameters governing the income process and the borrowing constraint. Third, we look at data on relative spreads to discipline a reasonable set of values for the transaction costs. Table 2 summarizes the calibration.

\(^{22}\) Naturally, whether the firm can borrow or not can affect the general equilibrium determination of \( \tilde{\beta} \) and \( \tilde{\delta} \). However, its qualitative properties do not change.
Table 2: Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor $\beta$</td>
<td>0.95</td>
</tr>
<tr>
<td>Risk aversion $\sigma$</td>
<td>2.00</td>
</tr>
<tr>
<td>Production weight on labor $\gamma$</td>
<td>0.80</td>
</tr>
<tr>
<td>Returns to scale $\psi$</td>
<td>0.95</td>
</tr>
<tr>
<td>Depreciation $\delta$</td>
<td>0.05</td>
</tr>
<tr>
<td>Borrowing limit $\bar{b}$</td>
<td>-1.00</td>
</tr>
<tr>
<td>Labor persistence $\rho_h$</td>
<td>0.50</td>
</tr>
<tr>
<td>Labor st dev $\sigma_h$</td>
<td>0.30</td>
</tr>
<tr>
<td>Transaction cost $\phi$</td>
<td>4.00</td>
</tr>
</tbody>
</table>

Standard parameters. There is a set of standard parameters that we borrow from previous papers. We set the discount factor equal to 0.95, which is standard for annual calibrations, and the coefficient of relative risk aversion equal to 2. For the production function, we assume $\gamma = 0.80$ and $\psi = 0.95$ (e.g., Gavazza et al., 2018). Finally, we set the depreciation rate at 5%.

Income process. Both the income process and the borrowing constraint are important as they determine the degree of market incompleteness, and therefore disagreement on IMRS across agents. We choose parameters so this disagreement lies on the conservative side. We do not want to have too much income risk because it generates larger disagreements.

Labor $h$ can take two values, interpreted as high and low income states. We approximate its distribution with the Rouwenhorst method with persistence $\rho_h = 0.5$ and a standard deviation of innovations $\sigma_h = 0.3$.\(^{23}\) Hence, the standard deviation of log income in the model is 0.3. The comparison with the data is not straightforward because we should only consider households participating in the stock market, which are less than 50% (Morelli, 2021). To get a rough comparison, the standard deviation of residual log income for the U.S. population is 0.9 (Guvenen et al., 2022). Finally, we set the borrowing limit equal to negative one, which corresponds to a household borrowing capacity equal to the average annual income.\(^{24}\)

\(^{23}\)For the transition matrix, we define $p$ as the probability of staying in the same income state. The Rouwenhorst method implies that $p = (1 + \rho_h)/2$. For the values of $\log(h)$ define $\Sigma_h = \sigma_h/\sqrt{1 - \rho_h^2}$. Then $\log(h)$ can take on values $\Sigma_h$ or $-\Sigma_h$. We then normalize $h$ so it has mean one.

\(^{24}\)Kaplan et al. (2018) also consider a borrowing constraint of 1 period of labor income but their
The key moment for the model is how incomplete markets are, which is reflected in the variance of log consumption relative to log income. Table 4 shows that this moment is about 0.2 in the model while it is 0.3 in the data (Krueger and Perri, 2006). Therefore, the calibration is on the conservative side as it generates a low degree of market incompleteness and disagreement. We then show that as we increase idiosyncratic uncertainty, the results in the model are amplified.

Transaction costs. To discipline the range for the transaction cost parameter $\phi$, we look at the data on relative spreads. We look at daily stock data from the Center for Research in Security Prices (CRSP) between January 2000 and March 2022. We follow Naes et al. (2011) and consider only ordinary common shares (variable SHRCD less or equal than 11) in the New York Stock Exchange (PRIMEXCH equal to N). For each day, we compute the relative spread as the quoted spread (the difference between the best ask and bid quotes) as a fraction of the midpoint price (the average of the best ask and bid quotes). We then define the quarterly data as the average within the quarter. Our final database has 3,369 firms, 89 quarters, with a total of 124,902 firm-quarter observations.

Table 3 shows the relative spreads in the data. In the entire sample the average relative spread is 3.37% and the median is 2.79%. Note that these moments are quite similar if we consider the period before the financial crisis (i.e., 2000Q1 to 2006Q1) or the period between the financial crisis and the COVID-19 crisis (i.e., 2010Q1 to 2019Q4). These estimates of relative spreads are consistent with the literature (e.g., Naes et al., 2011; Corwin and Schultz, 2012; Goyenko et al., 2009; Abdi and Ranaldo, 2017).

One potential concern is that relative spreads for large companies might be much smaller than the average. To address this issue, the second panel of Table 3 shows moments weighted by market capitalization. We observe that the average falls by about 1 percentage point while the median falls by about 80 basis points. Finally, Figure 1 shows a histogram of relative spreads. The histogram shows that most firms have a relative spread larger than 1 percent, and for some firms it can be as large as 10 percent. Given this data, we target a relative spread between 2.5% and 3%.

To generate relative spreads consistent with this evidence we consider values of $\phi$ in $[0, 10]$ for our numerical explorations. The left panel of Figure 2 shows relative model is quarterly, so effectively we are allowing more borrowing, which tends to dampen our results.
Table 3: Relative Spreads, %

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>St. dev.</th>
<th>p10</th>
<th>p50</th>
<th>p90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equal weight</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2000Q1-2022Q1</td>
<td>3.37</td>
<td>2.35</td>
<td>1.54</td>
<td>2.79</td>
<td>5.72</td>
</tr>
<tr>
<td>2000Q1-2006Q1</td>
<td>3.23</td>
<td>2.28</td>
<td>1.57</td>
<td>2.77</td>
<td>5.23</td>
</tr>
<tr>
<td>2010Q1-2019Q4</td>
<td>2.93</td>
<td>1.71</td>
<td>1.47</td>
<td>2.52</td>
<td>4.8</td>
</tr>
<tr>
<td>Weighted by market capitalization</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2000Q1-2022Q1</td>
<td>2.31</td>
<td>1.26</td>
<td>1.24</td>
<td>1.98</td>
<td>3.78</td>
</tr>
<tr>
<td>2000Q1-2006Q1</td>
<td>2.64</td>
<td>1.27</td>
<td>1.39</td>
<td>2.35</td>
<td>4.23</td>
</tr>
<tr>
<td>2010Q1-2019Q4</td>
<td>1.88</td>
<td>0.8</td>
<td>1.15</td>
<td>1.69</td>
<td>2.84</td>
</tr>
</tbody>
</table>

Data on relative spreads from CRSP.

Figure 1: Data: Relative Spreads

Spreads in the model. Relative spreads increase with the value of $\phi$ and are always below 5% for $\phi \leq 10$. Thus, the frictions associated with transaction costs in these exercises are within the range of the empirical estimates. For our benchmark we set $\phi = 4$, which generates a relative spread of 2.9%, which is consistent with the data presented above and equal to the average cost considered in Heaton and Lucas (1996).

The right panel of Figure 2 shows that the liquidity premium is always below 50 basis points for $\phi \leq 10$. We believe that this is a conservative value for the liquidity premium, although direct empirical counterparts are limited (see, e.g., Amihud et al., 2005). For comparison, the liquidity premium is much lower than in Kaplan et al. (2018), where it is equal to 370 basis points.\(^{26}\)

\(^{25}\)Relative spreads are $\mathbb{E} \left[ \frac{\phi}{2} \Delta^- \mid \Delta^- > 0 \right]$, where the expectation is over the cross-sectional dis-
Figure 2: Transaction Costs

<table>
<thead>
<tr>
<th>Relative spreads, %</th>
<th>Liquidity premium, basis points</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Graph" /></td>
<td><img src="image" alt="Graph" /></td>
</tr>
</tbody>
</table>

Note: The figures show the liquidity premium and relative spreads for the model without commitment for different transaction costs, $\phi$.

**Non-targeted moments.** Although the model is stylized, the composition of liquid and illiquid assets—which is not targeted in the calibration—is consistent with the data. The average share of illiquid assets to GDP is 3.5 in the model and 2.9 in the data, while the average share of liquid assets is 0.5 in the model and 0.3 in the data. Moreover, both in the model and data, the fraction of savers (i.e., $b_t > 0$) is equal to 50%.

Table 4: Non-targeted Moments

<table>
<thead>
<tr>
<th></th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variance log consumption / variance log income</td>
<td>0.2</td>
<td>0.3</td>
</tr>
<tr>
<td>Mean illiquid assets</td>
<td>3.5</td>
<td>2.9</td>
</tr>
<tr>
<td>Mean liquid assets</td>
<td>0.5</td>
<td>0.3</td>
</tr>
<tr>
<td>Frac. with $b &gt; 0$</td>
<td>0.5</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Note: The benchmark model has $\phi = 4$. Means are expressed as ratios to annual output. Consumption and income data from Krueger and Perri (2006). Asset data from SCF 2004 (see Kaplan et al., 2018).
6.2 Liquidity and Investment in General Equilibrium

Relative to a complete markets economy, our model features three forces impacting the firm’s discount factor: (i) precautionary savings, (ii) transaction costs, and (iii) lack of commitment. We now analyze the importance of each channel numerically.

Figure 3 shows the equilibrium allocations for economies with different values of the transaction cost, $\phi$. First, the black line shows the solution under complete markets, when the firm discounts at rate $r = 1/\beta - 1$ (independent of $\phi$). Second, the yellow point highlights the case with no transaction costs, i.e., $\phi = 0$, but with incomplete markets, so the model is analogous to the economy studied in Aiyagari (1994). The absence of trading frictions implies that the firm’s problem is time-consistent. In this economy, the interest rate is lower and capital is higher than with complete markets due to precautionary savings. In our parametrization, capital is 9% higher than under complete markets.

Third, the red line shows the solution when markets are incomplete, $\phi > 0$, and the firm cannot commit to future policies. In this case, capital is decreasing in $\phi$. For example, when $\phi = 4$, the liquidity premium is about 30 basis points, and capital is 10% lower than with complete markets. Hence, the combination of transaction costs and lack of commitment generates significant changes in aggregate capital relative to the standard Aiyagari (1994) economy. Lack of commitment is critical for this result. To see this, the blue line shows the solution when the firm can commit. In this case, capital has a moderate increase when $\phi$ increases. For example, when $\phi$ increases from 0 to 4 the liquidity premium increases from 0 to 36 basis points, but capital only increases by 3%, while without commitment capital decreases by 17%.

More generally, we compute the average elasticity of capital to the liquidity premium with and without commitment. In the economy without commitment, a 10 bps increase in the liquidity premium is associated with a 13% decrease in capital. In the economy with commitment, the effects are much smaller. A 10 bps increase of the liquidity premium is associated with only a 2% increase in capital.\footnote{We get similar results if we compute the elasticity of capital to relative spreads. A 1% increase in relative spreads implies a 2% increase in capital with commitment, and a 12% decrease in capital without commitment.}

To understand the discrepancy between firms with and without commitment, consider again Proposition 1. When the firm has commitment, the discount factor is $\tilde{\delta} = (1 - \Phi)/(1 + r)$, which in our numerical exercise has a modest increase with $\phi$. The
reason is that, as $\phi$ increases, bonds become more desirable assets than stocks: (i) the liquidity premium increases, but (ii) the interest rate decreases. As a result, both forces approximately offset each other, with a small net effect on the steady-state level of capital. When the firm lacks commitment, the disagreement among owners is captured by the difference between $\Phi$ and $\tilde{\Phi}$, which is increasing in the trading friction $\phi$, as captured by the last panel of Figure 3 ($\tilde{\beta}$). For example, when $\phi = 4$, $\tilde{\beta}$ is about 0.98. Recall that capital is increasing in $\tilde{\beta}$. Hence, without commitment, an increase in $\phi$ generates a decrease in $\tilde{\beta}$, which decreases capital.

Finally, a useful way to evaluate the return of the illiquid asset (i.e., the stock), and the liquid asset (i.e., the bond) is to compare their price-dividend ratios. For the stock, the price-dividend ratio is equal to $(r + \Phi)^{-1}$. For the bond, the price-dividend ratio is equal to $(r)^{-1}$. Hence, the illiquid-to-liquid ratio of the price-dividend ratios is $(1 + \frac{\Phi}{r})^{-1}$. Figure 3 shows that this ratio decreases as transaction cost increases because: (i) the liquidity premium increases, and (ii) the risk-free rate decreases. We return to this moment in Section 6.4.

Figure 3: Liquidity and Investment in General Equilibrium

![Graphs showing capital, liquidity premium, interest rate, price-dividend ratios, and time inconsistency](image)

Note: The figures show the allocations under different transaction costs, $\phi$.  

Complete markets  Aiyagari 94
No commitment  Commitment
These results imply that the transmission of trading frictions to the real economy, and capital accumulation in particular, crucially depends on the ability of firms to commit to future policies. If firms cannot commit, trading frictions can have large and negative effects on the real economy because of present bias. Interestingly, the economy can end up with less capital than with complete markets. That is, the over-accumulation of capital present in Aiyagari (1994) is overturned once we introduce trading frictions and lack of commitment. In contrast, if firms can commit, trading frictions affect asset pricing with almost no consequences for aggregate capital. These results illustrate that the assumptions about trading frictions and the firm’s problem are important for understanding both aggregate quantities and asset prices.

Our results are consistent with the literature on short-termism, which shows that public firms are concerned about meeting short-term targets. Laurence Fink, the CEO of BlackRock, one of the largest money managers, wrote that “the effects of the short-termist phenomenon are troubling ... more and more corporate leaders have responded with actions that can deliver immediate returns to shareholders, such as buybacks or dividend increases, while underinvesting in innovation, killed workforces or essential capital expenditures necessary to sustain long-term growth.” Short-termism is consistent with firms with present bias that cannot commit to future policies. Our theory shows that this short-termism can be attributed to trading frictions in financial markets and asset liquidity.

6.3 Liquidity and Investment: Cross-Section and Aggregate

In this subsection we numerically evaluate the extension with two type of firms from section 5.28

The first two panels of Figure 4 confirm our theoretical results about the cross-section. The first panel shows that, in the economy with commitment, firms with illiquid stock have less capital than firms with liquid stock. Moreover, the second panel shows that the difference in capital is more pronounced if the illiquid firms cannot commit. These results are consistent with Amihud and Levi (2022), who use cross-sectional data for US public firms to compare liquid and illiquid firms and conclude that illiquid firms invest less than liquid ones.

28We set the fraction of illiquid firms, $\omega$ equal to 0.975 so the economy is close to our benchmark calibration of $\omega = 1.0$. Results are robust to alternative values of $\omega$ as long as the amount of liquid assets in the economy is not too large, so there is a reasonable liquidity premium.
The cross-sectional evidence, however, is not enough to understand the aggregate effects of liquidity on investment. A naive reader would be tempted to conclude from the cross-sectional results that, independent of commitment issues, an economy with more illiquid assets would have lower aggregate capital. Instead, we can use the model to evaluate the aggregate implications and argue that this prediction is incorrect. The third panel of the figure shows how aggregate capital changes with transaction costs in this economy with both liquid and illiquid firms. As the transaction cost increases, capital decreases if firms cannot commit, while it increases if firms’ can commit. Hence, the aggregate effects are qualitatively different depending on whether firms are able to commit. These results highlight that: (i) the model is consistent with the cross-sectional evidence, and (ii) the cross-sectional evidence alone is not enough to make aggregate predictions.

Figure 4: Liquidity and Investment in the Cross-Section and the Aggregate

![Graph showing the relationship between transaction cost and capital for firms with and without commitment, and aggregate capital.]

Note: The figures show the allocations under different transaction costs, $\phi$.

### 6.4 Demand and Supply of Liquidity

We conclude this section with comparative statics with respect to changes in the demand and supply of assets. In particular, we consider the effects of changes in the level of idiosyncratic risk and the supply of government bonds.

**Demand of Liquidity.** First, we evaluate the role of increasing uncertainty, which is akin to increasing the demand for liquidity. In particular, we consider increases in $\sigma_h$, which increases the uncertainty of the income process while keeping the average income and the persistence of the process constant (i.e., a mean preserving spread).

A higher level of uncertainty increases the agents’ precautionary savings motive and—as a result—represents a force toward a higher level of capital, as in Aiyagari...
This effect dominates when firms can commit to an investment plan. However, when firms cannot commit, more uncertainty implies more time-inconsistency problems, and—as a result—represents a force towards a lower level of capital. The reason is that higher liquidity needs exacerbate the disagreement between current and future owners. Current owners increase their preference for immediate dividend distribution, leading to lower levels of investment.

Figure 5 shows how the economy responds to changes in $\sigma_h$. The first panel shows that more uncertainty implies more time-inconsistency problems: $\tilde{\beta}$ decreases with volatility, $\sigma_h$. The second panel shows the ratio of capital with commitment relative to no commitment. An increase in uncertainty implies more capital with commitment, while less without commitment. As a consequence, the capital ratio increases with volatility.

Finally, the third panel shows that the illiquid-to-liquid ratio of the price-dividend ratios, $\left(1 + \frac{\rho}{r}\right)^{-1}$, decreases with volatility. These results reflect a “flight to liquidity”: as uncertainty increases, investors shift their portfolios into liquid assets, consistent with the empirical evidence in Naes et al. (2011).

**Figure 5: Uncertainty**

<table>
<thead>
<tr>
<th>Time inconsistency: $\tilde{\beta}$</th>
<th>K: Commit/ No commit</th>
<th>Price-dividend ratios: illiquid to liquid</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Graph" /></td>
<td><img src="image2.png" alt="Graph" /></td>
<td><img src="image3.png" alt="Graph" /></td>
</tr>
</tbody>
</table>

Note: The figures show the allocations under different volatility, $\sigma_h$. The first and last figures are without commitment. The middle figure is the ratio of commitment to no commitment.

**Supply of Liquidity.** Now suppose the government can issue one-period bonds, denoted by $B^G$, and it uses lump-sum taxes to pay for the debt services. As $B^G$ increases, the economy has a larger supply of liquid assets.\(^\text{29}\)

\(^{29}\)We assume that the return on government bonds is the risk-free rate $r$. While government bonds trade in over-the-counter markets, short-term U.S. bonds are among the most liquid securities.
As the supply of bonds increases, bond prices decrease and the interest rate increases. Not surprisingly, then, the liquidity premium decreases. The first panel of Figure 6 shows the decrease in liquidity premium as the supply of liquid assets increases.

The effect on aggregate capital crucially depends on the firm’s ability to commit. On the one hand, when firms can commit, a higher supply of liquid assets implies a lower residual demand for private assets and—as a result—less capital. On the other hand, when firms cannot commit, a higher supply of liquid assets implies a less severe time-inconsistency problem and—as a result—a higher level of capital. The second panel of Figure 6 shows that the ratio of capital with—relative to without—commitment decreases as the supply of liquid assets increases.

Figure 6: Shortage of Liquid Assets

![Graph showing the relationship between liquidity premium and capital commitment](image)

Note: The figures show the allocations under different government bonds, $B^G$.

In summary, with an increase in the demand or the supply of liquid assets, the aggregate response of capital crucially depends on whether firms can commit to an investment plan. On the one hand, an increase in the demand of liquid assets exacerbates the time inconsistency problem and, as a consequence, the difference in capital with—relative to without—commitment increases. On the other hand, an increase in the supply of liquid assets ameliorates the time inconsistency problem and, as a consequence, the difference in capital with—relative to without—commitment decreases.
7 Conclusion

This paper studies the implications of trading frictions in financial markets for firm investment and dividend choices. The main result is that when equity shares trade in frictional asset markets, the firm’s problem is time-inconsistent, and the empirically relevant direction of inconsistency is present bias. Financial transaction costs cause buyers to be patient and value future firm investment, and sellers to be impatient and prefer immediate dividend distribution. Hence, future owners (buyers) make investment plans, yet current owners (including sellers) readjust plans for immediate dividend payout, resulting in a problem of the firm featuring present bias. Furthermore, under a reasonable set of assumptions, the firm problem features quasi-hyperbolic discounting.

When firms are able to commit to future investments, present bias has large effects on asset prices but small consequences on real variables. However, without commitment, firms over-issue dividends and under-invest in capital. The no-commitment case is consistent with empirical findings pertaining to liquidity and investment in the cross-section, over the business cycle, and with the short-termism narrative for publicly traded firms.

References


A  Proofs

Proof of Lemma 1. In what follows, the dependence of each variable on \((\theta, b, h)\) is suppressed for brevity. The first-order condition for \(\Delta_t^+\) when household are buyers \((\bar{\eta}_t = 0)\) is given by:

\[
\mu_t = \lambda_t q_t
\]

The first-order condition for \(\Delta_t^-\) when households are sellers is given by:

\[
\mu_t = \lambda_t q_t (1 - \phi \Delta_t^-) - \bar{\eta}_t
\]

Expression (6) follows from plugging these conditions into the envelope condition:

\[
\bar{q}_t = d_t + \frac{\mu_t + \bar{\eta}_t}{\lambda_t}
\]

\[\square\]

Proof of Propositions 1 and 4. We prove Proposition 4, which collapses to Proposition 1 when \(w(\theta, b, h) = \theta\). Begin with the definitions of \(\Phi\) and \(\Phi\), suppressing dependence on \((\theta, b, h)\) whenever it does not lead to confusion.

\[
\Phi = \int \left[ \mathbb{E}_t[\Delta_{t+1}^+] + \frac{\text{cov}_t(u'(C_{t+1}), \Delta_{t+1}^-)}{\mathbb{E}_t[u'(C_{t+1})]} \right]
\]

\[
\Phi = \phi \sum_h \int \int w(\theta, b, h) \Delta^-_t(\theta, b, h) d\Gamma(\theta, b, h)
\]

By stationarity, \(\Phi\) also equals:

\[
\Phi = \phi \sum_h \sum_{h'} \int \int w(\theta_{t+1}, b_{t+1}, h') \Delta^-_{t+1}(\theta_{t+1}, b_{t+1}, h') d\Gamma(\theta_{t+1}, b_{t+1}, h')
\]

\[
= \phi \sum_h \sum_{h'} \int \int w(\Theta_{t+1}, B_{t+1}, h') \Delta^-_{t+1}(\Theta_{t+1}, B_{t+1}, h') dF(h'|h) d\Gamma(\theta, b, h)
\]

where the second equality follows from plugging in policy functions \(\Theta_{t+1}\) and \(B_{t+1}\).

Because we focus on equilibria where buyers are unconstrained, \(\Phi\) does not depend
on \((\theta, b, h)\) for buyers. Hence we make use of the following tautology:

\[
\Phi = \sum_h \sum_{h'} \int_{\theta} \int_b w(\Theta_{t+1}, B_{t+1}, h') \Phi_1(\Delta^+_{t>0}) dF(h'|h) d\Gamma(\theta, b, h) \\
\sum_h \sum_{h'} \int_{\theta} \int_b w(\Theta_{t+1}, B_{t+1}, h') 1_{\{\Delta^+_{t>0}\}} dF(h'|h) d\Gamma(\theta, b, h)
\]

We call the denominator \(B\). Now take the difference between \(\Phi\) and \(\Phi\):

\[
\Phi - \Phi = \frac{1}{B} \sum_h \sum_{h'} \int_{\theta} \int_b w(\Theta_{t+1}, B_{t+1}, h') \Phi_1(\Delta^+_{t>0}) dF(h'|h) d\Gamma(\theta, b, h) - \Phi
\]

\[
= \frac{\phi}{B} \sum_h \sum_{h'} \int_{\theta} \int_b w(\Theta_{t+1}, B_{t+1}, h') \mathbb{E}_t[\Delta^+_{t-1}] 1_{\{\Delta^+_{t>0}\}} dF(h'|h) d\Gamma(\theta, b, h)
\]

\[
+ \frac{\phi}{B} \sum_h \sum_{h'} \int_{\theta} \int_b w(\Theta_{t+1}, B_{t+1}, h') \frac{\text{cov}_t(w(C_{t+1}), \Delta^+_{t-1})}{\mathbb{E}_t[w(C_{t+1})]} 1_{\{\Delta^+_{t>0}\}} dF(h'|h) d\Gamma(\theta, b, h)
\]

\[
- \frac{\phi}{B} \sum_h \sum_{h'} \int_{\theta} \int_b w(\Theta_{t+1}, B_{t+1}, h') \Delta^+_{t-1} dF(h'|h) d\Gamma(\theta, b, h)
\]

(20)

where we have plugged in the definitions of \(\Phi\) and \(\Phi\). Notice that the second term in condition (20) is the risk premium. Focusing on the first and third term in (20):

Persistence effect + Weight covariance

\[
= \phi \sum_h \sum_{h'} \int_{\theta} \int_b w_{t+1} \mathbb{E}_t[\Delta^+_{t-1}] \frac{1_{\{\Delta^+_{t>0}\}}}{B} dF(h'|h) d\Gamma(\theta, b, h)
\]

\[
- \phi \sum_h \int_{\theta} \int_b \left( \mathbb{E}_t[w_{t+1}] \mathbb{E}_t[\Delta^+_{t-1}] + \text{cov}_t(w_{t+1}, \Delta^+_{t-1}) \right) d\Gamma(\theta, b, h)
\]

\[
= \phi \left( \mathbb{E}_t[\Delta^+_{t-1}] | \text{buyer} \right) - \mathbb{E}_t[\Delta^+_{t-1}] \right] - \phi \sum_h \int_{\theta} \int_b \text{cov}_t(w_{t+1}, \Delta^+_{t-1}) d\Gamma(\theta, b, h)
\]

This completes the proof of Proposition 4. To specialize this result to Proposition 1, let \(w(\Theta_{t+1}, B_{t+1}, h) = \Theta_{t+1}\), which is not measurable with respect to \(h'\). This makes the weight covariance equal to zero.

\(\square\)

**Proof of Proposition 2.** We prove the proposition for a more general firm problem with any twice-differentiable production function \(y_t = f(l_t, k_t)\) that is weakly concave
in both variables (jointly), and strictly concave in $k_t$. Profits are given by

$$\pi_t = y_t - w_t l_t$$

so the intratemporal decision satisfies

$$f_1(l_t, k_t) = w_t$$

and the subscript denotes the partial derivative. Condition (21) implicitly defines the labor function, which we denote $l(k_t)$. Next we solve the firm’s intertemporal problem with no commitment. The first-order condition of the firm’s problem is given by

$$F_2(k, k') + \tilde{\beta} \delta W'(k') = 0$$

where

$$W'(k') \equiv F_1(k', g(k')) + g'(k') \left( F_2(k', g(k')) + \tilde{\delta} W'(g(k')) \right)$$

Plugging in the function $F(k, k') = f(l(k), k) - w l(k) + (1 - \delta) k - k'$:

$$\frac{1}{\beta \delta} = \pi'(k') + 1 - \delta + g'(k') \left( -1 + \frac{1}{\beta} \right)$$

(22)

where

$$\pi'(k) = f_1(H, k)l_1(k) + f_2(H, k) - w l_1(k)$$

$$= f_2(H, k)$$

and the second equality follows from the intratemporal condition (21), and $H$ is the unconditional expectation of the labor process. Because $f$ is strictly concave in $k$, $\pi'(k)$ is a strictly decreasing function. First we guess that $g'(k') = 0$, yielding the steady-state level of capital:

$$k^N = (\pi')^{-1} \left( \frac{1}{\beta \delta} - 1 + \delta \right)$$

where $\pi'$ has an inverse because $\pi'$ was shown to be strictly decreasing. By the same reasoning, $(\pi')^{-1}$ must be strictly decreasing. Similar logic for the commitment case
yields
\[
k^C = (\pi')^{-1} \left( \frac{1}{\delta} - 1 + \delta \right)
\]
and so we have shown that \( k^N < k^C \) if and only if the firm has present bias, \( \tilde{\beta} < 1 \). Now say \( g'(k') \neq 0 \), and one of two scenarios can occur. First, (22) could pin down an optimal value of \( k' \) (notice that this value does not depend on the value of \( k \)). But since \( g'(k') \neq 0 \), \( g(k') \) is not a constant function. By definition of MPE, \( \zeta(k) \) is also not a constant function, which contradicts the optimal choice \( k' \) (which was irrespective of the value of \( k \)). Second, (22) could be satisfied for any value of \( k' \). The \( g(k') \) required for this indeterminacy can be found by solving (22) for \( g'(k') \):

\[
g'(k') = \frac{1 + \tilde{\beta} \delta [\delta - 1 - \pi'(k')]}{\delta (1 - \beta)}
\]

Integrating this last expression yields:

\[
g(k') = \frac{k' + \tilde{\beta} \delta [k'(\delta - 1) - \pi(k')]}{\delta (1 - \beta)} + c
\]

which leads to indeterminacy of the steady-state, based on this choice of integration constant \( c \). Now consider a resource-exhausting finite-horizon version of the same problem.\(^{30}\) Because there is zero investment in the final period, the firm objective in the preceding period would be

\[
F(k, k') + \tilde{\beta} \delta F(k', 0)
\]

and first-order conditions yield \( k^N \). Using backward induction, the firm in each preceding period would also choose \( k^N \) and hence our claim is proved. \( \square \)

**Proof of Proposition 3.** The problem of the firm that can borrow up to a limit is given by

\[
V^F(k_t, b_t^-, b_t^+) = \max_{\{k_{t+s}, b_{t+s}, b_{t+s}^+\}, s \geq 1} d_t + \tilde{\beta} \sum_{s=1}^{\infty} \tilde{\delta}^s d_{t+s}
\]

\(^{30}\)Laibson (1997) uses the same equilibrium refinement.
subject to

\[ d_t = F(k_t, k_{t+1}) + \frac{b_{t+1}^- (1 - \tilde{\phi})}{1 + r} - b_t^- - \frac{b_{t+1}^+}{1 + r} + b_t^+ , \quad 0 \leq b_{t+1}^- \leq \bar{b}, \quad 0 \leq b_{t+1}^+ \]

The firm can borrow the amount $b_{t+1}^-$, up to a limit $\bar{b}$. The parameter $\tilde{\phi} \geq 0$ captures the illiquidity of corporate bonds. Firms can save the amount $b_{t+1}^+$ at the interest rate $r$. First-order conditions for capital do not change from Proposition 2. First-order conditions for bonds are

\[
\begin{align*}
(b_{t+1}^-) : & \quad \frac{1 - \tilde{\phi}}{1 + r} - \tilde{\beta}\tilde{\delta} + \mu_t^1 - \mu_t^2 = 0 \\
(b_{t+s}^-) : & \quad \frac{1 - \tilde{\phi}}{1 + r} - \tilde{\delta} + \mu_{t+s-1}^1 - \mu_{t+s-1}^2 = 0 \\
(b_{t+1}^+ ) : & \quad -\frac{1}{1 + r} + \tilde{\beta}\tilde{\delta} + \mu_t^3 = 0 \\
(b_{t+s}^+ ) : & \quad -\frac{1}{1 + r} + \tilde{\delta} + \mu_{t+s-1}^3 = 0
\end{align*}
\]

where $\mu_t^1, \mu_t^2$, and $\mu_t^3$ are the Lagrange multipliers on the three inequality constraints, respectively. Because $\tilde{\beta}\tilde{\delta} = \frac{1 - \Phi}{1 + r} < \frac{1}{1 + r}$ and $\tilde{\delta} = \frac{1 - \Phi}{1 + r} < \frac{1}{1 + r}$, we have that $b_{t+s}^+ = 0$ for all $s \geq 1$. In other words, no savings occurs. The firm’s borrowing decision depends on the illiquidity parameter $\tilde{\phi}$. We consider the empirically relevant case of present bias, $\Phi > \Phi$.

**Case 1:** $\tilde{\phi} > \Phi$ and, hence, corporate bonds are extremely illiquid. Then $\tilde{\beta}\tilde{\delta} = \frac{1 - \tilde{\phi}}{1 + r} > \frac{1 - \Phi}{1 + r}$ and $\tilde{\delta} = \frac{1 - \Phi}{1 + r} > \frac{1 - \phi}{1 + r}$ so $b_{t+s}^- = 0$ for all $s \geq 1$.

**Case 2:** $\tilde{\phi} < \Phi$ and, hence, corporate bonds are extremely liquid. Then $\tilde{\beta}\tilde{\delta} = \frac{1 - \tilde{\phi}}{1 + r} < \frac{1 - \Phi}{1 + r}$ and $\tilde{\delta} = \frac{1 - \Phi}{1 + r} < \frac{1 - \phi}{1 + r}$ so $b_{t+s}^- = \bar{b}$ for all $s \geq 1$.

**Case 3a:** $\Phi = \tilde{\phi} < \Phi$ and, hence, corporate bonds are characterized by intermediate liquidity. Then $\tilde{\beta}\tilde{\delta} = \frac{1 - \tilde{\phi}}{1 + r} < \frac{1 - \Phi}{1 + r}$ yet $\tilde{\delta} = \frac{1 - \Phi}{1 + r} = \frac{1 - \phi}{1 + r}$, so $b_{t+1}^- = \bar{b}$ yet $0 \leq b_{t+s}^- \leq \bar{b}$ for $s > 1$. This problem is time-inconsistent.

**Case 3b:** $\Phi < \tilde{\phi} < \Phi$ and, again, corporate bonds are characterized by intermediate liquidity. Then $\tilde{\beta}\tilde{\delta} = \frac{1 - \tilde{\phi}}{1 + r} < \frac{1 - \phi}{1 + r}$ yet $\tilde{\delta} = \frac{1 - \Phi}{1 + r} > \frac{1 - \phi}{1 + r}$, so $b_{t+1}^- = \bar{b}$ yet $b_{t+s}^- = 0$ for
s > 1. This problem is time-inconsistent.

Case 3c: Φ < ˜φ = Φ and, again, corporate bonds are characterized by intermediate liquidity. Then ˜βδ = 1−Φ 1+r = 1−φ 1+r yet δ = 1−Φ 1+r > 1−φ 1+r, so 0 ≤ b_{t+1} ≤ ˜b yet b_{t+s} = 0 for s > 1. This problem is time-inconsistent.

In the time-inconsistent parameter range, we must consider a MPE when the firm cannot commit. Letting g(b) denote the policy function, the first-order condition with respect to b_{t+1} is

\[
\frac{1−φ}{1+r} + \mu_1 − \mu_2 + ˜βδW'(b') = 0
\]

where

\[
W'(b') \equiv -1 + g'(b') \left[ \frac{1−φ}{1+r} + \mu' − \mu'' + ˜δW'(g(b')) \right]
\]

and one of the solutions is found by setting g'(b') = 0. This collapses the first-order condition into those of Case 3 above. One can apply similar arguments to those in Proposition 2 to show that this is, in fact, the only equilibrium that is the limit of a finite-horizon version of the same problem.

\[\square\]

B Generalized Weights

B.1 Theoretical Analysis

In this appendix, we generalize Assumption 1 to encompass three important voting rules: that of Grossman and Hart (1979), Dreze (1974), and DeMarzo (1993). Grossman and Hart (1979) is the familiar case, discussed in the main text, of the firm maximizing an ownership-weighted valuation. Dreze (1974) makes a similar assumption, except that the weighting is done by the next period’s (that is, future) ownership. Finally, DeMarzo (1993) suggests a majority voting rule that works in the following way. A majority stable allocation is defined by the absence of an alternative allocation preferred by at least half of the shares. Due to the continuum of traders in our setting, for tractability, we strengthen this definition to the absence of any alternative allocation preferred by more than half of the shares.
Assumption 3. The firm maximizes a weighted valuation given by:

\[ \int_{\theta,b,h} w(\theta, b, h) \left[ d_t + (1 - \phi \Delta^- \theta, b, h)q_t \right] d\Gamma_t(\theta, b, h) = d_t + (1 - \Phi_t)q_t, \]  

where \( \Gamma_t(\theta, b, h) \) denotes the cross-section distribution over the portfolio holdings and employment status, \( d_t = F_t(k_t, k_{t+1}) \), and

\[ \Phi_t \equiv \phi \int_{\theta,b,h} w(\theta, b, h) \Delta^- \theta, b, h) d\Gamma_t(\theta, b, h). \]  

Weights \( w(\theta, b, h) \) are assumed to be non-negative and integrate to one. Current shareholder weighting from Grossman and Hart (1979) is given by \( w(\theta, b, h) = \theta \), and future shareholder weighting from Dreze (1974) is given by \( w(\theta, b, h) = \Theta_{t+1}(\theta, b, h) \). For the case of DeMarzo (1993), consider the median \( m \) satisfying

\[ \sum_h \int_{\theta} \int_{b} 1_{\{\Delta^- \theta, b, h \leq m\}} \theta d\Gamma_t(\theta, b, h) = \frac{1}{2} \]

and let

\[ w(\theta, b, h) = \begin{cases} 
    k & \text{if } \Delta^- \theta, b, h = m \\
    0 & \text{otherwise}
\end{cases} \]

where \( k > 0 \) is chosen so that weights integrate to one. Then the optimum of (23) satisfies majority stability. To see why, fix two households such that \( \Delta^- \theta, b, h > m \) and \( \Delta^- \theta, b, h < m \). Then if the above-median household prefers a particular deviation \((\tilde{d}_t, \tilde{q}_t)\),

\[ \tilde{d}_t + (1 - \Delta^- \theta, b, h)) \tilde{q}_t > d_t + (1 - \Delta^- \theta, b, h))q_t \]

where \((\tilde{d}_t, \tilde{q}_t)\) satisfy constraints of the firm problem and \((d_t, q_t) \in \text{argmax } d_t + (1 - m)q_t \) subject to constraints of the firm problem, then the below-median household must not,

\[ \tilde{d}_t + (1 - \Delta^- \theta, b, h)) \tilde{q}_t < d_t + (1 - \Delta^- \theta, b, h))q_t \]

For if this were not the case, we could take an appropriate convex combination of the two inequalities above to contradict the initial optimality of \((d_t, q_t)\). Similar arguments apply to deviations by the below-median household.
Next we show that our theoretical results from the main text hold under this
generalized setup. This next proposition generalizes Proposition 1, which decomposed
the sources of time inconsistency.

**Proposition 4.** The difference $\Phi - \bar{\Phi}$ is equal to the sum of a persistence effect, a risk premium, and a weight covariance:

$$
\Phi - \bar{\Phi} = \phi \left[ \mathbb{E}_t \left[ \mathbb{E}_t \left[ \Delta_{t+1} \right] \right] \right] - \bar{\Phi} \left[ \mathbb{E}_t \left[ \Delta_{t+1} \right] \right]
$$

$$
+ \phi \tilde{\mathbb{E}}_t \left[ \frac{\text{cov}_t \left( u' \left( \mathcal{C}_{t+1} \right), \Delta_{t+1} \right)}{\mathbb{E}_t \left[ u' \left( \mathcal{C}_{t+1} \right) \right]} \right] - \phi \sum_b \int_{\theta} \int_{\theta} \text{cov}_t \left( w_{t+1}, \Delta_{t+1} \right) d\Gamma(\theta, b, h)
$$

where tilde moments are taken with respect to the cross-sectional weighted density $w(\Theta_{t+1}, \mathcal{B}_{t+1}, h')dF(h'|h)d\Gamma(\theta, b, h)$, and the non-tilde moments are taken with respect to the density $dF(h'|h)$.

Just like in the special case of Grossman and Hart (1979) from the main text, Proposition 4 shows that time-inconsistency can only emerge in an economy with transaction costs, i.e., $\phi > 0$. The new term in the decomposition above is the weight covariance. Recall that in the Grossman and Hart (1979) setup, $w(\theta, b, h) = \theta$. In this case, $w_{t+1}(\Theta_{t+1}, \mathcal{B}_{t+1}, h') = \Theta_{t+1}(\theta, b, h)$, which is not measurable with respect to $h'$. This makes the weight covariance equal to zero. The weight covariance only appears under more complex weighting schemes such as that of Dreze (1974). In the Dreze (1974) case, the weight covariance is negative because those who sell ($\Delta_{t+1}$ large) have smaller portfolios ($\Theta_{t+2}$ small) by construction. This represents a force towards future bias.

In the case of majority voting from DeMarzo (1993), instead of studying the decomposition in Proposition 4, we directly consider whether the median shareholder is a seller. In numerical simulations, we find that sixty to seventy percent of shares belong to sellers. That is, a large number of households sell small amounts, while a small number of households buy large amounts; this is partly due to the quadratic nature of transaction costs. The result is a large value of $\bar{\Phi}$. When buyers tend to stay buyers, $\Phi$ is small; altogether we get a force towards present bias.

The net effect of all three competing forces depends on parameter values. However, in our numerical simulations, we find that the risk premium and weight covariance
are negligible in comparison to the persistence effect, leading to present bias.

B.2 Quantitative Analysis

In this section, we quantitatively solve the general equilibrium model under alternative assumptions for the weights in the firms’ problem. First, we consider the case with commitment. Note that the solution with commitment is independent of the specific weights assumed in the problem of the firm. The reason is that the specific weights matter for $\tilde{\beta}$ but not for $\tilde{\delta}$, and with commitment the firm does not use $\tilde{\beta}$. Second, we solve the model without commitment for three alternative weights: (i) current owners (the benchmark model, Grossman and Hart, 1979), (ii) future owners (Dreze, 1974), and (iii) median owner (DeMarzo, 1993).

Figure 7 shows the equilibrium allocations for capital, and the discount factors $\tilde{\delta}$ and $\tilde{\beta}$ for economies with different values of the transaction cost, $\phi$. The solution with commitment as well as the solution without commitment and current owners’ weight correspond to the benchmark case considered in the main text. The main result of this exercise is that both the case of future owners as well as the median owner generates qualitatively the same results of present-bias and under-accumulation of capital due to trading frictions and lack of commitment.

Figure 7: Generalized Weights

![Graph showing equilibrium allocations for capital, and the discount factors $\tilde{\delta}$ and $\tilde{\beta}$ for economies with different values of the transaction cost, $\phi$.]

Note: The figures show the allocations under different transaction costs, $\phi$. 

51
C 3-Period Model

C.1 Stationary Model

Consider a stationary version of the 3-period model laid out in Section 3. Like before, there are only three time periods \( t = 0, 1, 2 \) and \( c_t = 0 \) for \( t \geq 3 \); however \( q_2 \neq 0 \). The reason households hold stock beyond period 2 is due to an exogenous “continuation value” we append to the household’s objective. This trick allows us to induce a stationary 3-period equilibrium with constant asset prices. The household objective becomes

\[
\sum_{t=0}^{2} \beta^t u(c_t^j) + \beta^2 \mu^j \theta_3^j
\]

The Euler equation in \( t = 2 \) becomes

\[
q_2 u'(c_2^j)(1 - \phi \Delta_{j}^{-}) = \mu^j
\]

If \( \mu^j \) and initial assets \( \theta_0^j \) are selected appropriately, then the resulting household equilibrium will be stationary. In this stationary equilibrium, consumption will simply fluctuate (i.e. \( c^t, c^h, c^l \)), asset sales will also fluctuate (i.e. \( \Delta^{−}, 0, \Delta^{−} \)), and asset prices will be a constant \( q \). Using this notation, the required \( \mu^j \) are given by

\[
\mu^l = \beta u'(c^h)(d + q), \quad \text{and} \quad \mu^h = \beta u'(c^l)(d + (1 - \phi \Delta^{−})q)
\]

We now re-solve the numerical example in Section 3 with all the same parameter values but one. Moving to this stationary setup, asset prices are larger in magnitude (because we mimic the first-order conditions of an infinite-horizon problem). With larger asset prices, asset trades are smaller and time-inconsistency issues become more difficult to discern. For no other reason than to reduce asset prices, we let \( d_t = 0.1 \) for all \( t \). The household equilibrium is summarized in Table 5, and the stationary asset price is given by \( q = 1.9480 \).

<table>
<thead>
<tr>
<th>Household type</th>
<th>( c_0 )</th>
<th>( c_1 )</th>
<th>( c_2 )</th>
<th>( \Delta_0^{-} )</th>
<th>( \Delta_1^{-} )</th>
<th>( \Delta_2^{-} )</th>
<th>( \theta_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>low</td>
<td>2.0385</td>
<td>2.1389</td>
<td>2.0385</td>
<td>0.4810</td>
<td>0.0000</td>
<td>0.4810</td>
<td>1.2405</td>
</tr>
<tr>
<td>high</td>
<td>2.1389</td>
<td>2.0385</td>
<td>2.1389</td>
<td>0.0000</td>
<td>0.4810</td>
<td>0.0000</td>
<td>0.7595</td>
</tr>
</tbody>
</table>
The problem of the firm is defined as before, which, in period 0, becomes

\[ V^F_0(k_0) = \max_{k_1, k_2 \geq 0} d_0 + 0.9401d_1 + 0.9025d_2 \]

and, in period 1, becomes

\[ V^F_1(k_1) = \max_{k_2 \geq 0} d_1 + 0.9401d_2 \]

Due to the stationary nature of our new equilibrium, the one-period-ahead discount rate is always the same, 9.401, and the two-period-ahead discount rate is simply \( \beta^2 \) (that is, consumption at time \( t \) and time \( t + 2 \) agree). From its period 0 perspective, the firm discounts between time \( t = 1 \) and \( t = 2 \) at the rate 0.9600, which is the ratio between 0.9025 and 0.9401. When period 1 arrives, the firm discounts between time \( t = 1 \) and \( t = 2 \) at the rate 0.9401. Hence the problem of the firm is time-inconsistent and, like in Section 3, the direction of inconsistency is towards present bias.

### C.2 Symmetric Transaction Costs

Consider a 3-period economy, as in Section 3, but with symmetric transaction costs. That is, the budget constraint (2) now has one additional term \( \frac{\phi}{2}(\Delta^+_t)^2q_t \), which denotes the transaction cost incurred for buying the stock. The new budget constraint is

\[ c_t + q_t\Delta^+_t + \frac{b_{t+1}}{1+r_t} \leq w_t h_t + d_t \theta_t + \left( \Delta^-_t - \frac{\phi}{2}(\Delta^-_t)^2 - \frac{\phi}{2}(\Delta^+_t)^2 \right) q_t + b_t \]

and the Euler equation of the household becomes

\[ (1 + \phi \Delta^+_t)q_t = \beta \frac{u'(c_{t+1}^j)}{u'(c_t^j)} d_{t+1} + \beta \frac{u'(c_{t+1}^j)}{u'(c_t^j)} (1 + \phi \Delta^+_{t+1})q_{t+1}, \quad t \in \{0, 1\} \]

where \( \Delta^+_t \equiv (\Delta^{++}_t - \Delta^{--}_t) \). Intuitively, buyers now have an even higher valuation of the stock than in the asymmetric setting because they must incur a transaction cost to buy. Calculations similar to those of Lemma 1 yield a firm objective equal to

\[ \sum_{j \in \{l, h\}} \frac{\theta^j}{2} [d_t + (1 + \phi \Delta^+_t)q_t] \]
Table 6: Equilibrium with Symmetric Transaction Costs

<table>
<thead>
<tr>
<th>Household type</th>
<th>$c_0$</th>
<th>$c_1$</th>
<th>$c_2$</th>
<th>$\Delta_0^{-}$</th>
<th>$\Delta_1^{-}$</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>low</td>
<td>2.5939</td>
<td>2.7925</td>
<td>2.5635</td>
<td>0.3253</td>
<td>0.0000</td>
<td>0.6747</td>
<td>1.5635</td>
</tr>
<tr>
<td>high</td>
<td>3.3865</td>
<td>3.1324</td>
<td>3.4365</td>
<td>0.0000</td>
<td>0.8888</td>
<td>1.3253</td>
<td>0.4365</td>
</tr>
</tbody>
</table>

With this new setup, we solve a numerical example using the same parameter values as in Section 3. The household equilibrium is summarized in Table 6, and asset prices are given by $q_0 = 1.8560$ and $q_1 = 0.9504$.

The problem of the firm in period 0 becomes

$$V_0^F(k_0) = \max_{k_1, k_2 \geq 0} d_0 + 0.9547d_1 + 0.9013d_2$$

and the problem of the firm in period 1 becomes

$$V_1^F(k_1) = \max_{k_2 \geq 0} d_1 + 0.9229d_2$$

so that, from its period 0 perspective, the firm discounts between time $t = 1$ and $t = 2$ at the rate 0.9441, which is the ratio between 0.9013 and 0.9547. When period 1 arrives, the firm discounts between time $t = 1$ and $t = 2$ at the rate 0.9229. Hence the problem of the firm remains time-inconsistent, with a direction of inconsistency towards present bias. Transaction costs on buyers further restrict asset trading (beyond the case of asymmetric transaction costs), which creates larger differences in IMRS across agents; this exacerbates problems of time-inconsistency.

### D Liquid Firms

Consider an economy with both liquid and illiquid stocks. The proportion of liquid stocks is $(1 - \omega)$, and the proportion of illiquid stocks is $\omega$ so that the total mass of stocks is still normalized to one. The special case of $\omega = 1$ corresponds to our main model. With two types of firms in the economy (with potentially differing discount rates), we must restrict our analysis to decreasing returns technologies to retain an interior solution to both firms’ problems. The household budget constraint becomes

$$c_t + q_t\Delta_t^+ + \frac{b_{t+1}}{1 + r_t} + \hat{q}_t\hat{\theta}_{t+1} \leq w_t h_t + d_t \theta_t + (\hat{d}_t + \hat{q}_t)\hat{\theta}_t + \left(\Delta_t^+ - \frac{\phi}{2}(\Delta_t^-)^2\right) q_t + b_t$$
where $\hat{q}_t$ denotes the price of the liquid stock, $\hat{\theta}_{t+1}$ denotes the amount of liquid stock purchased, and $\hat{d}_t$ denotes dividends paid by the liquid firm. Beyond the standard constraints on illiquid stocks, we have the short-selling constraint on the liquid stock and the borrowing constraint

$$\hat{\theta}_{t+1} \geq 0, \quad b_{t+1} \geq \bar{b}$$

To simplify this problem, we note that liquid stocks and bonds are redundant, and hence we can combine them into one state variable

$$W_{t+1} \equiv (\hat{d}_{t+1} + \hat{q}_{t+1})\hat{\theta}_{t+1} + b_{t+1}$$

so that the budget constraint can be rewritten

$$c_t + q_t\Delta_t^+ + \frac{W_{t+1}}{1 + r_t} \leq w_t h_t + d_t \theta_t + W_t + \left(\Delta_t^- - \frac{\phi}{2}(\Delta_t^-)^2\right) q_t$$

The constraints (25) can be combined

$$W_{t+1} \geq \bar{b}$$

Market clearing conditions for bonds, illiquid stocks, and labor become

$$\int_{j \in [0,1]} W_{j,t} dj = (1 - \omega)(\hat{d}_t + \hat{q}_t)$$

$$\int_{j \in [0,1]} \theta_{j,t} dj = \omega$$

$$\int_{j \in [0,1]} h_{j,t} dj = \omega l_t + (1 - \omega)\hat{l}_t$$

On the first line above, the equality follows from the fact that the proportion of liquid stocks is $(1 - \omega)$. On the third line above, $\hat{l}_t$ denotes labor demanded by the liquid firm, which is defined by the intratemporal first-order condition,

$$\hat{l}_t = \psi \gamma \frac{\hat{y}_t}{w_t}, \text{ where } \hat{y}_t = [(\hat{l}_t)^\gamma (\hat{k}_t)^{1-\gamma}]^\psi.$$
Labor demanded by the illiquid firm, $l_t$, is defined by similar conditions,

$$l_t = \psi \gamma \frac{y_t}{w_t},$$

where $y_t = [(l_t)^\gamma (k_t)^{1-\gamma}]^\psi$.

Combining the expression for output with the optimality condition for labor, we get

$$l_t = (\gamma \psi)^{\frac{1}{1-\gamma}} \frac{k_t^{(1-\gamma)\psi}}{w_t^{1-\gamma\psi}}, \quad \text{and} \quad \dot{l}_t = (\gamma \psi)^{\frac{1}{1-\gamma}} \frac{\dot{k}_t^{(1-\gamma)\psi}}{w_t^{1-\gamma\psi}}.$$

Thus, market clearing in the labor market implies

$$\omega l_t + (1 - \omega) \dot{l}_t = H = \omega (\gamma \psi)^{\frac{1}{1-\gamma}} \frac{k_t^{(1-\gamma)\psi}}{w_t^{1-\gamma\psi}} + (1 - \omega) (\gamma \psi)^{\frac{1}{1-\gamma}} \frac{\dot{k}_t^{(1-\gamma)\psi}}{w_t^{1-\gamma\psi}}.$$

or

$$w_t = \frac{\gamma \psi}{H^{1-\gamma\psi}} \left( \omega k_t^{(1-\gamma)\psi} + (1 - \omega) \dot{k}_t^{(1-\gamma)\psi} \right)^{1-\gamma\psi}.$$

Now, let’s use the optimality conditions for capital. Focusing on a steady-state equilibrium, the analysis in Section 5, implies

$$M \left( z\alpha k^{\alpha - 1} + (1 - \delta) \right) = 1$$

for the illiquid firms, and

$$\hat{M} \left( z\alpha \dot{k}^{\alpha - 1} + (1 - \delta) \right) = 1$$

for the liquid firms, where $M = \tilde{\delta}$ if the firms can commit to an investment plan and $M = \tilde{\delta} \tilde{\delta}$ if they cannot, $\hat{M} = \frac{1}{1+r}$, and

$$z = (1 - \gamma \psi) \left( \frac{\gamma \psi}{w} \right)^{\gamma \psi \frac{1}{1-\gamma\psi}}$$

$$= \frac{(1 - \gamma \psi) H^{\gamma \psi}}{\left( \omega k^{(1-\gamma)\psi} + (1 - \omega) \dot{k}^{(1-\gamma)\psi} \right)^{\gamma \psi}}.$$
Combining the two optimality conditions, we get
\[ \hat{k} = \left( \frac{\frac{1}{\hat{M}} - (1 - \delta)}{\frac{1}{M} - (1 - \delta)} \right)^{\frac{1}{1 - \alpha}} k. \]

Since, \( M < \hat{M} \), then \( \hat{k} > k \). To solve for \( k \), we plug the expressions for \( z \) and \( \alpha = \frac{(1 - \gamma)\psi}{1 - \gamma \psi} \) into the optimality condition for illiquid firms,

\[
k = \left[ \frac{1}{\omega + (1 - \omega) \left( \frac{1}{\hat{M}} - (1 - \delta) \right)^{\frac{(1 - \gamma)\psi}{1 - \gamma \psi}}} \right]^{\frac{(1 - \gamma)\psi M}{1 - \hat{M} \left(1 - \delta\right)H^\psi}}.
\]