Monetary Policy and Government Debt

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Abstract

We study how the level of government debt affects the effectiveness of monetary policy, i.e., the elasticity of economic aggregates to interest rate changes. We build a New Keynesian model where fiscal policy is non-Ricardian and government debt is risk-free. Wealth effects generated by government bonds weaken the transmission of changes in the policy rate to output. Using data on private ownership of public debt for the U.S., we find that when the government debt-to-GDP ratio is one standard deviation above its mean, the response of industrial production and unemployment to a monetary shock is reduced by 0.75pp and 0.1pp, respectively, out to a three-year horizon.

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1 Introduction

Government debt has been rising in many advanced economies, and it is projected to continue increasing in the next decades (Yared, 2019). For example, U.S. government debt currently represents more than 100% of GDP, while it was less than 50% in the 1990s. Moreover, the CBO projects that the number will surpass 200% by 2051. The importance of public debt in shaping economic outcomes is widely recognized in macroeconomics. Its relevance covers a variety of questions, from its role as a tool to smooth the government's fiscal needs (Barro 1979) to generating a burden (D'Erasmo et al. 2016) and triggering recessions or slowing growth (Reinhart et al. 2012). In this paper, we explore the role of government debt in the monetary transmission mechanism.

Monetary policy has become the main macroeconomic stabilization policy tool in advanced economies. However, little is known about how the effectiveness of monetary policy interacts with the level of government debt. The textbook analysis implies that government debt has no impact on the effect of monetary policy on the real economy (see Woodford, 2001; Galí, 2015). In contrast, models that emphasize the importance of monetary and fiscal interactions highlight the relevance of government debt in the dynamics of the economy but do not consider the consequences of high debt levels for the effectiveness of monetary policy. This is the focus of our paper.

We study the role of government debt in a New Keynesian model in continuous time. Since we focus on developed economies, we abstract from default risk and assume that government debt is safe in nominal terms.[1] Moreover, we assume that fiscal policy is non-Ricardian or, in Leeper (1991) terminology, the economy is in an "active fiscal/passive monetary" policy regime. In this setting, the government's budget constraint becomes a relevant equilibrium condition, and government debt affects the real economy through wealth effects that are not fully offset by tax policy. Our main theoretical result is that monetary policy is less effective in economies with a higher level of public debt, meaning that the output response to changes in the nominal interest rate is attenuated relative to low-debt economies. We then explore the model's predictions empirically and find that

[1] See Arellano et al. (2020) for a model of monetary policy and sovereign default risk.

they are consistent with the U.S. data.

To understand the intuition behind the results, consider an economy where the monetary authority increases the nominal interest rate. In the presence of nominal rigidities, this implies an increase in the real interest rate and a reduction in initial consumption. The magnitude of the effect depends on two forces. First, there is the standard intertemporal substitution effect: when interest rates go up, households reduce present consumption in favor of future consumption. Second, there is the change in the households' wealth generated by the change in policy.

Households' wealth depends on their labor income and their financial assets. Wages, employment, and profits from ownership of firms respond to monetary policy only indirectly from the general equilibrium forces in the economy. In contrast, holdings of government bonds are directly affected by changes in monetary policy. Suppose all government debt is short-term. Then, an increase in the nominal interest rate represents a *positive wealth effect* from the bond holdings, as households get a higher return for their savings.[2] Absent a fiscal offset, this channel *weakens* the recessionary effects of contractionary monetary policy interventions. Crucially, the wealth effect generated by government debt is proportional to the stock of debt held by households, where a larger stock generates a larger wealth effect.

Our results are in sharp contrast to the predictions obtained from the standard equilibrium selection (the so-called "Taylor equilibrium"), in which fiscal variables are irrelevant to the determination of equilibrium. Notably, this stark difference is not driven by differences in the wealth effects associated with government bonds. Monetary policy *always* affects the valuation and return of government debt, independently of the equilibrium selection criterion. However, the standard selection neutralizes these wealth effects by assuming offsetting lump-sum transfers, such that the net effect is always zero. Thus, different government debt levels affect the fiscal response to changes in monetary policy, but they do not affect the dynamics of households' wealth and, therefore, the consumption response to changes in the policy rate. In contrast, transfers do not offset these wealth

[2] Note that if government debt is positive, the household sector is a *net saver* in the aggregate, so it benefits from an increase in the interest rate when debt is short term.

effects in our non-Ricardian setting, opening the possibility that the level of government debt affects the dynamics of the economy.

We then extend the main results to an economy with long-term government debt. In this case, monetary policy generates an additional wealth effect that operates through the repricing of assets. An increase in the policy rate reduces the price of long-term government bonds, generating a *negative wealth effect*. Whether this repricing channel is sufficient to overturn the positive effect of higher returns on households' savings depends on the duration of the debt. While the positive effect is independent of the duration of government debt, the negative effect is stronger the longer the duration. Whether the net effect is positive or negative ultimately depends on whether a higher interest rate increases or reduces the government debt burden since a positive wealth effect is the counterpart of an increase in the government debt burden (and vice versa). Thus, if contractionary monetary policy increases the government's debt burden, households will experience a positive wealth effect, and monetary policy becomes weaker with the level of government debt. Notably, the net effect is more likely to be positive the more sticky prices are. In the extreme case in which prices are fully rigid, the wealth effect of a contractionary monetary shock is positive for any duration lower than that of a consol.

Finally, we explore the validity of the model's predictions on U.S. data. We study the interaction between identified monetary policy shocks using the Romer and Romer (2004) narrative approach and the public debt position of private investors using the data from Hall et al. (2018). We extend the Jordà (2005) local projections method to study this interaction in a dynamic setting, and we find that high levels of government debt attenuate the effects of monetary policy on industrial production and the unemployment rate. When the privately-held government debt-to-GDP ratio is one standard deviation above its mean, the response of industrial production is diminished by 0.75pp, and the response of the unemployment rate is reduced by 0.1pp, at a three-year horizon. We also show that our results are robust to considering the monetary shock series estimated by Miranda-Agrippino and Ricco (2021) and the Blinder-Oaxaca decomposition suggested by Cloyne et al. (2020). These results suggest that, in contrast to the standard analysis, the level of government debt is an important source of time variation in the monetary transmission, such that higher levels of debt weaken the transmission of monetary policy.[3]

Literature Review. This paper is related to several strands of literature. First, the paper is connected to the literature that studies the real effects of government debt.[4] Ball and Mankiw (1995) study the crowding-out effect of government debt, while Reinhart et al. (2012) argue that high debt levels are associated with lower long-run growth. Our paper identifies the relationship between public debt and monetary policy as a new channel through which government debt can affect the economy.

Our paper also relates to discussions of sustainable public debt and stabilization policies. D'Erasmo et al. (2016) study empirical and theoretical models of sovereign default and show the conditions under which public debt can be considered sustainable when the government cannot commit to repaying its debts. Leeper et al. (2016) show that in the absence of commitment, optimal monetary policy faces an inflation bias, partly to stabilize the real value of government debt. Davig et al. (2011) study the theoretical limits of a government's ability to finance its debt through taxation and find that the tail events associated with this limit imply an upward bias in inflation expectations that present a challenge to monetary policymakers. These mechanisms highlight the interdependence of monetary policy and the structure of government finances.

Additionally, our paper contributes to the theoretical literature on New Keynesian models and the Fiscal Theory of the Price Level (FTPL). Leeper (1991), Sims (1994) and Woodford (2001) are early developments of the FTPL. Kim (2003) provides an analysis combining studying the effects of the FTPL in a New Keynesian model. Caramp and Silva (2023) show that fiscal policy is a crucial determinant of the wealth effects in the monetary transmission mechanism.[5] We extend their analysis and focus on the role of government debt in shaping the effectiveness of monetary policy. Closely related is Cochrane (2001), who identifies the importance of maturity in determining the path of inflation under the FTPL. Our analysis differs from his in that we study the interaction between the debt level

[3]Moreover, the empirical results reject the standard formulation of active monetary regimes.

[4]See Elmendorf and Mankiw (1999) for a comprehensive review.

[5]Bianchi and Melosi (2019) and Bianchi et al. (2020) are recent contributions studying the consequences of monetary/fiscal interactions in economies with high levels of government debt.

and monetary policy's effectiveness.

Finally, our paper builds upon recent advances in econometric methods to examine the interaction between monetary policy and government debt. Estimating the effects of monetary policy has a long history in macroeconomics.[6] In the spirit of Tenreyro and Thwaites (2016); Angrist et al. (2018); Barnichon et al. (2022) and others, we augment the Jordà (2005) local projections model with nonlinear interactions to study the effect of government debt on the transmission mechanism of monetary policy.[7]

Outline. The rest of the paper is organized as follows. Section 2 describes the model and Section 3 studies the equilibrium dynamics. Section 4 presents the paper's main results: the relationship between the level of debt and monetary policy. Section 5 conducts the empirical analysis and Section 6 concludes. All the proofs are in Appendix A.

2 The Model

Time is continuous and denoted by $t \in \mathbb{R}_+$.[8] The economy is populated by a large number of identical, infinitely-lived households and a continuum of firms that produce final and intermediate goods. Final good producers operate in a perfectly competitive market and combine intermediate goods using a CES aggregator with elasticity $\epsilon > 1$. Intermediate-goods producers use labor as the only factor of production to produce a differentiated good that is traded in monopolistically competitive markets. We assume that intermediate-goods firms face a pricing friction à la Calvo. Moreover, there is an infinitely-lived government that sets monetary and fiscal policy.

We study the determination of equilibrium of an economy in which fiscal policy is described by a non-Ricardian rule, in the sense that primary surpluses do not automatically adjust to satisfy the budget constraint *for every* sequence of endogenous and exogenous variables (see Woodford, 2001). We shall see that this assumption is crucial to obtain that the level of government debt matters for the economy's response to policy changes.

[6] See Ramey (2016) for a literature review.

[7] See also Broner et al. (2022), Ottonello and Winberry (2020) and Alessandri and Venditti (2022).

[8] See Werning (2011) and Cochrane (2017) for formulations of the New Keynesian model in continuous time.

Households. Households have preferences given by

$$\int_0^\infty e^{-\rho t} \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi} \right] dt, \tag{1}$$

where C_t denotes consumption in period t, N_t is hours worked, $\rho > 0$ is the instantaneous discount factor, and $\sigma, \phi \geq 0$. They face an intertemporal budget constraint given by

$$\int_0^\infty e^{-\int_0^t i_s ds} P_t C_t dt \leq B_0 + \int_0^\infty e^{-\int_0^t i_s ds} \left[W_t N_t + \Pi_t + P_t T_t \right] dt, \tag{2}$$

where i_t represents the nominal interest rate, B_t is a short-term (instantaneous) nominal bond, W_t is the nominal wage, Π_t is aggregate nominal profits, T_t is a government lump-sum transfer, and P_t is the price level.

The households' objective is to choose sequences $[C_t, N_t]_{t \geq 0}$ to maximize (1) subject to (2), given B_0. The households' optimality conditions are given by

$$N_t^\phi C_t^\sigma = \frac{W_t}{P_t},$$
$$\frac{\dot{C}_t}{C_t} = \sigma^{-1}(i_t - \pi_t - \rho).$$

Firms. There are two types of firms in the economy: final goods producers and intermediate goods producers. Final goods producers operate in a perfectly competitive market and combine a unit mass of intermediate goods $Y_t(i)$, for $i \in [0, 1]$, using the production function

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}. \tag{3}$$

The problem of a final goods producer is given by

$$\max_{[Y_t(i)]_{i \in [0,1]}} P_t Y_t - \int_0^1 P_t(i) Y_t(i) di,$$

subject to (3). The solution to this problem gives the standard CES demand

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} Y_t, \tag{4}$$

where $P_t \equiv \left(\int_0^1 P_t(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}$ is the aggregate price level.

Intermediate goods are produced using the following technology:

$$Y_t(i) = N_t(i)^{1-\gamma},$$

with $\gamma \in [0, 1)$. Intermediate goods firms choose the price for their good, $P_t(i)$, subject to the demand for their good, given by (4), taking the aggregate price level, P_t, and aggregate output, Y_t, as given. As is standard in New Keynesian models, we assume that firms are subject to a pricing friction à la Calvo: firms are allowed to reset their prices with Poisson intensity ρ_δ. Moreover, we assume that the government levies a constant sales tax τ. Let P_t^* denote the price chosen by a firm that can set their price in period t. Then, P_t^* is the solution to the following problem:

$$\max_{P_t^*} \int_0^\infty e^{-(\rho+\rho_\delta)s} \left(\frac{C_{t+s}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+s}} \left[(1-\tau) P_t^* Y_{t+s|t} - W_{t+s} Y_{t+s|t}^{\frac{1}{1-\gamma}} \right] ds,$$

where $e^{-\rho s} \left(\frac{C_{t+s}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+s}}$ is the households' stochastic discount factor for nominal payoffs, $Y_{t+s|t}$ represents the demand function faced at period $t+s$ by a producer that last set price in period t, that is

$$Y_{t+s|t} = \left(\frac{P_t^*}{P_{t+s}} \right)^{-\epsilon} Y_{t+s},$$

Y_t denotes the aggregate demand at period t, and we used that $N_{t+s}(i) = Y_{t+s|t}^{\frac{1}{1-\gamma}}$. The first-order condition associated with this problem is given by

$$\int_0^\infty e^{-(\rho+\rho_\delta)s} \left(\frac{C_{t+s}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+s}} \left[(1-\tau) P_t^* Y_{t+s|t} - \frac{\epsilon}{\epsilon-1} \frac{1}{1-\gamma} W_{t+s} Y_{t+s|t}^{\frac{1}{1-\gamma}} \right] ds = 0.$$

Since P_0 is predetermined in this continuous time setting, we normalize it to one, i.e., $P_0 = 1$.

Government. The government's intertemporal budget constraint is given by

$$D_0^g = \int_0^\infty e^{-\int_0^t i_s ds} \left(\tau P_t Y_t - P_t T_t\right) dt,$$

where D_0^g denotes the government debt level in period 0. Note that we have assumed that government debt is short-term (instantaneous) here. We extend the analysis to long-term bonds in Section 4. Moreover, Appendix B shows that our results extend to a setting with government spending.

An important feature for the determination of equilibrium is that fiscal policy is described by a non-Ricardian rule, in the sense that primary surpluses do not automatically adjust to satisfy the budget constraint *for every* sequence of endogenous and exogenous variables. In particular, we follow Leeper (1991) and assume that the fiscal authority adjusts the lump-sum transfers in response to the level of real government debt outstanding, that is,

$$T_t = \gamma_0 - \gamma_d \frac{D_t^g}{P_t}, \tag{5}$$

while the monetary authority sets the nominal interest rate as a function of current inflation, that is,

$$i_t = \rho + \phi_\pi \pi_t + u_t, \tag{6}$$

where u_t represents an innovation of the rule relative to its systematic response to inflation. A non-Ricardian regime, also known as an "active fiscal/passive monetary" regime, requires that $\gamma_d \in [0, \rho)$ and $\phi_\pi \in [0, 1)$.[9] In contrast, a Ricardian regime, or "active monetary/passive fiscal" regime, requires that $\gamma_d > \rho$ and $\phi_\pi > 1$. For ease of exposition, we focus on a non-Ricardian regime with $\gamma_d = \phi_\pi = 0$ in the main text. This implies that lump-sum transfers are constant and the nominal interest rate responds one-to-one to the monetary shock. We show in Appendix C that all our results generalize to any $\gamma_d \in [0, \rho)$ and $\phi_\pi \in [0, 1)$. Moreover, Appendix C shows that our results survive a generalization of the fiscal rule that includes interest payments as long as the lump-sum transfers do not

[9] Note that if $\gamma_d \in (0, \rho)$, there are solutions of the system that feature bounded paths for consumption and inflation but an unbounded debt-to-output ratio and still satisfy the transversality condition. Here we adopt the convention in Leeper (1991) and focus on equilibria with a bounded debt-to-output ratio. For a discussion of this point, see Cochrane (2023), Chapter 5.4.

adjust to fully neutralize the wealth effects generated by government bonds.[10] For concreteness, we study an economy hit by a monetary shock that leads to a mean-reverting process for $[u_t]_{t\geq 0}$, that is, we assume that

$$u_t = e^{-\psi_m t} u_0, \quad u_0 \text{ given,}$$

with $\psi_m > 0$.

Market clearing and the aggregate price level. The market clearing condition for goods and bonds are given by

$$C_t = Y_t, \qquad B_t = D_t^g.$$

Applying an appropriate law of large numbers, we get that the aggregate price level is an average of prices set in different periods:

$$P_t = \left(\int_{-\infty}^{t} \rho_\delta e^{-\rho_\delta (t-s)} (P_s^*)^{1-\epsilon} ds \right)^{\frac{1}{1-\epsilon}} \iff P_t^{1-\epsilon} = \int_{-\infty}^{t} \rho_\delta e^{-\rho_\delta (t-s)} (P_s^*)^{1-\epsilon} ds.$$

Differentiating the expression above, we get

$$(1-\epsilon) P_t^{1-\epsilon} \frac{\dot{P}_t}{P_t} = \rho_\delta (P_t^*)^{1-\epsilon} - \rho_\delta P_t^{1-\epsilon}.$$

Defining the inflation rate as $\pi_t \equiv \frac{\dot{P}_t}{P_t}$, we get

$$\pi_t = \frac{\rho_\delta}{\epsilon - 1} \left[1 - \left(\frac{P_t^*}{P_t} \right)^{1-\epsilon} \right].$$

Steady-state equilibrium and the irrelevance of government debt. Let the variables without subscript denote their value in the zero-inflation steady state. In this equilibrium, policy is such that: i) the fiscal variables are constant, i.e., $T_t = T$ for all t; ii) the nominal

[10] In particular, we consider fiscal rules of the type $T_t = \gamma_0 - \gamma_d \frac{D_t^g}{P_t} - \gamma_r \frac{D_t^g}{P_t}(i_t - \pi_t)$. Our results require that $\gamma_r < 1$, i.e. the lump-sum transfer does not completely finance the change in interest payments.

interest rate is $i_t = \rho$ for all t. The steady-state allocation satisfies

$$C = Y = \left[\frac{1}{(1-\tau)(1-\gamma)}\frac{\epsilon}{\epsilon-1}\right]^{\frac{1-\gamma}{\gamma+\varphi-\sigma(1-\gamma)}}, \tag{7}$$

$$N = Y^{\frac{1}{1-\gamma}}, \tag{8}$$

$$D^g = \frac{\tau Y - T}{\rho}. \tag{9}$$

These equations lead to the following result.

Proposition 1. *Given τ, the steady-state level of output, consumption, and labor are independent of the level of government debt, D^g.*

The steady-state levels of output, consumption, and labor are determined by equations (7)-(8), which are independent of the level of debt, conditional on τ. Then, equation (9) determines the combination of lump-sum transfers and debt levels consistent with the government's budget constraint. For example, a higher level of steady-state debt is associated with a lower level of lump-sum transfers (recall that these are transfers *to* the agents), which are used to pay the interest on the debt. The following corollary provides a benchmark for the analysis that follows.

Corollary 1.1. *Consider two economies like the one described here, with the same preferences and technologies. If the steady-state level of distortionary taxes coincides, their steady-state level of output, consumption, and labor also coincide.*

This result provides a useful benchmark for our exercises in the following sections. It states that two economies that differ only in their steady-state level of debt feature the same steady-state allocation. However, we will show that, despite this, their dynamics after a monetary shock may differ.

3 Equilibrium Dynamics

To study the dynamics of the economy, we log-linearize the equilibrium conditions around a steady state that features a constant path for the policy variables and zero inflation. Let

$$c_t = \log(C_t) - \log(C) \quad \text{and} \quad y_t = \log(Y_t) - \log(Y).$$

Given the path of the nominal interest rate, $[i_t]_{t \geq 0}$, the equilibrium is characterized by

$$\dot{c}_t = \sigma^{-1}(i_t - \pi_t - \rho), \tag{10}$$

$$\dot{\pi}_t = \rho \pi_t - \kappa c_t, \tag{11}$$

and the intertemporal budget constraint

$$\int_0^\infty e^{-\rho t} c_t dt = \int_0^\infty e^{-\rho t} \left[(1-\tau) y_t + \varsigma_d (i_t - \pi_t - \rho)\right] dt, \tag{12}$$

where κ is a positive constant defined in the appendix, and ς_d is the debt-to-output ratio in the steady state (recall that the lump-sum transfers are constant, i.e., $T_t = T \; \forall t$).[11] Equation (10) is the households' Euler equation and equation (11) is the Phillips curve, which arises from the intermediate-goods firms' optimal pricing decisions. Finally, equation (12) is the households' budget constraint, which states that the present value of consumption equals the present value of *after-tax* income from wages and profits, plus the interest income from government bond holdings. Noting that the Euler equation implies

$$\int_0^\infty e^{-\rho t} \rho \sigma \left(c_t - c_0\right) dt = \int_0^\infty e^{-\rho t} \left(i_t - \pi_t - \rho\right) dt,$$

we can rewrite the budget constraint as

$$\int_0^\infty e^{-\rho t} c_t dt = \int_0^\infty e^{-\rho t} \left[(1-\tau)\, y_t + \rho \varsigma_d \sigma \left(c_t - c_0\right)\right] dt, \tag{13}$$

[11] See Appendix B for the full derivation.

where the last term on the right-hand side represents the change in the real rate of return on government bonds.

Next, we solve the model. We start with the case of rigid prices, which allows a simple characterization. After that, we solve the general case with sticky prices.

Rigid prices. Before solving the full model, let's consider the case with rigid prices, i.e., $\kappa = 0$ so $\pi_t = 0 \; \forall t$. The households' Euler equation implies

$$c_t = \underbrace{c_0}_{\text{level}} + \underbrace{\sigma^{-1} \int_0^t (i_s - \rho) ds}_{\text{slope}} . \tag{14}$$

The Euler equation determines the *slope* of the consumption path, which depends on the path of the nominal interest rate and the EIS, σ^{-1}. The *level* of the consumption path is determined by the households' budget constraint. Plugging equation (14) into the intertemporal budget constraint (13), and using the market-clearing condition in the goods market and the dynamics of the nominal interest rate after a monetary shock, we get

$$c_0 = -\sigma^{-1} \frac{\tau - \rho \varsigma_d \sigma}{\tau} \frac{u_0}{\rho + \psi_m}.$$

Note that the debt-to-output ratio ς_d is a crucial component of the determination of c_0.

Sticky prices. It is useful to define the following two constants (which are the eigenvalues of the system given by (10) and (11)):

$$\overline{\omega} = \frac{\rho + \sqrt{\rho^2 + 4\sigma^{-1}\kappa}}{2} > 0, \quad \underline{\omega} = \frac{\rho - \sqrt{\rho^2 + 4\sigma^{-1}\kappa}}{2} < 0.$$

The next proposition characterizes the solution of the system (10) and (11), given the path for the monetary shock u_t, in closed form.

Proposition 2. *The equilibrium path for consumption is given by*

$$c_t = e^{\underline{\omega} t} c_0 + c_t^m,$$

where

$$c_t^m \equiv \sigma^{-1} \frac{\rho + \psi_m}{(\overline{\omega} + \psi_m)(\underline{\omega} + \psi_m)} \left(e^{\underline{\omega} t} - e^{-\psi_m t} \right) u_0,$$

and the initial value c_0 is given by

$$c_0 = -\sigma^{-1} \frac{\tau - \rho \varsigma_d \sigma}{\tau - \underline{\omega} \varsigma_d \sigma} \frac{u_0}{\overline{\omega} + \psi_m}.$$

Given u_0, the path of consumption, $[c_t]_{t \geq 0}$, is uniquely determined.

Proposition 2 characterizes the equilibrium path of consumption in the non-Ricardian regime in which lump-sum transfers are constant. This solution differs from the standard equilibrium selection, which relies on an interest rate rule that satisfies the "Taylor principle." The standard selection typically drops the budget constraint (13) and instead assumes an interest rate rule of the form of (6) with $\phi_\pi > 1$. Then, the equilibrium of the economy is the solution to the system of equations given by (10), (11), and (6). However, there is no guarantee that such a solution will satisfy the budget constraint (13). This problem is resolved by assuming that the path of the lump-sum transfer $[T_t]_{t \geq 0}$ automatically adjusts to satisfy the constraint, for example by assuming that $\gamma_d > \rho$ in the fiscal rule (5). Crucially, the equilibrium paths of consumption and inflation are *independent* of the level of government debt. In contrast, the equilibrium in Proposition 2 is obtained by assuming a monetary rule with $\phi_\pi = 0$ and assuming that the path of the lump-sum transfer *does not* react to the change in monetary policy.[12] A key feature of this solution is that c_0 depends on the debt-to-output ratio, ς_d.

In what follows, we make the following assumption.

Assumption 1. *$\tau > \rho \varsigma_d \sigma$.*

The left-hand side of Assumption 1 captures the first-order effect of an increase in consumption on tax revenues. The right-hand side captures the first-order effect of the increase in consumption in $t > 0$ on the interest payments on the debt. An increase in consumption pushes real interest rates up by σ, while the interest payments on the debt

[12] Appendix C shows that all the results extend to non-Ricardian regimes with the more general policy rules (5) and (6).

in the steady-state are given by $\rho\varsigma_d$. Hence, Assumption 1 implies that a boom in consumption increases government revenues by more than it increases the financing costs, so that it improves the government's finances overall.[13] Notably, $\tau > 0$ is a necessary condition for the assumption to hold. Under this condition, we get the following result.

Proposition 3. *Suppose Assumption 1 holds. Then,*

$$\frac{\partial c_0}{\partial u_0} < 0.$$

Proposition 3 establishes that the model generates standard comparative statics with respect to a monetary policy shock, that is, that a contractionary monetary shock reduces consumption in period 0. Assumption 1 is crucial in delivering this result as strong wealth effects could overturn it. To understand why this is the case, note that a monetary shock triggers two effects. First, we have the standard intertemporal substitution effect, which operates through changes in the relative price of current and future consumption, namely, the real interest rate. Through this channel, an increase in the nominal interest rate tilts the path of consumption upward.[14] Thus, fixing the households' wealth, the new path for the nominal interest rate will induce a lower level of consumption in period 0. This is the standard channel emphasized in the New Keynesian literature. Second, monetary policy generates *wealth effects*. Households' wealth depends on their labor income and their financial assets. Wages, employment, and profits from the ownership of firms respond to monetary policy only indirectly. In contrast, holdings of government bonds are directly affected by changes in monetary policy. The increase in the real rate *increases* the households' interest income which, because of the non-Ricardian fiscal policy, is not offset by a change in lump-sum transfers. Thus, this becomes a *positive* wealth effect for the households. Assumption 1 guarantees that this positive wealth effect does not overturn the substitution effect. It does so by guaranteeing that an increase in initial consumption is not affordable: the increase in its cost would be greater than the increase in the house-

[13] More formally, note that the government's budget constraint is $\int_0^\infty e^{-\rho t} \left[\tau c_t - \rho\varsigma_d\sigma\left(c_t - c_0\right)\right] dt = 0$. Taking the derivative of the right-hand side with respect to c_t, we get $e^{-\rho t}\left(\tau - \rho\varsigma_d\sigma\right)$. Thus, Assumption 1 implies that the revenue effect of a consumption boom outweighs the increase in interest payments from the change in the real rate.

[14] From the Euler equation we have $\dot{c}_t > 0 \iff i_t - \pi_t > \rho$.

holds' after-tax income (that is, $1 > (1-\tau) + \sigma\varsigma_d\rho$). Thus, consumption in period 0 has to decline.

The next section presents the paper's main theoretical result, namely, that the level of government debt affects the effectiveness of monetary policy. As we will see, the wealth effects emphasized above will be a crucial component for the results.

4 Monetary Policy and Government Debt

In this section, we explore how the level of government debt affects the effectiveness of monetary policy interventions, that is, the effect of government debt on the elasticity of output to interest rate changes. As a benchmark, we begin by presenting the irrelevance of the level of government debt in the standard Taylor equilibrium.

Irrelevance of debt in the Taylor equilibrium. Consider the equilibrium of an economy characterized by equations (10), (11) and (6), with $\phi_\pi > 1$ (and $\gamma_d > \rho$ so that equation (13) is also satisfied). The next proposition states that the level of debt is irrelevant to the economy's response to monetary shocks.

Proposition 4. *Consider the equilibrium of an economy described by* (10), (11) *and* (6), *with* $\phi_\pi > 1$. *Then,*

$$\frac{\partial^2 c_0}{\partial u_0 \partial \varsigma_d} = 0.$$

Proposition 4 formalizes a well-known result from the literature: fiscal variables do not affect the economy's response to monetary shocks in the standard equilibrium. Note, however, that this result does not imply that the wealth effects emphasized in the previous section are absent in this equilibrium. On the contrary, these wealth effects are present but neutralized by an automatic (or *passive*) adjustment of the lump-sum transfers. In particular, we have that

$$\frac{\partial^2 \int_0^\infty e^{-\rho t}\left[\varsigma_d(i_t - \pi_t - \rho) + T_t\right] dt}{\partial u_0 \partial \varsigma_d} = \int_0^\infty e^{-\rho t}\left[\left(\frac{\partial i_t}{\partial u_0} - \frac{\partial \pi_t}{\partial u_0}\right) + \frac{\partial^2 T_t}{\partial u_0 \partial \varsigma_d}\right] dt = 0,$$

that is, the change in (the present value of) lump-sum transfers after a monetary shock moves one-to-one with the change in total interest payments given a change in the level of government debt. This is not the case in the non-Ricardian regime.

Government debt in the non-Ricardian regime. Consider two economies with the same technology, preferences, distortionary taxes, and pricing frictions but that differ in their steady-state level of government debt. As we showed in Proposition 1, both economies have the same equilibrium allocation in steady-state. The next proposition shows that the consumption response to policy shocks is attenuated in the economy with a higher level of government debt.

Proposition 5. *Suppose Assumption 1 holds. Then, the effect of monetary policy is decreasing in the level of government debt, that is*

$$\frac{\partial^2 c_0}{\partial u_0 \partial \varsigma_d} > 0.$$

Proposition 3 established that an increase in the nominal interest rate reduces initial consumption. We explained that there were two effects: a substitution effect and a wealth effect. Note that the substitution effect is independent of the level of government debt; it only depends on the elasticity of intertemporal substitution, σ^{-1}. In contrast, the wealth effect depends on the level of government debt: the effect is stronger the larger the households' holdings. And since the wealth effect is *positive* after a *contractionary* monetary shock, the impact of monetary policy on initial consumption *decreases* with the level of government debt.[15]

It is important to note that while we have primarily focused on the effects of policy changes on period-0 consumption, the conclusions apply to the whole path. Recall that

$$c_t = e^{\underline{\omega} t} c_0 + c_t^m.$$

From Proposition 2 we know that c_t^m is independent of ς_d. Thus, by finding the effect of debt on initial consumption, we obtain the effect on the entire consumption path.

To summarize, we have shown that the efficacy of monetary policy decreases with the level of government debt. An important limitation of the results is that we have assumed

[15] Note that as long as Assumption 1 is satisfied, contractionary monetary policy always reduces initial consumption.

that government debt is short-term. In reality, most government debt is long-term (e.g., the average maturity of U.S. debt is around five years). Next, we explore how the presence of long-term government bonds affects the results.

Long-Term Bonds. Let's assume now that the government can also issue long-term nominal debt. The long-term bond is a perpetuity with exponentially decaying coupons, as in Woodford (2001). Formally, one unit of the bond at date t corresponds to a promise to pay $e^{-\rho_L(s-t)}$ in nominal terms at every date $s \geq t$. The price of the bond is given by

$$Q_{L,t} = \int_t^\infty e^{-\int_t^s i_z dz} e^{-\rho_L(s-t)} ds = \int_t^\infty e^{-\int_t^s (i_z+\rho_L)dz} ds,$$

and the bond duration in steady state is $\frac{1}{\rho+\rho_L}$. Hence, by varying ρ_L, we can study how the results change with the duration of government debt. For future reference, note that a higher value of ρ_L implies a *lower* duration of the debt.

The households' per-period budget constraint is now given by

$$\dot{B}_{S,t} + Q_{L,t}\dot{B}_{L,t} = i_t B_{S,t} + (1 - Q_{L,t}\rho_L)B_{L,t} + W_t N_t + \Pi_t + P_t T_t - P_t C_t,$$

where $(1 - Q_{L,t}\rho_L)B_{L,t}$ represents the coupon payment net of the "depreciation" of the bond. Then, the households' intertemporal budget constraint is given by

$$\int_0^\infty e^{-\int_0^t i_s ds} P_t C_t dt = D^g_{S,0} + Q_{L,0} D^g_{L,0} + \int_0^\infty e^{-\int_0^t i_s ds} \left(W_t N_t + \Pi_t + P_t T_t\right) dt,$$

where we have already imposed market-clearing in the bonds market, and $D^g_{S,0}$ and $D^g_{L,0}$ denote the stock of short-term and long-term government bonds, respectively. Notably, initial debt now depends on the price of the long-term bond. This is the only difference with respect to the previous model. The following result provides the benchmark for this economy with long-term bonds.

Proposition 6. *Given τ, the steady-state level of output, consumption, and labor are independent of the level and duration of government debt.*

This result is an extension of Proposition 1 and Corollary 1.1. It says that not only the steady-state level of debt, D_S^g and D_L^g, is irrelevant for the steady-state allocation, but the duration of long-term debt, ρ_L, as well.

Let's now consider the response of the economy to monetary policy shocks. The Euler equation and the Phillips curve are still given by equations (10) and (11), respectively. The only difference is in the intertemporal budget constraint, which in its log-linear form is now given by

$$\int_0^\infty e^{-\rho t} c_t dt = \int_0^\infty e^{-\rho t} \left[(1-\tau)y_t + \sigma\rho\varsigma_d(c_t - c_0)\right] dt + d_0^g \varsigma_d,$$

where, up to first order,

$$d_0^g = \varsigma_L q_{L,0},$$

and

$$q_{L,t} = -\int_t^\infty e^{-(\rho+\rho_L)(s-t)}(i_s - \rho) ds$$

is the first-order approximation of the bond price, $Q_{L,t}$, and where ς_L denotes the steady-state fraction of debt that is long-term, that is $\varsigma_L \equiv \frac{Q_L D_L^g}{D_S^g + Q_L D_L^g}$. Plugging these expressions into the budget constraint, we get

$$\int_0^\infty e^{-\rho t} c_t dt = \int_0^\infty e^{-\rho t} \left[(1-\tau)y_t + \sigma\rho\varsigma_d(c_t - c_0)\right] dt - \varsigma_d\varsigma_L \int_0^\infty e^{-(\rho+\rho_L)t}(i_t - \rho) dt.$$

Hence, the budget constraint has an additional term that depends on the nominal interest rate, i_t, the fraction of long-term bonds, ς_L, and the bond's duration, ρ_L.

Solving the new system of equations, we get

$$c_0 = -\sigma^{-1} \frac{\tau - \rho\varsigma_d\sigma}{\tau - \underline{\omega}\varsigma_d\sigma} \frac{u_0}{\overline{\omega} + \psi_m} - \frac{\overline{\omega}\varsigma_d\varsigma_L}{\tau - \underline{\omega}\varsigma_d\sigma} \frac{u_0}{\rho + \rho_L + \psi_m}.$$

Note that the first term of this expression coincides with initial consumption in the model with only short-term debt (see Proposition 2). The next proposition extends Proposition 5 to the model with long-term government debt.

Proposition 7. *Suppose Assumption 1 holds. Then, an increase in the nominal interest rate reduces initial consumption, and the effect is stronger the higher the fraction of long-term debt and* *the higher the bond duration, that is,*

$$\frac{\partial c_0}{\partial u_0} < 0, \quad \frac{\partial^2 c_0}{\partial u_0 \partial \varsigma_L} < 0, \quad \frac{\partial^2 c_0}{\partial u_0 \partial \rho_L} > 0.$$

Moreover, if $\rho_L > |\underline{\omega}|$,

$$\frac{\partial^2 c_0}{\partial u_0 \partial \varsigma_d} > 0.$$

Long-term bonds introduce a new channel to the model in Section 2: the response of the bond price to interest rate changes. Note that increases in the nominal interest rate always reduce the bond price. Thus, long-term bonds reinforce the contractionary effects of higher nominal rates. Moreover, this effect is stronger the higher the fraction of long-term debt and the longer its duration. Crucially, there is a threshold duration of government debt such that if the duration of government debt is *lower* than the threshold, the positive effect of the change in the rate of return of bonds dominates the negative effect of repricing, and higher government debt leads to weaker monetary policy. This threshold is given by the absolute value of the negative eigenvalue of the New Keynesian system of differential equations. To get some intuition behind this condition, consider the effects of the degree of price flexibility, κ. It is straightforward to show that the threshold $|\underline{\omega}|$ is increasing in κ. Then, a lower degree of price flexibility reduces the lower bound on the duration of government bonds that guarantees the result that monetary policy weakens as the level of government debt increases. In the extreme case of rigid prices, that is, if $\kappa = 0$, we have that $\underline{\omega} = 0$, so any duration shorter than a consol generates a weaker transmission. In contrast, only the shortest duration leads to this result as prices become fully flexible. Intuitively, this comparative statics reflects the fact that monetary policy has weaker real effects when prices are more flexible.[16] As prices become more flexible, the real rate follows more closely the nominal rate, a manifestation of the Neo-Fisherian forces present in this model.[17] Thus, as κ increases, the change in the real rate following a monetary shock becomes smaller, generating a smaller increase in (real) interest income. In contrast, the initial repricing of long-term bonds depends only on the path of the nom-

[16] It is straightforward to see that $\left|\frac{\partial c_0}{\partial u_0}\right|$ is decreasing in κ.

[17] For a detailed analysis of Neo-Fisherianism in the New Keynesian model see Garín et al. (2018).

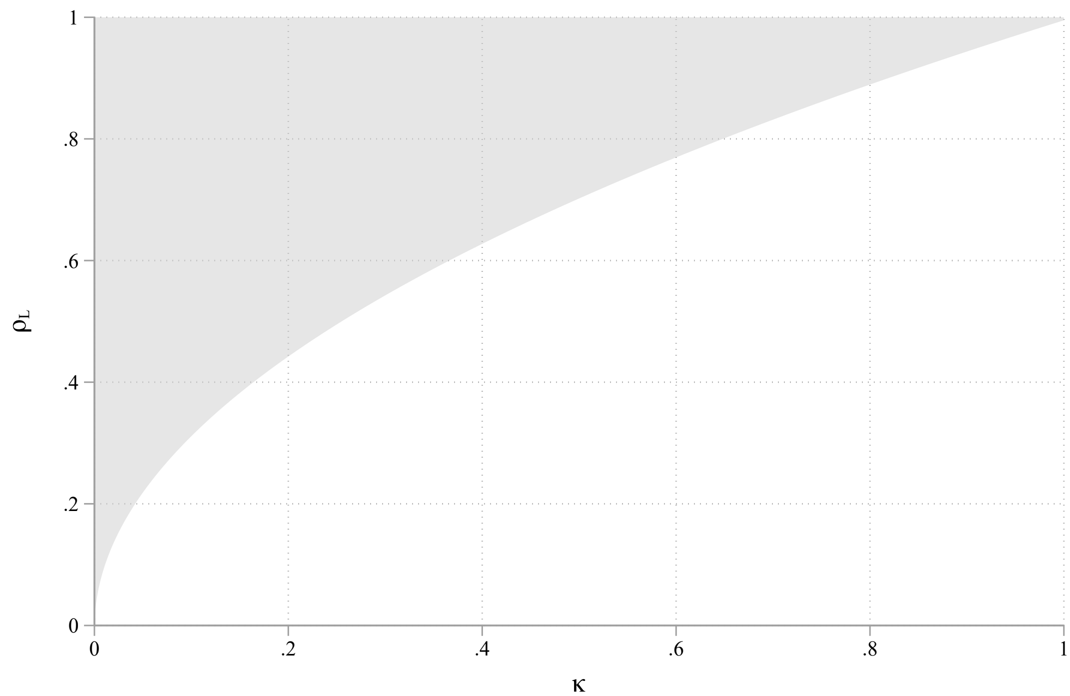

FIGURE 1: Duration threshold for $\frac{\partial^2 c_0}{\partial u_0 \partial \varsigma_d} > 0$ as a function of κ.

Notes: The gray area corresponds to $\rho_L > |\underline{\omega}| = -\frac{\rho - \sqrt{\rho^2 + 4\sigma^{-1}\kappa}}{2}$, i.e., the region where $\frac{\partial^2 c_0}{\partial u_0 \partial \varsigma_d} > 0$.

inal rate, as the price level in period 0 is predetermined in the continuous time setting. As a result, the repricing effect will tend to dominate under higher price flexibility. Figure 1 shows the combination of ρ_L and κ that lead to the interest income effect or the repricing effect to dominate.

From an economic perspective, the result depends on whether an increase in the nominal interest increases or reduces the government's debt burden. Note that the positive wealth effect of government bonds we have emphasized until now is the counterpart of a *negative* effect on the government's budget, that is, an increase in the debt burden. Similarly, if an increase in the nominal interest reduced the government debt burden, this would imply a *negative* wealth effect for the households. As noted above, when prices are more sticky (or the Phillips curve is relatively flat, as argued to be the case for the U.S., see Hazell et al., 2022), almost any finite duration of government debt implies that higher nominal interest rates increase the government's debt burden and, therefore, the level of government debt weakens the effect of monetary policy.

To summarize, we have found that if the duration of government debt is not too long, the efficacy of monetary policy decreases in the stock of government debt. In the next section, we empirically test the model's predictions by exploring the connection between the level of government debt and the effectiveness of monetary policy.

5 Empirical Evidence

In this section, we evaluate the validity of the model's predictions on U.S. data. Section 5.1 describes the data. Section 5.2 presents the econometric specification and reports the empirical results. Section 5.3 conducts some robustness checks.

5.1 Data

Our baseline sample runs from March 1969 to December 2007. Most of the macroeconomic series we use are taken from standard sources: the industrial production index (Federal Reserve Board of Governors release G.17 Industrial Production and Capacity Utilization); the U-3 measure of the unemployment rate (BLS Current Population Survey); the consumer price index for all urban consumers (BLS Consumer Price Index); the producer price index for all commodities (BLS Producer Price Index); and the federal funds effective rate (Federal Reserve Board of Governors H.15 Selected Interest Rates).

As the measure of the stock of government debt, we use data on privately held U.S. government debt provided by Hall et al. (2018).[18] Figure 2 plots the path of the market value of privately held U.S. government debt spanning our sample period. We divide this measure by monthly estimates of nominal GDP (Stock and Watson, 2010).[19] Figure 3 plots the resulting debt measure. The debt ratio reaches a trough from 1970 until the early 1980s, before rising until the mid-90s during the Clinton administration, when it decreases steadily until it stabilizes in the early 2000s. We then standardize the debt-to-GDP ratio to have a mean of zero and a standard deviation of one.

[18] The most recent vintage of this data set may be found on George J. Hall's website: https://people.brandeis.edu/ ghall/

[19] The results are unchanged if we divide government debt by industrial production instead.

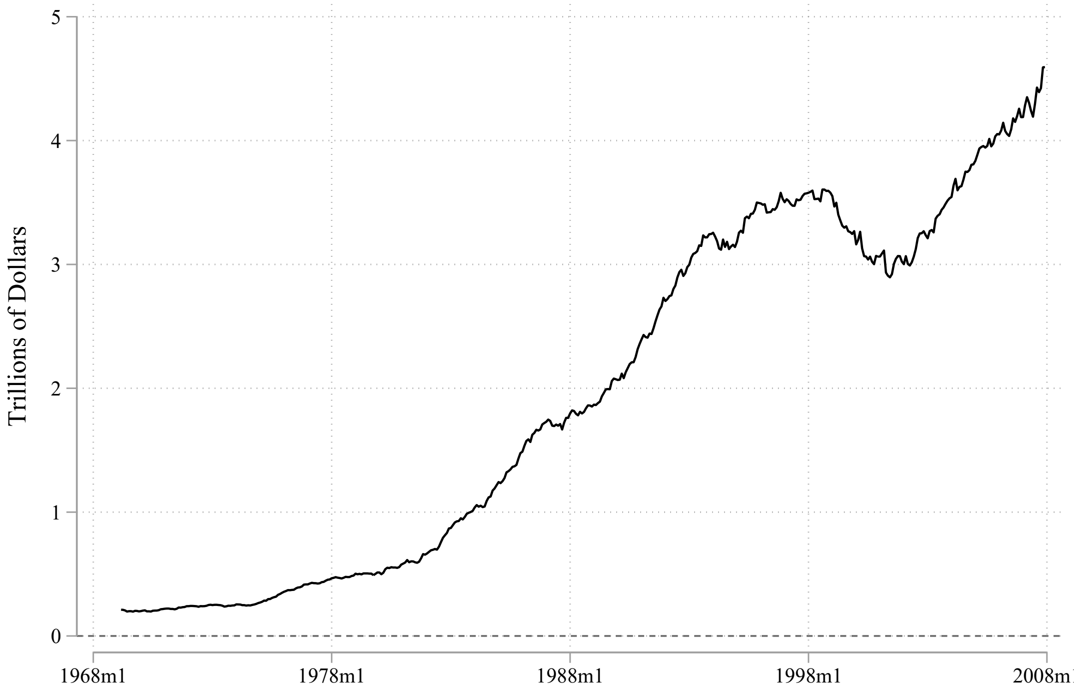

FIGURE 2: Market Value of Privately-Held U.S. Government Debt

Notes: From Hall et al. (2018).

Monetary policy changes are typically endogenous to changes in the macroeconomic outlook. Following the literature, we rely on the Romer and Romer (2004) narrative measure of monetary policy shocks. We use an extended sample of shocks available from March 1969 through December 2007 as estimated in Wieland and Yang (2020).[20] The Romer and Romer measure is estimated by regressing changes in the federal funds rate on internal Federal Reserve Greenbook forecasts of the unemployment rate, industrial production, and CPI inflation. The residual of this regression is taken to represent changes in the stance of monetary policy purged of systematic responses to current and expected future economic news. Figure 4 plots the shock measure. We limit our sample to before the Global Financial Crisis of 2008 to avoid issues related to the zero lower bound on the federal funds rate.

[20]This series and the code for its estimation are maintained at https://www.openicpsr.org/openicpsr/project/135741/version/V1/view.

FIGURE 3: Debt-to-GDP

Notes: From Hall et al. (2018), Stock and Watson (2010), and authors' calculations. The value is the ratio of the market value of privately-held U.S. government debt divided by monthly nominal GDP.

5.2 Empirical Methodology

For our empirical exercise, we employ a nonlinear variant of the Jordà (2005) local projections estimator studied by Gonçalves et al. (2021), in which we incorporate a role for privately-held government debt in the transmission of monetary policy shocks. This method has been used by Tenreyro and Thwaites (2016) and Angrist et al. (2018) to study the asymmetric effects of monetary policy over the business cycle, and by Barnichon et al. (2022) to study asymmetries and state-dependence in the propagation of credit shocks. The methods we use are similar to those of Broner et al. (2022), who study whether variation in the share of public debt held by foreigners can explain the magnitude of government spending multipliers.

Let Z_t be our standardized measure of privately-held U.S. government debt, ϵ_t^{MP} be our identified monetary policy shock series, and $\mathbf{X}_t$ be a vector of controls. Our baseline nonlinear local projections specification consists of the sequence of linear regressions

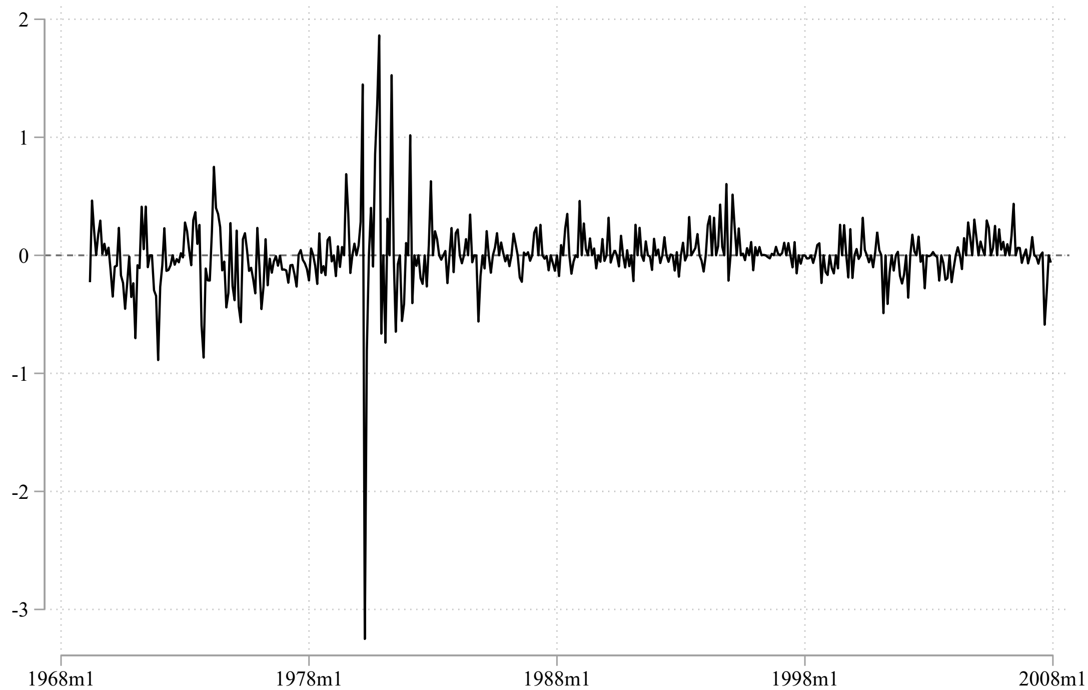

FIGURE 4: Identified Monetary Policy Shocks

Notes: Estimated by Wieland and Yang (2020) based on the methodology of Romer and Romer (2004).

given by

$$\Delta^h y_{t+h} = \alpha^h + \beta^h \epsilon_t^{MP} + \delta^h Z_{t-1} + \gamma^h Z_{t-1} \epsilon_t^{MP} + \sum_{i=1}^{I} \mathbf{X}_{t-i} \boldsymbol{\theta}^h + \omega_{t+h}, \tag{15}$$

where $h = 0 \ldots H$. As the debt-to-GDP ratio we are interested in is a predetermined (state) variable at time t, we introduce the debt variable with a lag. Our control variables include lags of the shock series, the log of industrial production, the log of the consumer price index, the log of the producer price index, and the federal funds rate. These variables enter with a lag so as not to impose any restrictions on the contemporaneous response to monetary policy shocks. In our baseline specification, we set $H = 36$ and $I = 12$.[21] Throughout our analysis, we estimate standard errors using the approach of Newey and

[21] Our results are robust to altering the number of lags, which we demonstrate in Section 5.3.1. Moreover, in Section 5.3.3 we consider a specification with a full set of interaction terms between the controls and the debt variable, as proposed by Cloyne et al. (2020) in their Oaxaca-Blinder decomposition. All our results are robust to this alternative.

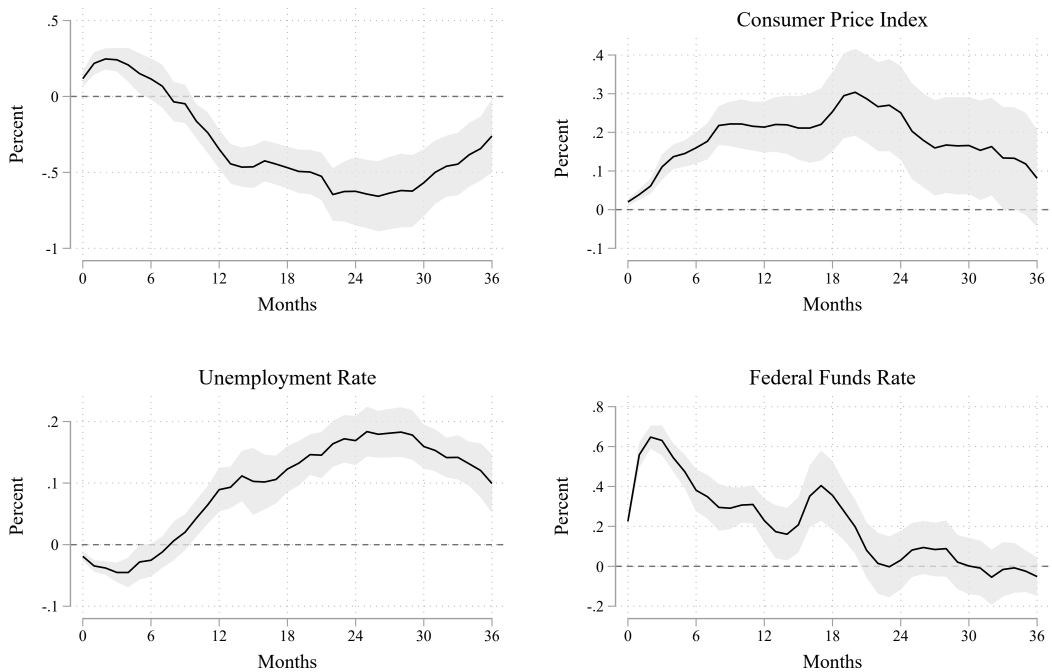

FIGURE 5: Monetary Policy Shocks in the Linear Model

Notes: Cumulative impulse responses to a one standard deviation increase in the Romer and Romer (2004) monetary policy shock measure. 90% confidence intervals are provided.

West (1987) to correct for serial correlation.

The cumulative impulse response of the monetary policy shock at time t on our outcome variables out to horizon h is a function of the debt measure and equal to

$$IRF(Z_{t-1}) = \beta^h + \gamma^h Z_{t-1}.$$

As the debt measure is standardized, we obtain the impulse response at the *average debt level* by setting $Z_{t-1} = 0$, in which case the cumulative impulse response function is simply the sequence $\{\beta^h\}_{h=0}^{H}$. Additionally, we consider the case in which the standardized debt measure is one standard deviation above its sample mean by setting $Z_{t-1} = 1$, in which case the cumulative impulse response function is the sequence $\{\beta^h + \gamma^h\}_{h=0}^{H}$. The sequence $\{\gamma^h\}_{h=0}^{H}$ then represents the cumulative interaction between publicly-held government debt and monetary policy.

As a benchmark, the results in Figure 5 show the cumulative impulse responses estimated via local projections excluding the debt interaction term in equation (15), and are largely consistent with the results for the Romer and Romer (2004) shock series as presented by Ramey (2016) in Figure 2, panel B. As one standard deviation increase in the Romer and Romer (2004) measure induces an increase in the Federal funds rate by over 0.6pp within six months, reducing industrial production by over half of a percentage point within two years, while unemployment rises by nearly 0.2pp. As documented by Ramey (2016) among others, the Romer and Romer series produces several puzzles, including an apparently expansionary effect on industrial production and unemployment on impact, as well as a significant and persistent "price puzzle".[22] Finally, we also estimate the impact response of the real value of government debt to the monetary shock, which represents a quantification of the repricing effect of long-term bonds highlighted in the previous section. We find that a monetary contraction of one standard deviation induces a reduction in the real value of debt of 0.16pp.

Results for equation (15) are presented in Figure 6. The impulse response functions in blue show the effects of a one standard deviation Romer and Romer shock when our debt measure is at its sample mean. Consistent with the literature, these responses show a drop in industrial production of nearly 0.5pp within one year, together with an increase in the unemployment rate of 0.1pp at the two-year mark. As noted, the impulse response of the CPI appears to exhibit the price puzzle, rising by 0.2pp out to two years. Shown in red are the same impulse responses when privately-held government debt is one standard deviation above the sample mean entering period t. By contrast, these impulse responses show a diminished response of industrial production and unemployment. Industrial production falls in line with the mean case but recovers more quickly after the one-year mark. Likewise, unemployment recovers more quickly in the case with high debt, returning to the mean within two years.

Figure 7 plots the cumulative interaction between privately-held government debt and the Romer and Romer shock, which is equal to the difference between the impulse response functions presented in Figure 6. As noted, the level of debt causes a statistically

[22] As we demonstrate in Section 5.3.2, our results survive the use of shocks identified via high-frequency movements in financial markets as proposed by Gurkaynak et al. (2004) and Bernanke and Kuttner (2005), which do not exhibit the price puzzle.

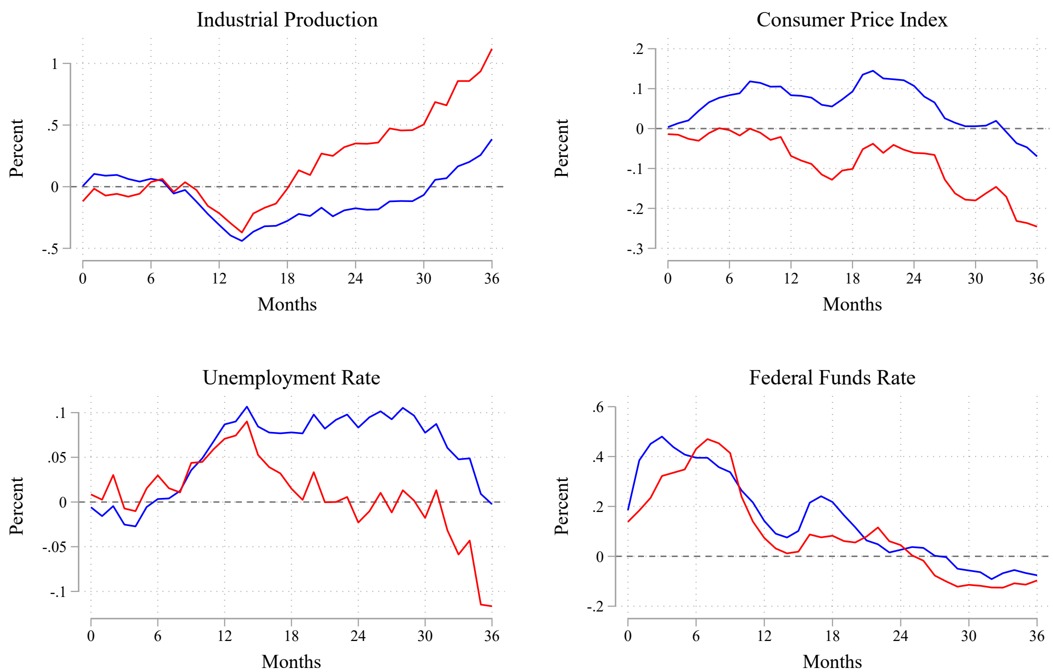

FIGURE 6: Monetary Policy Shocks in the Nonlinear Model

Notes: Cumulative impulse responses to a one standard deviation increase in the Romer and Romer (2004) monetary policy shock measure. The impulse response function in blue represents the case where privately-held government debt is at the sample mean. The red impulse response function represents the case where the debt measure is one standard deviation above the sample mean.

significant difference in the impulse response functions of industrial production and the unemployment rate to the Romer and Romer shock. When our debt measure is one standard deviation above its sample mean, the response of industrial production reflects a nearly 0.75pp increase relative to the mean case within three years. Similarly, the increase in the unemployment rate is over 0.1pp lower in the high-debt case out to three years.

These findings are consistent with the predictions of the model laid out in Sections 2 to 4. Namely, when government debt is higher, the effectiveness of monetary policy, measured as the elasticity of output to changes in the path of the nominal interest rate, decreases.

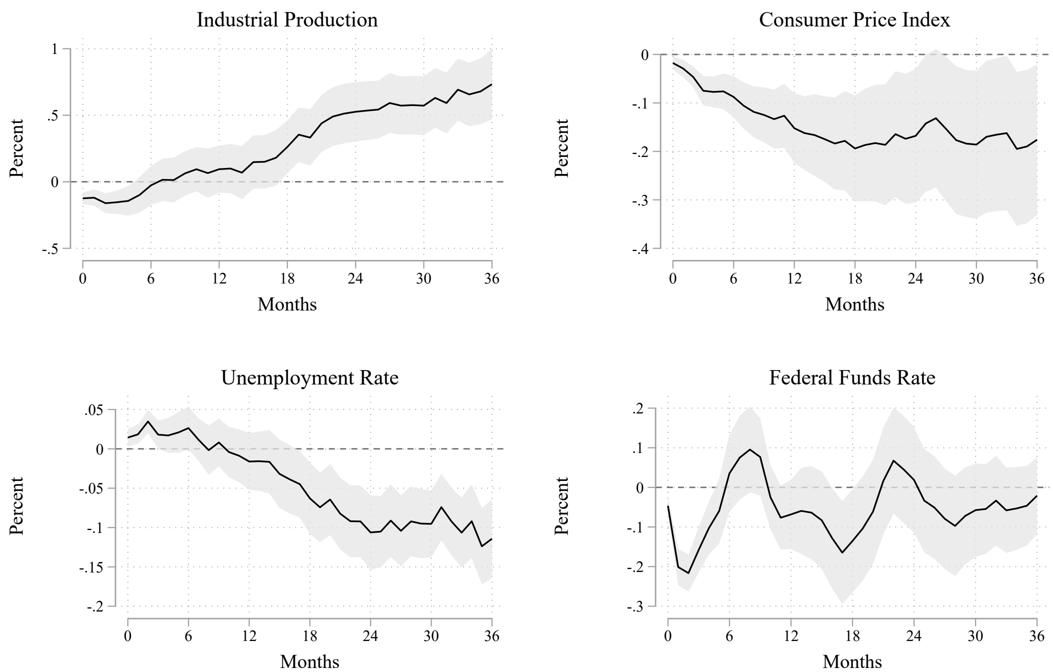

FIGURE 7: Cumulative Interactions in the Nonlinear Model

Notes: The cumulative interaction between monetary policy shocks and the debt measure after a one standard deviation increase in the Romer and Romer (2004) monetary policy shock measure. 90% confidence intervals are provided.

5.3 Robustness

5.3.1 Robustness to Alternative Lag Lengths

We consider whether our empirical results are sensitive to the lag length, I, in equation (15). Figures 8 and 9 replicate Figures 5 and 7, varying the number of lags used of both the control variables and the monetary policy shock. As seen in the figures, the estimated cumulative impulse response functions and the cumulative interaction are remarkably insensitive to the choice of lag length.

5.3.2 Robustness to Alternative Monetary Policy Shocks

We consider whether our empirical results are sensitive to an alternative method of identifying monetary policy shocks. In the spirit of Gurkaynak et al. (2004) and Bernanke and

FIGURE 8: Robustness to Alternative Lag Lengths in the Linear Model

Notes: Cumulative impulse responses to a one standard deviation increase in the Romer and Romer (2004) monetary policy shock measure.

Kuttner (2005), we consider monetary policy shocks identified by high-frequency variation in federal funds futures markets. The key identifying assumption underlying these methods is that any variation in the three-month ahead fed funds futures rate within a narrow window of time bracketing an announcement by the Federal Open Market Committee should reflect the announcement alone rather than news about macroeconomic events. We use the shock series estimated by Miranda-Agrippino and Ricco (2021) which purges the raw financial market shocks of the "information effect" of central bank announcements by regressing the measure on Greenbook forecasts of macroeconomic data available to the Federal Reserve officials at the time of an announcement. Figure 10 plots this shock measure.

Figures 11 and 12 replicate Figures 5 and 7, replacing the Romer and Romer (2004) shock measure with the measure identified by Miranda-Agrippino and Ricco, which spans January 1991 to December 2009. We estimate cumulative impulse responses using

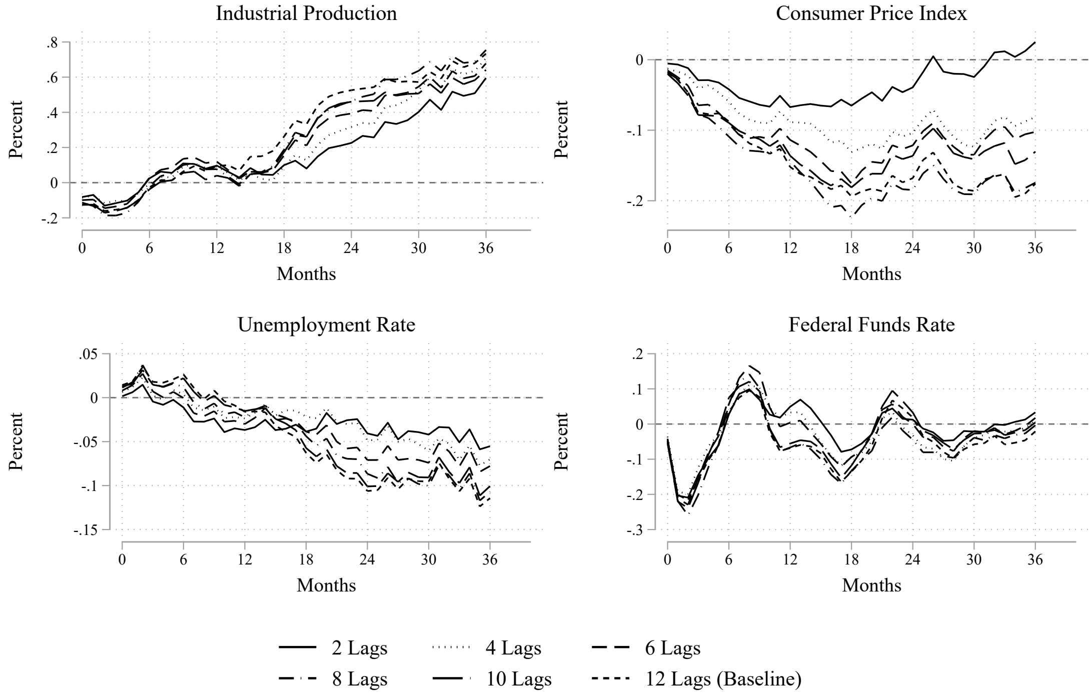

FIGURE 9: Robustness to Alternative Lag Lengths in the Nonlinear Model

Notes: The cumulative interaction between monetary policy shocks and the debt measure after a one standard deviation increase in the Romer and Romer (2004) monetary policy shock measure.

equation (15) with twelve lags of the following control variables: the log of industrial production, the log of the consumer price index, the Gilchrist and ZakrajÅąek (2012) excess bond premium, and the one-year Treasury rate. Of additional note, we follow Miranda-Agrippino and Ricco in using the one-year Treasury rate as our indicator of the stance of monetary policy rather than the federal funds rate.

In Figure 11, we note two observations. First, the Miranda-Agrippino and Ricco shocks induce contractionary responses of industrial production and the unemployment rate that are similar in magnitude to those induced by the Romer and Romer measure despite the minimal overlap in the two samples. Second, unlike the responses using the Romer and Romer series, the response of the consumer price index exhibits no significant price puzzle.

As noted, Figure 12 plots the estimated cumulative interaction between privately-held government debt and monetary policy using the high-frequency identified shocks. An

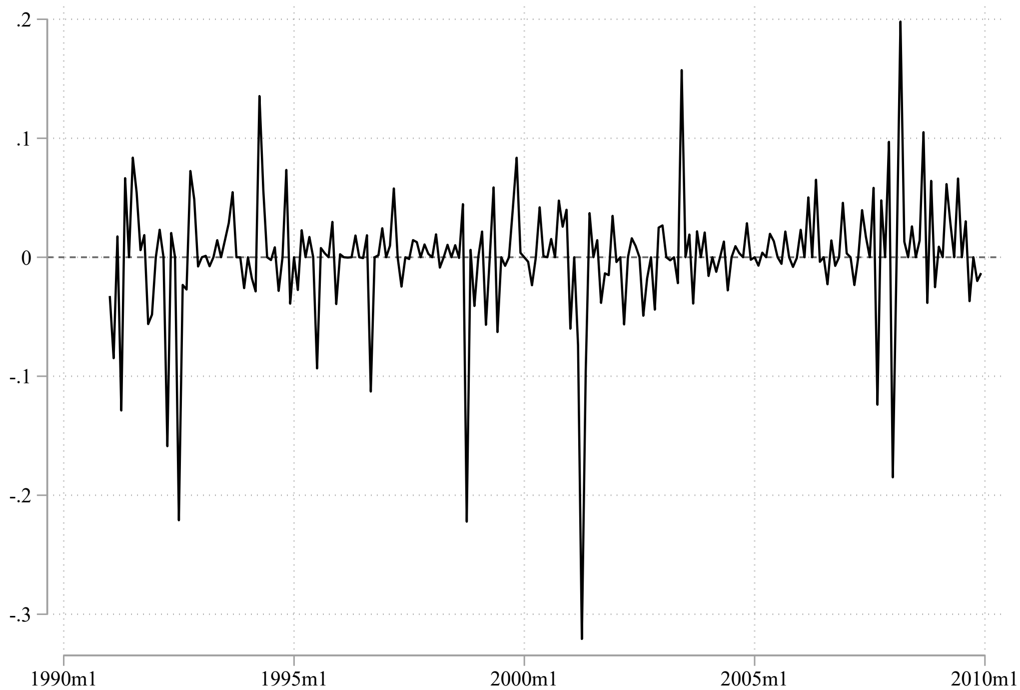

FIGURE 10: Identified Monetary Policy Shocks

Notes: Estimated by Miranda-Agrippino and Ricco (2021).

economy with privately-held government debt one standard deviation above the mean exhibits less severe responses of industrial production, with declines dampened by between 0.2 and 0.3pp out to two years. Furthermore, the unemployment rate rises by nearly 0.75pp less in the high-debt case than in the mean debt case within two years. These results have the same direction as those under the Romer and Romer shocks and provide evidence for the dampening mechanism explored in the main text.

5.3.3 Kitagawa-Oaxaca-Blinder Local Projections

As an additional robustness test, we alter equation (15) following Cloyne et al. (2020) to admit a Kitagawa-Oaxaca-Blinder decomposition of estimated impulse responses (Kitagawa, 1955; Oaxaca, 1973; Blinder, 1973). As noted by Cloyne et al., the Kitagawa-Oaxaca-Blinder framework is used in applied microeconomics to decompose the effects of a policy innovation into three separate determinants: 1) a direct effect, or the average treatment effect of a policy innovation on the outcome variable, 2) a composition effect, or a bias

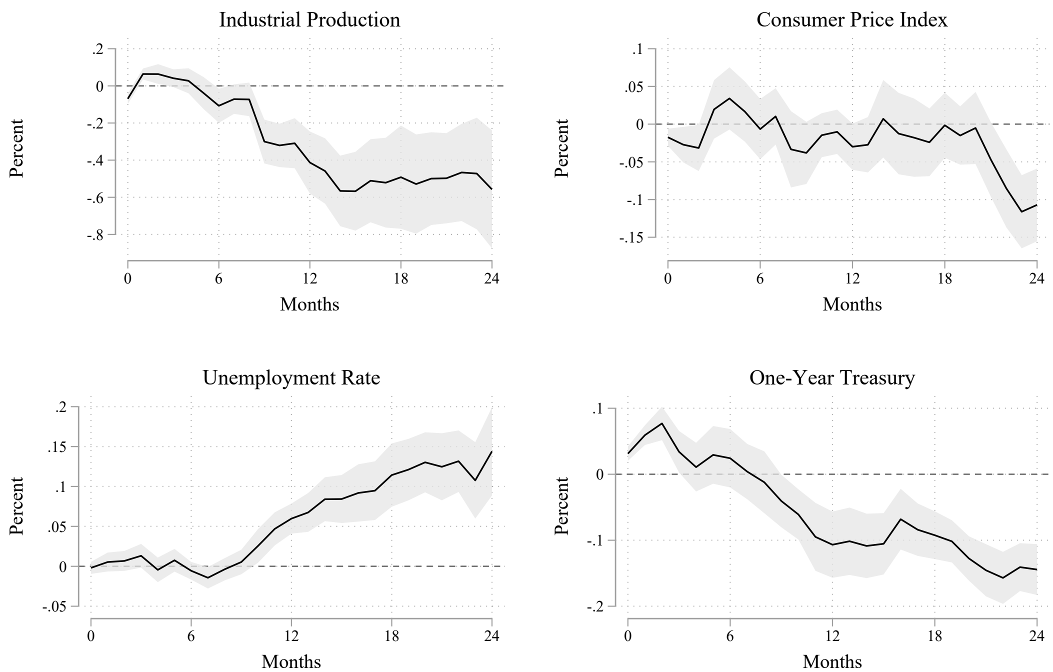

FIGURE 11: High-Frequency Identification in the Linear Model

Notes: Cumulative impulse responses to a one standard deviation increase in the Miranda-Agrippino and Ricco (2021) monetary policy shock measure.

introduced by non-random assignment of the treatment, and 3) an indirect effect of the policy innovation altering the relationship between the outcome and control variables. Let $\boldsymbol{X}_t$ be a vector of control variables, which now includes the debt measure, and let ϵ_t^{MP} be our identified monetary policy shock series. The Kitagawa-Oaxaca-Blinder specification is given by

$$\Delta^h y_{t+h} = \alpha^h + \underbrace{\beta^h \epsilon_t^{MP}}_{\text{direct effect}} + \underbrace{(\mathbf{X}_t - \bar{\mathbf{X}})\epsilon_t^{MP}\boldsymbol{\Gamma}^h}_{\text{indirect effect}} + \underbrace{(\mathbf{X}_t - \bar{\mathbf{X}})\boldsymbol{\Theta}^h}_{\text{composition effect}} + \omega_{t+h}. \tag{16}$$

Adapting this decomposition to the present setting, the Kitagawa-Oaxaca-Blinder local projections setup can be used to decompose the impulse response of macroeconomic time series into analogous channels. The indirect effect we estimate will include the cumulative interaction between private ownership of government debt and the transmission of monetary policy shocks. In this setting, we return to using the Romer and Romer (2004)

FIGURE 12: High-Frequency Identification in the Nonlinear Model

Notes: The cumulative interaction between monetary policy shocks and the debt measure after a one standard deviation increase in the Miranda-Agrippino and Ricco (2021) monetary policy shock measure.

measure of identified monetary policy shocks and include as controls twelve lags of each of the following: the log of industrial production, the log of the consumer price index, the log of the producer price index, and the Federal funds rate.

Under the Kitagawa-Oaxaca-Blinder decomposition, the cumulative impulse response of a monetary policy shock a time t on the outcome variable out to horizon h is a function of the state at time t, which includes the levels of each control variable:

$$IRF(\mathbf{X}_t) = \beta^h + (\mathbf{X}_t - \bar{\mathbf{X}})\mathbf{\Gamma}^h.$$

As we are interested in the average treatment effect of stabilization policy conditional on the level of debt, we estimate impulse responses where $\mathbf{x}_t = \bar{\mathbf{x}}$ for each control variable except for our debt measure, which we set equal to zero, representing the sample-mean-debt case, or one, representing the case where the debt measure is elevated by one standard deviation relative to the sample mean.

FIGURE 13: Cumulative Interactions in the Kitagawa-Oaxaca-Blinder Model

Notes: The cumulative interaction between monetary policy shocks and the debt measure after a one standard deviation increase in the Romer and Romer (2004) monetary policy shock measure.

Figure 13 replicates Figure 7 using the Kitagawa-Oaxaca-Blinder specification.[23] The figure demonstrates that the main results of the paper are supported. When the cumulative interaction is significant, we see that the response of industrial production shows a substantial dampening of approximately 3pp out to two years relative to the mean case when the debt measure is elevated by one standard deviation. Additionally, the unemployment rate rises by approximately 0.3pp less in the high-debt case than in the mean-debt case out to three years, although there is a period within one year for which the interaction is significantly more contractionary.

[23] Note that we do not reproduce the linear case, as a linear Kitagawa-Oaxaca-Blinder specification coincides with the original linear local projections specification.

6 Conclusion

This paper explores the role of government debt in the monetary transmission mechanism. We build a New Keynesian model where fiscal variables affect the determination of equilibrium. We find that the effectiveness of monetary policy becomes weaker in high-debt economies. Behind this result, there is a wealth effect from the revaluation of public debt after a change in the nominal interest rate. We test the model's implications empirically and find that high government debt levels attenuate the effects of monetary policy on industrial production and the unemployment rate, consistent with the model.

This analysis has important implications for the conduct of monetary policy. Most advanced economies are currently experiencing high levels of debt. Our findings imply that the efficacy of monetary policy decreases in these environments, calling for stronger interventions to stabilize the economy. However, this recommendation conflicts with the secular decline of policy rates, which limits the room for monetary policy accommodation. In light of this, future research should focus on understanding how other policy tools (e.g., unconventional monetary policy and fiscal policy) are affected by government debt.

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A Proofs

Proof of Proposition 1.

Equations (7)-(8) determine $\{Y, C, N\}$, which are independet of D^g conditional on τ.

■

Proof of Corrolay 1.1.

Immediate from Proposition 1.

■

Proof of Proposition 2.

The system given by (10) and (11) can be written in matrix form:

$$\begin{bmatrix}\dot{\pi}_t \\ \dot{c}_t\end{bmatrix} = \begin{bmatrix}\rho & -\kappa \\ -\sigma^1 & 0\end{bmatrix}\begin{bmatrix}\pi_t \\ c_t\end{bmatrix} + \begin{bmatrix}0 \\ m_t\end{bmatrix}$$

where $m_t \equiv \sigma^{-1}(i_t - \rho) = \sigma^{-1}u_t$.

Let the eigenvalues of the coefficient matrix be denoted by

$$\overline{\omega} = \frac{\rho + \sqrt{\rho^2 + 4\sigma^1\kappa}}{2} \quad \text{and} \quad \underline{\omega} = \frac{\rho - \sqrt{\rho^2 + 4\sigma^1\kappa}}{2}.$$

The matrix of coefficients can be decomposed as

$$\begin{bmatrix}\rho & -\kappa \\ -\sigma^1 & 0\end{bmatrix} = \begin{bmatrix}1 & 1 \\ -(\sigma\overline{\omega})^{-1} & -(\sigma\underline{\omega})^{-1}\end{bmatrix}\begin{bmatrix}\overline{\omega} & 0 \\ 0 & \underline{\omega}\end{bmatrix}\begin{bmatrix}1 & 1 \\ -(\sigma\overline{\omega})^{-1} & -(\sigma\underline{\omega})^{-1}\end{bmatrix}^{-1}.$$

Note that $\overline{\omega} + \underline{\omega} = \rho$, $\overline{\omega}\underline{\omega} = -\sigma^1\kappa$, $\overline{\omega} - \underline{\omega} = \sqrt{\rho^2 + 4\sigma^1\kappa}$, and that if prices are rigid, i.e. $\kappa = 0$, then $\underline{\omega} = 0$.

Define the following transformation of our original variables

$$Z_t = \begin{bmatrix}Z_{1,t} \\ Z_{2,t}\end{bmatrix} \equiv \frac{\kappa}{\overline{\omega} - \underline{\omega}}\begin{bmatrix}-(\sigma\underline{\omega})^{-1} & -1 \\ (\sigma\overline{\omega})^{-1} & 1\end{bmatrix}\begin{bmatrix}\pi_t \\ c_t\end{bmatrix}$$

The system in the new coordinates can be written as

$$\begin{bmatrix}\dot{Z}_{1,t} \\ \dot{Z}_{2,t}\end{bmatrix} = \begin{bmatrix}\overline{\omega} & 0 \\ 0 & \underline{\omega}\end{bmatrix}\begin{bmatrix}Z_{1,t} \\ Z_{2,t}\end{bmatrix} + \begin{bmatrix}\eta_{1,t} \\ \eta_{2,t}\end{bmatrix}$$

where

$$\begin{bmatrix}\eta_{1,t} \\ \eta_{2,t}\end{bmatrix} \equiv \frac{\kappa}{\overline{\omega} - \underline{\omega}}\begin{bmatrix}-(\sigma\underline{\omega})^{-1} & -1 \\ (\sigma\overline{\omega})^{-1} & 1\end{bmatrix}\begin{bmatrix}0 \\ m_t\end{bmatrix} = \begin{bmatrix}-\frac{\kappa}{\overline{\omega}-\underline{\omega}}m_t \\ \frac{\kappa}{\overline{\omega}-\underline{\omega}}m_t\end{bmatrix}.$$

Since we are focusing on bounded solutions, we can solve the first equation forward and the second backward to get

$$Z_{1,t} = -\int_t^\infty e^{-\overline{\omega}(s-t)}\eta_{1,s}ds,$$
$$Z_{2,t} = e^{\underline{\omega}t}Z_{2,0} + \int_0^t e^{\underline{\omega}(t-s)}\eta_{2,s}ds.$$

In terms of the original variables, we have

$$\begin{bmatrix}\pi_t \\ c_t\end{bmatrix} = \begin{bmatrix}1 & 1 \\ -(\sigma\overline{\omega})^{-1} & -(\sigma\underline{\omega})^{-1}\end{bmatrix}\begin{bmatrix}-\int_t^\infty e^{-\overline{\omega}(s-t)}\eta_{1,s}ds \\ e^{\underline{\omega}t}Z_{2,0} + \int_0^t e^{\underline{\omega}(t-s)}\eta_{2,s}ds\end{bmatrix},$$

or

$$\pi_t = e^{\underline{\omega}t}Z_{2,0} + \int_0^t e^{\underline{\omega}(t-s)}\eta_{2,s}ds - \int_t^\infty e^{-\overline{\omega}(s-t)}\eta_{1,s}ds,$$
$$c_t = -e^{\underline{\omega}t}\frac{Z_{2,0}}{\sigma\underline{\omega}} - \int_0^t e^{\underline{\omega}(t-s)}\frac{\eta_{2,s}}{\sigma\underline{\omega}}ds + \int_t^\infty e^{-\overline{\omega}(s-t)}\frac{\eta_{1,s}}{\sigma\overline{\omega}}ds.$$

Evaluating in $t = 0$ we get

$$\pi_0 = Z_{2,0} - \int_0^\infty e^{-\overline{\omega}t}\eta_{1,t}dt,$$
$$c_0 = -\frac{Z_{2,0}}{\sigma\underline{\omega}} + \int_0^\infty e^{-\overline{\omega}t}\frac{\eta_{1,t}}{\sigma\overline{\omega}}dt,$$

and therefore, we can rewrite the system as

$$\pi_t = e^{\underline{\omega}t}\pi_0 + e^{\underline{\omega}t}\int_0^t\left(e^{-\overline{\omega}s}\eta_{1,s} + e^{-\underline{\omega}s}\eta_{2,s}\right)ds - \left(e^{\overline{\omega}t} - e^{\underline{\omega}t}\right)\int_t^\infty e^{-\overline{\omega}s}\eta_{1,s}ds,$$
$$c_t = e^{\underline{\omega}t}c_0 - \sigma^{-1}e^{\underline{\omega}t}\int_0^t\left(e^{-\overline{\omega}s}\frac{\eta_{1,s}}{\overline{\omega}} + e^{-\underline{\omega}s}\frac{\eta_{2,s}}{\underline{\omega}}\right)ds + \sigma^{-1}\frac{e^{\overline{\omega}t} - e^{\underline{\omega}t}}{\overline{\omega}}\int_t^\infty e^{-\overline{\omega}s}\eta_{1,s}ds.$$

Writing the system in terms of the original shocks, we obtain

$$c_t = c_t^m + e^{\underline{\omega}t}c_0, \tag{17}$$

where $c_t^m \equiv \frac{\sigma^{-1}}{\overline{\omega}-\underline{\omega}}e^{\underline{\omega}t}\left[\int_0^t\left(\overline{\omega}e^{-\underline{\omega}s} - \underline{\omega}e^{-\overline{\omega}s}\right)u_sds + \underline{\omega}\left(e^{(\overline{\omega}-\underline{\omega})t} - 1\right)\int_t^\infty e^{-\overline{\omega}s}u_sds\right]$, or, using that $u_t = e^{-\psi_m t}u_0$, $c_t^m = \sigma^{-1}\frac{\rho+\psi_m}{(\underline{\omega}+\psi_m)(\overline{\omega}+\psi_m)}\left(e^{\underline{\omega}t} - e^{-\psi_m t}\right)u_0$.

It remains to determine c_0. Plugging (17) in the budget constraint (13), we get $c_0 = -\frac{\tau-\sigma\rho\varsigma_d}{\tau-\sigma\underline{\omega}\varsigma_d}\overline{\omega}\int_0^\infty e^{-\rho t}c_t^m dt$. Note that

$$\int_0^\infty e^{-\rho t}c_t^m dt = \frac{\sigma^{-1}}{\overline{\omega}}\int_0^\infty e^{-\rho t}e^{\underline{\omega}t}u_t dt = \frac{\sigma^{-1}}{\overline{\omega}}\frac{u_0}{\overline{\omega}+\psi_m}.$$

Then, the intertemporal budget constraint can then be written as $c_0 = -\sigma^{-1}\frac{\tau-\rho\varsigma_d\sigma}{\tau-\underline{\omega}\varsigma_d\sigma}\frac{u_0}{\overline{\omega}+\psi_m}$.

■

Proof of Proposition 3.

Note that $\frac{\partial c_0}{\partial u_0} = -\sigma^{-1}\frac{\tau-\rho\varsigma_d\sigma}{\tau-\underline{\omega}\varsigma_d\sigma}\frac{1}{\overline{\omega}+\psi_m}$. Since $\underline{\omega} < 0$, it is immediate that $\frac{\partial c_0}{\partial u_0} < 0$ if and only if Assumption 1 is satisfied.

■

Proof of Proposition 4.

The Taylor equilibrium is the unique bounded solution to

$$\dot{c}_t = \sigma^{-1}(i_t - \pi_t - \rho),$$
$$\dot{\pi}_t = \rho\pi_t - \kappa c_t,$$
$$i_t = \rho + \phi_\pi \pi_t + u_t, \quad \phi_\pi > 1.$$

This system is independent of ς_d, hence the solution is independent of ς_d. Moreover, the government's flow budget constraint is given by

$$\dot{d}_t^g = \underbrace{(\rho - \gamma_d)}_{<0} d_t^g - \frac{\tau}{\varsigma_d} c_t,$$

or, integrating backward,

$$d_t^g = -\frac{\tau}{\varsigma_d}\int_0^t e^{(\rho-\gamma_d)(t-s)} c_s ds.$$

Then,

$$\lim_{t\to\infty} d_t^g = -\frac{\tau}{\varsigma_d}\lim_{t\to\infty}\frac{\int_0^t e^{-(\rho-\gamma_d)s} c_s ds}{e^{-(\rho-\gamma_d)t}}.$$

Since $\lim_{t\to\infty} e^{-(\rho-\gamma_d)t} = \infty$, if $\lim_{t\to\infty}\int_0^t e^{-(\rho-\gamma_d)s} c_s ds < \infty$, then $\lim_{t\to\infty} d_t^g = 0$. If $\int_0^t e^{-(\rho-\gamma_d)s} c_s ds \to \infty$, we can apply L'Hôpital's rule and obtain

$$\lim_{t\to\infty} d_t^g = \frac{1}{\rho-\gamma_d}\frac{\tau}{\varsigma_d}\lim_{t\to\infty}\frac{e^{-(\rho-\gamma_d)t} c_t}{e^{-(\rho-\gamma_d)t}} = \frac{1}{\rho-\gamma_d}\frac{\tau}{\varsigma_d}\lim_{t\to\infty} c_t = 0.$$

That is, government debt is bounded for any bounded path of consumption.

■

Proof of Proposition 5.

We have $\frac{\partial^2 c_0}{\partial i_t \partial \varsigma_d} = \frac{\overline{\omega}\tau}{(\tau-\underline{\omega}\varsigma_d\sigma)^2}\frac{1}{\overline{\omega}+\psi_m} > 0$.

■

Proof of Proposition 6.

Immediate from Proposition 1, replacing D^g by $D_S^g + \frac{D_L^g}{\rho+\rho_L}$.

■

Proof of Proposition 7.

We have $\frac{\partial c_0}{\partial u_0} = -\sigma^{-1}\frac{\tau-\rho\varsigma_d\sigma}{\tau-\underline{\omega}\varsigma_d\sigma}\frac{1}{\overline{\omega}+\psi_m} - \frac{\overline{\omega}\varsigma_d\varsigma_L}{\tau-\underline{\omega}\varsigma_d\sigma}\frac{1}{\rho+\rho_L+\psi_m} < 0$ if Assumption 1 holds. More-

over, fixing ς_d, we have $\frac{\partial^2 c_0}{\partial u_0 \partial \varsigma_L} = -\frac{\overline{\omega}\varsigma_d}{\tau - \underline{\omega}\varsigma_d\sigma}\frac{1}{\rho+\rho_L+\psi_m} < 0$, and $\frac{\partial^2 c_0}{\partial u_0 \partial \rho_L} = \frac{\overline{\omega}\varsigma_d\varsigma_L}{\tau - \underline{\omega}\varsigma_d\sigma}\frac{1}{(\rho+\rho_L+\psi_m)^2} > 0$. Finally, we have $\frac{\partial^2 c_0}{\partial u_0 \partial \varsigma_d} = \frac{\overline{\omega}\tau}{(\tau - \underline{\omega}\varsigma_d\sigma)^2}\frac{\rho_L + \underline{\omega} + (1-\varsigma_L)(\overline{\omega}+\psi_m)}{(\overline{\omega}+\psi_m)(\rho+\rho_L+\psi_m)}$. Since $\varsigma_L \in [0,1]$, if $\rho_L + \underline{\omega} > 0$, then $\frac{\partial^2 c_0}{\partial u_0 \partial \varsigma_d} > 0$.
■

B Model log-linearization

This section provides the log-linearization of the model in Section 2 augmented to incorporate a constant path of government spending.

The intertemporal Euler equation is given by

$$\frac{\dot{C}_t}{C_t} = \sigma^{-1}(i_t - \pi_t - \rho).$$

Since $c_t = \log\left(\frac{C_t}{C}\right)$, $\dot{c}_t = \frac{\dot{C}_t}{C_t}$, and then, up to first order,

$$\dot{c}_t = \sigma^{-1}(i_t - \pi_t - \rho).$$

The intratemporal Euler equation is

$$\frac{W_t}{P_t} = N_t^{\phi} C_t^{\sigma}.$$

Hence, up to first order,

$$w_t - p_t = \phi n_t + \sigma c_t. \tag{18}$$

The aggregate resource constraint is given by

$$C_t + G = Y_t,$$

hence,

$$\varsigma_c c_t = y_t, \tag{19}$$

where $\varsigma_c \equiv \frac{C}{Y}$.

The intermediate-goods firms' production function is

$$Y_t(i) = N_t(i)^{1-\gamma}.$$

Then, up to first order,

$$y_t(i) = (1-\gamma) n_t(i).$$

Noting that $\int_0^1 y_t(i)di = y_t$ and $\int_0^1 n_t(i)di = n_t$, we have

$$y_t = (1-\gamma) n_t. \tag{20}$$

The inflation rate is given by

$$\pi_t = \frac{\rho_\delta}{\epsilon - 1}\left[1 - \left(\frac{P_t^*}{P_t}\right)^{1-\epsilon}\right].$$

Then, up to first order,

$$\pi_t = \rho_\delta(p_t^* - p_t). \tag{21}$$

The optimal pricing equation is given by

$$\int_0^\infty e^{-(\rho+\rho_\delta)s}\left(\frac{C_{t+s}}{C_t}\right)^{-\sigma}\left[(1-\tau)\left(\frac{P_t}{P_{t+s}}\right)^{1-\epsilon}\left(\frac{P_t^*}{P_t}\right)^{1-\epsilon}Y_{t+s} - \right.$$
$$\left.\frac{\epsilon}{\epsilon-1}\frac{1}{1-\gamma}\frac{W_{t+s}}{P_{t+s}}\left(\frac{P_t}{P_{t+s}}\right)^{-\frac{\epsilon}{1-\gamma}}\left(\frac{P_t^*}{P_t}\right)^{-\frac{\epsilon}{1-\gamma}}Y_{t+s}^{\frac{1}{1-\gamma}}\right]ds = 0.$$

Then, up to first order,

$$\int_0^\infty e^{-(\rho+\rho_\delta)s}\left[(1-\tau)Y\left((1-\epsilon)\left(p_t - p_{t+s} + p_t^* - p_t\right) + y_{t+s}\right) - \frac{\epsilon}{\epsilon-1}\frac{1}{1-\gamma}\frac{W}{P}Y^{\frac{1}{1-\gamma}}\right.$$
$$\left.\left((w_{t+s} - p_{t+s}) - \frac{\epsilon}{1-\gamma}\left(p_t - p_{t+s} + p_t^* - p_t\right) + \frac{1}{1-\gamma}y_{t+s}\right)\right]ds = 0.$$

Noting that $(1-\tau)Y = \frac{\epsilon}{\epsilon-1}\frac{1}{1-\gamma}\frac{W}{P}Y^{\frac{1}{1-\gamma}}$, we can rewrite this equation as

$$\int_0^\infty e^{-(\rho+\rho_\delta)s}\left[\frac{1-\gamma+\epsilon\gamma}{1-\gamma}\left(p_t - p_{t+s} + p_t^* - p_t\right) - \frac{\gamma}{1-\gamma}y_{t+s} - (w_{t+s} - p_{t+s})\right]ds = 0.$$

Combining with equations (18), (19) and (20), we get

$$\int_0^\infty e^{-(\rho+\rho_\delta)s}\left[\frac{1-\gamma+\epsilon\gamma}{1-\gamma}\left(p_t - p_{t+s} + p_t^* - p_t\right) - \left(\sigma + \frac{\gamma+\phi}{1-\gamma}\varsigma_c\right)c_{t+s}\right]ds = 0.$$

And using equation (21), we can rewrite this equation as

$$\pi_t = \rho_\delta\left(\rho+\rho_\delta\right)\int_0^\infty e^{-(\rho+\rho_\delta)s}\left[(p_{t+s} - p_t) + \frac{1-\gamma}{1-\gamma+\epsilon\gamma}\left(\sigma + \frac{\gamma+\phi}{1-\gamma}\varsigma_c\right)c_{t+s}\right]ds.$$

Differentiating over time, we get

$$\dot{\pi}_t = -\rho_\delta\left(\rho+\rho_\delta\right)\frac{1-\gamma}{1-\gamma+\epsilon\gamma}\left(\sigma + \frac{\gamma+\phi}{1-\gamma}\varsigma_c\right)c_t + \left(\rho+\rho_\delta\right)\pi_t -$$
$$\left(\rho+\rho_\delta\right)\rho_\delta\int_t^\infty e^{-(\rho+\rho_\delta)(s-t)}\dot{p}_t ds.$$

Noting that $\dot{p}_t = \pi_t$, we obtain the log-linear New Keynesian Phillips curve:

$$\dot{\pi}_t = \rho\pi_t - \kappa\omega_c c_t,$$

where $\kappa \equiv \rho_\delta (\rho + \rho_\delta) \frac{1-\gamma}{1-\gamma+\epsilon\gamma} \left(\sigma + \frac{\gamma+\phi}{1-\gamma}\right)$ and $\omega_c \equiv \frac{\sigma + \frac{\gamma+\phi}{1-\gamma}\varsigma_c}{\sigma + \frac{\gamma+\phi}{1-\gamma}}$. Note that if $G = 0$, $\omega_c = 1$.

Finally, note that the households' intertemporal budget constraint is given by

$$\int_0^\infty e^{-\int_0^t (i_s - \pi_s) ds} C_t dt = \frac{B_0}{P_0} + \int_0^\infty e^{-\int_0^t (i_s - \pi_s) ds} \left[(1-\tau) Y_t + T_t\right] dt,$$

where we used that $\frac{W_t N_t}{P_t} + \frac{\Pi_t}{P_t} = (1-\tau) Y_t$. Then, up to first order,

$$\int_0^\infty e^{-\rho t} C c_t dt - \frac{C}{\rho} \int_0^\infty e^{-\rho t} (i_t - \pi_t - \rho) ds = \int_0^\infty e^{-\rho t} (1-\tau) Y y_t dt - \frac{(1-\tau) Y + T}{\rho} \int_0^\infty e^{-\rho t} (i_t - \pi_t - \rho) dt,$$

where we used that $T_t = T \; \forall t$. Noting that $(1-\tau) Y + T - C = T + G - \tau Y = -\rho D^g$, and letting $\varsigma_d \equiv \frac{D^g}{Y}$, we get

$$\int_0^\infty e^{-\rho t} \varsigma_c c_t dt = \int_0^\infty e^{-\rho t} \left[(1-\tau) y_t + \varsigma_d (i_t - \pi_t - \rho)\right] dt.$$

Let

$$\overline{\omega}_g = \frac{\rho + \sqrt{\rho^2 + 4\sigma^1 \kappa\omega_c}}{2} \quad \text{and} \quad \underline{\omega} = \frac{\rho - \sqrt{\rho^2 + 4\sigma^1 \kappa\omega_c}}{2}.$$

The following proposition extends Proposition 2 to the setting with positive government spending.

Proposition 8. *The equilibrium path for consumption is given by*

$$c_t = e^{\underline{\omega}_g t} c_0 + c_t^m,$$

where

$$c_t^m \equiv \sigma^{-1} \frac{\rho + \psi_m}{\left(\overline{\omega}_g + \psi_m\right)\left(\underline{\omega}_g + \psi_m\right)} \left(e^{\underline{\omega}_g t} - e^{-\psi_m t}\right) u_0,$$

and the initial value of c_0 is given by

$$c_0 = -\sigma^{-1} \frac{\varsigma_c \tau - \sigma\rho\varsigma_d}{\varsigma_c \tau - \sigma\underline{\omega}_g \varsigma_d} \frac{u_0}{\overline{\omega}_g + \psi_m}.$$

Proof. A simple extension to the proof of Proposition 2. ■

We make the following assumption.

Assumption 2. *$\varsigma_c \tau > \rho\varsigma_d \sigma$.*

Then, we get the following result.

Proposition 9. *Suppose Assumption 2 holds. Then,*

$$\frac{\partial c_0}{\partial u_0} < 0, \quad \frac{\partial^2 c_0}{\partial u_0 \partial \varsigma_d} > 0.$$

Proof. We have

$$\frac{\partial c_0}{\partial u_0} = -\sigma^{-1} \frac{\varsigma_c \tau - \sigma \rho \varsigma_d}{\varsigma_c \tau - \sigma \underline{\omega}_g \varsigma_d} \frac{1}{\overline{\omega}_g + \psi_m},$$

which is negative if Assumption 2 holds. Moreover,

$$\frac{\partial^2 c_0}{\partial u_0 \partial \varsigma_d} = \frac{\varsigma_c \tau \overline{\omega}_g}{\left(\varsigma_c \tau - \sigma \underline{\omega}_g \varsigma_d\right)^2} \frac{1}{\overline{\omega}_g + \psi_m} > 0.$$

■

C Policy Rules

Assume that the fiscal authority adjusts lump-sum transfers in response to the level of real government debt outstanding and interest payments such that

$$T_t = \gamma_0 - \gamma_d \frac{D_t^g}{P_t} - \gamma_r \frac{D_t^g}{P_t}(i_t - \pi_t).$$

The first-orden approximation around the zero-inflation steady-state equilibrium is given by

$$\hat{T}_t = -\tilde{\gamma}_d \varsigma_d d_t^g - \gamma_r \varsigma_d (i_t - \pi_t - \rho), \tag{22}$$

where $\tilde{\gamma}_d \equiv \gamma_d + \gamma_r \rho$, $\hat{T}_t \equiv \frac{T_t - T}{Y}$, and we used that $T = \gamma_0 - \gamma_d D^g - \gamma_r \rho D^g$. Note that the last term (partially) neutralizes the wealth effects generated by monetary policy.

Moreover, assume that the monetary authority sets the nominal interest rate following the rule

$$i_t = \rho + \phi_\pi \pi_t + u_t,$$

where u_t represents an innovation of the rule relative to its systematic response to inflation, and $\phi_\pi \geq 0$. In particular, we assume that u_t follows a mean reverting process after a one time unexpected shock, that is, $\dot{u}_t = -\rho_m u_t$ with u_0 given.

Consider the government's budget constraint. The flow budget constraint is given by

$$\dot{\left(\frac{D_t^g}{P_t}\right)} = -\left(\tau Y_t - T_t\right) + \left(i_t - \pi_t\right) \frac{D_t^g}{P_t}.$$

Let $d_t^g \equiv \log\left(\frac{\frac{D_t^g}{P_t}}{D^g}\right)$. Then

$$\dot{d}_t^g = \rho d_t^g - \frac{1}{\varsigma_d}\left(\tau y_t - \hat{T}_t\right) + \left(i_t - \pi_t - \rho\right).$$

Using the fiscal rule (22) and rearranging, we get

$$\dot{d}_t^g = \left(\rho - \tilde{\gamma}_d\right) d_t^g + \left(1 - \gamma_r\right)\left(i_t - \pi_t - \rho\right) - \frac{\tau}{\varsigma_d} c_t,$$

where we replaced y_t for c_t using the resource constraint.

Then, the equilibrium of the economy can be characterized by the system of differential equations

$$\begin{aligned}
\dot{c}_t &= \sigma^{-1}\left(i_t - \pi_t - \rho\right)\\
\dot{\pi}_t &= \rho\pi_t - \kappa c_t\\
\dot{d}_t^g &= \left(\rho - \tilde{\gamma}_d\right) d_t^g + \left(1 - \gamma_r\right)\left(i_t - \pi_t - \rho\right) - \frac{\tau}{\varsigma_d} c_t,
\end{aligned}$$

and the interest rate rule

$$i_t = \rho + \phi_\pi \pi_t + u_t.$$

Using the interest rate rule to replace for the nominal interest rate, we can write the system of differential equations in matrix form as

$$\begin{bmatrix} \dot{c}_t \\ \dot{\pi}_t \\ \dot{d}_t^g \end{bmatrix} = \begin{bmatrix} 0 & -\sigma^{-1}\left(1 - \phi_\pi\right) & 0 \\ -\kappa & \rho & 0 \\ -\frac{\tau}{\varsigma_d} & -\left(1 - \gamma_r\right)\left(1 - \phi_\pi\right) & \left(\rho - \tilde{\gamma}_d\right) \end{bmatrix} \begin{bmatrix} c_t \\ \pi_t \\ d_t^g \end{bmatrix} + \begin{bmatrix} \sigma^{-1} u_t \\ 0 \\ \left(1 - \gamma_r\right) I' u_t \end{bmatrix}$$

The eigenvalues of the system are

$$\omega_1 = \rho - \tilde{\gamma}_d$$

$$\omega_2 = \frac{\rho + \sqrt{\rho^2 + 4\sigma^{-1}\left(1 - \phi_\pi\right)\kappa}}{2}$$

$$\omega_3 = \frac{\rho - \sqrt{\rho^2 + 4\sigma^{-1}\left(1 - \phi_\pi\right)\kappa}}{2}$$

The system has a unique bounded solution if two eigenvalues are positive and one is negative. Hence, if $\phi_\pi < 1$, we need $\rho > \tilde{\gamma}_d$. Note that if $\tilde{\gamma}_d > 0$ there are solutions of the system that feature a bounded path for consumption and inflation but an unbounded debt-to-output ratio and still satisfy the transversality condition. Here we follow Leeper (1991) and focus on equilibria with bounded debt-to-output ratio. For a discussion of this point, see Cochrane (2023), Chapter 5.4. Then, assuming $\tilde{\gamma}_d < \rho$ and focusing on bounded solutions we can solve the debt equation forward to obtain

$$\int_0^\infty e^{-(\rho-\tilde{\gamma}_d)t}\left[\dot{d}_t^g-(\rho-\tilde{\gamma}_d)\,d_t^g\right]dt=\int_0^\infty e^{-(\rho-\tilde{\gamma}_d)t}\left[(1-\gamma_r)\,(i_t-\pi_t-\rho)-\frac{\tau}{\varsigma_d}c_t\right]dt$$

Using that the Euler equation implies

$$\int_0^\infty e^{-(\rho-\tilde{\gamma}_d)t}\,(\rho-\tilde{\gamma}_d)\,\sigma\,(c_t-c_0)\,dt=\int_0^\infty e^{-(\rho-\tilde{\gamma}_d)t}\,(i_t-\pi_t-\rho)\,dt,$$

we get

$$0=\int_0^\infty e^{-(\rho-\tilde{\gamma}_d)t}\left[(1-\gamma_r)\,\varsigma_d\,(\rho-\tilde{\gamma}_d)\,\sigma\,(c_t-c_0)-\tau c_t\right]dt$$

or

$$\left[\tau-(1-\gamma_r)\,\underline{\omega}\varsigma_d\sigma\right]\frac{1}{\overline{\omega}-\tilde{\gamma}_d}c_0=-\left[\tau-(1-\gamma_r)\,\varsigma_d\,(\rho-\tilde{\gamma}_d)\,\sigma\right]\int_0^\infty e^{-(\rho-\tilde{\gamma}_d)t}c_t^m dt.$$

Note that

$$\int_0^\infty e^{-(\rho-\tilde{\gamma}_d)t}c_t^m dt=\sigma^{-1}\frac{\rho+\psi_m}{(\overline{\omega}+\psi_m)\,(\rho+\psi_m-\tilde{\gamma}_d)}\frac{1}{\overline{\omega}-\tilde{\gamma}_d}u_0$$

Then

$$c_0=-\sigma^{-1}\frac{\tau-(1-\gamma_r)\,\varsigma_d\,(\rho-\tilde{\gamma}_d)\,\sigma}{\tau-(1-\gamma_r)\,\underline{\omega}\varsigma_d\sigma}\frac{\rho+\psi_m}{\rho+\psi_m-\tilde{\gamma}_d}\frac{u_0}{\overline{\omega}+\psi_m}.$$

The next proposition presents the main result of this section.

Proposition 10. *Suppose $\tilde{\gamma}_d\in[0,\rho)$ and $\phi_\pi\in[0,1)$. If $\gamma_r<1$ and $\tau>(1-\gamma_r)\,\varsigma_d\,(\rho-\tilde{\gamma}_d)\,\sigma$, we have*

$$\frac{\partial c_0}{\partial u_0}<0,\quad \frac{\partial^2 c_0}{\partial u_0\partial\varsigma_d}>0$$

Proof. We have

$$\frac{\partial c_0}{\partial u_0}=-\sigma^{-1}\frac{\tau-(1-\gamma_r)\,\varsigma_d\,(\rho-\tilde{\gamma}_d)\,\sigma}{\tau-(1-\gamma_r)\,\underline{\omega}\varsigma_d\sigma}\frac{\rho+\psi_m}{\rho+\psi_m-\tilde{\gamma}_d}\frac{1}{\overline{\omega}+\psi_m}<0$$

and

$$\frac{\partial^2 c_0}{\partial u_0\partial\varsigma_d}=(1-\gamma_r)\,\tau\frac{\overline{\omega}-\tilde{\gamma}_d}{\left[\tau-(1-\gamma_r)\,\underline{\omega}\varsigma_d\sigma\right]^2}\frac{\rho+\psi_m}{\rho+\psi_m-\tilde{\gamma}_d}\frac{1}{\overline{\omega}+\psi_m}>0.$$

■

Long-term bonds. Now, consider the economy with long-term bonds. We now assume that $\gamma_d=\gamma_r=0$. The price of these bonds are now given by

$$q_{L,0}=-\int_0^\infty e^{-(\rho+\rho_L)t}(i_t-\rho)dt=-\int_0^\infty e^{-(\rho+\rho_L)t}(\phi_\pi\pi_t+u_t)dt.$$

From the Phillips Curve, we have

$$\dot{\pi}_t = \rho\pi_t - \kappa c_t \implies \pi_t = \kappa \int_t^\infty e^{-\rho(s-t)} c_s ds.$$

Introducing this expression into the price of the bond, we get

$$q_{L,0} = -\frac{\phi_\pi}{\rho_L}\kappa \int_0^\infty \left(e^{-\rho t} - e^{-(\rho+\rho_L)t}\right) c_t dt - \int_0^\infty e^{-(\rho+\rho_L)t} u_t dt.$$

The households' intertemporal budget constraint is given by

$$\int_0^\infty e^{-\rho t} c_t dt = \int_0^\infty e^{-\rho t} \left[(1-\tau)\, y_t + \varsigma_d \rho\sigma \left(c_t - c_0\right)\right] dt + \varsigma_d \varsigma_L q_{L,0},$$

Introducing the expression for the price of the bond, we get

$$\int_0^\infty e^{-\rho t} c_t dt = \int_0^\infty e^{-\rho t} \left[(1-\tau)\, y_t + \varsigma_d \rho\sigma \left(c_t - c_0\right)\right] dt -$$
$$\varsigma_d \varsigma_L \left[\frac{\phi_\pi}{\rho_L}\kappa \int_0^\infty \left(e^{-\rho t} - e^{-(\rho+\rho_L)t}\right) c_t dt + \int_0^\infty e^{-(\rho+\rho_L)t} u_t dt\right],$$

or

$$\left[\tau - \sigma\underline{\omega}\varsigma_d + \varsigma_d\varsigma_L\phi_\pi\kappa\frac{1}{\overline{\omega}+\rho_L}\right]\frac{1}{\overline{\omega}}c_0 = -\left[\tau - \sigma\rho\varsigma_d + \varsigma_d\varsigma_L\phi_\pi\kappa\frac{1}{\rho_L}\right]\int_0^\infty e^{-\rho t} c_t^m dt +$$
$$\varsigma_d\varsigma_L\frac{\phi_\pi}{\rho_L}\kappa\int_0^\infty e^{-(\rho+\rho_L)t} c_t^m dt - \varsigma_d\varsigma_L\int_0^\infty e^{-(\rho+\rho_L)t} u_t dt.$$

Note that

$$\int_0^\infty e^{-\rho t} c_t^m dt = \frac{\sigma^{-1}}{\overline{\omega}}\frac{u_0}{\overline{\omega}+\psi_m},$$

and

$$\int_0^\infty e^{-(\rho+\rho_L)t} c_t^m dt = \frac{\sigma^{-1}}{\left(\overline{\omega}+\rho_L\right)\left(\underline{\omega}+\rho_L\right)}\left(\frac{\rho_L}{\rho+\rho_L+\psi_m} + \frac{\underline{\omega}}{\overline{\omega}+\psi_m}\right)u_0.$$

Then, we can write the budget constraint as

$$\left[\tau - \sigma\underline{\omega}\varsigma_d + \varsigma_d\varsigma_L\phi_\pi\kappa\frac{1}{\overline{\omega}+\rho_L}\right]\frac{1}{\overline{\omega}}c_0 =$$
$$-\sigma^{-1}\left[\tau - \sigma\rho\varsigma_d + \varsigma_d\varsigma_L\phi_\pi\kappa\frac{\rho+\rho_L}{\left(\overline{\omega}+\rho_L\right)\left(\underline{\omega}+\rho_L\right)}\right]\frac{1}{\overline{\omega}}\frac{u_0}{\overline{\omega}+\psi_m} -$$
$$\varsigma_d\varsigma_L\left[1 - \sigma^{-1}\phi_\pi\kappa\frac{1}{\left(\overline{\omega}+\rho_L\right)\left(\underline{\omega}+\rho_L\right)}\right]\frac{u_0}{\rho+\rho_L+\psi_m},$$

and therefore

$$c_0 = -\sigma^{-1} \frac{\tau - \sigma\rho\varsigma_d + \varsigma_d\varsigma_L\phi_\pi\kappa\frac{\rho+\rho_L}{(\overline{\omega}+\rho_L)(\underline{\omega}+\rho_L)}}{\tau - \sigma\underline{\omega}\varsigma_d + \varsigma_d\varsigma_L\phi_\pi\kappa\frac{1}{\overline{\omega}+\rho_L}} \frac{u_0}{\overline{\omega}+\psi_m} - \varsigma_d\varsigma_L\overline{\omega}\frac{1-\sigma^{-1}\phi_\pi\kappa\frac{1}{(\overline{\omega}+\rho_L)(\underline{\omega}+\rho_L)}}{\tau - \sigma\underline{\omega}\varsigma_d + \varsigma_d\varsigma_L\phi_\pi\kappa\frac{1}{\overline{\omega}+\rho_L}} \frac{u_0}{\rho+\rho_L+\psi_m}.$$

Then, we have

$$\frac{\partial c_0}{\partial u_0} = -\sigma^{-1} \frac{\tau - \sigma\rho\varsigma_d + \varsigma_d\varsigma_L\phi_\pi\kappa\frac{\rho+\rho_L}{(\overline{\omega}+\rho_L)(\underline{\omega}+\rho_L)}}{\tau - \sigma\underline{\omega}\varsigma_d + \varsigma_d\varsigma_L\phi_\pi\kappa\frac{1}{\overline{\omega}+\rho_L}} \frac{1}{\overline{\omega}+\psi_m} - \varsigma_d\varsigma_L\overline{\omega}\frac{1-\sigma^{-1}\phi_\pi\kappa\frac{1}{(\overline{\omega}+\rho_L)(\underline{\omega}+\rho_L)}}{\tau - \sigma\underline{\omega}\varsigma_d + \varsigma_d\varsigma_L\phi_\pi\kappa\frac{1}{\overline{\omega}+\rho_L}} \frac{1}{\rho+\rho_L+\psi_m},$$

which is negative if Assumption 1 holds, and

$$\frac{\partial^2 c_0}{\partial u_0 \partial \varsigma_d} = \frac{\tau\overline{\omega}}{\left[\tau - \sigma\underline{\omega}\varsigma_d + \varsigma_d\varsigma_L\phi_\pi\kappa\frac{1}{\overline{\omega}+\rho_L}\right]^2} \times \left[\frac{1}{\overline{\omega}+\psi_m} - \frac{\varsigma_L}{\rho+\rho_L+\psi_m} - \sigma^{-1}\phi_\pi\kappa\frac{\varsigma_L}{(\overline{\omega}+\psi_m)(\rho+\rho_L+\psi_m)}\frac{1}{\overline{\omega}+\rho_L}\right].$$

A sufficient condition for $\frac{\partial^2 c_0}{\partial u_0 \partial \varsigma_d} > 0$ is

$$\frac{1}{\overline{\omega}+\psi_m} - \frac{1}{\rho+\rho_L+\psi_m} - \sigma^{-1}\phi_\pi\kappa\frac{1}{(\overline{\omega}+\psi_m)(\rho+\rho_L+\psi_m)}\frac{1}{\overline{\omega}+\rho_L} > 0$$

which holds if and only if

$$\rho_L > -\frac{\rho - \sqrt{\rho^2 + 4\sigma^{-1}\kappa}}{2}.$$

This condition is the same as then one in Proposition 7 once we note that, in Section 4, $\underline{\omega} = -\frac{\rho - \sqrt{\rho^2+4\sigma^{-1}\kappa}}{2}$.