Fiscal Policy and the Monetary Transmission Mechanism*

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Abstract

The economy's response to monetary policy depends on its fiscal backing. We present a novel decomposition of the equilibrium that links the wealth effect, i.e. the revaluation of households' financial and human wealth, to the fiscal response to monetary policy. When monetary policy has fiscal consequences, monetary variables affect the timing of aggregate output while fiscal variables shape its present value and the wealth effect. Consequently, a contractionary monetary policy reduces inflation only if followed by contractionary fiscal policy. The slope of the Phillips curve determines the importance of monetary-fiscal coordination for the effectiveness of monetary policy.

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1 Introduction

The monetary transmission mechanism is often described as the effect that changes in a policy instrument, usually the stock of money or the short-term interest rate, have on aggregate variables such as inflation, output, consumption, and investment.[1] This description limits the scope of the monetary transmission mechanism to *monetary policy*, i.e. actions generally undertaken by a central bank. However, this characterization depicts an incomplete account of all the policy actions involved, as monetary policy usually has fiscal consequences: it affects the value of government debt, debt servicing costs, and primary surpluses (through changes in revenues and other automatic stabilizers). This paper revisits the monetary transmission mechanism with a focus on monetary and fiscal interactions. The analysis isolates the role that the different policy instruments play in shaping the economy's equilibrium, with a focus on the wealth effect, i.e. the revaluation of households' financial and human wealth.

The fiscal response to monetary policy is almost entirely overlooked in textbook formulations of the monetary transmission mechanism.[2] This approach usually acknowledges the importance of an appropriate fiscal backing in supporting the equilibrium, but its role is relegated to an adjustment in the background. An example of this is the so-called *Taylor equilibrium*, characterized by an interest rate rule that satisfies the principles of Taylor (1993). Alternative formulations put fiscal policy at the forefront and emphasize the role of government debt and primary surpluses in determining the equilibrium. Much of this literature's focus has been on the determination of the price level, which is why it is generally known as the *Fiscal Theory of the Price Level* (FTPL).[3] This paper presents a unifying framework that identifies the channels through which the different policy instruments (monetary and fiscal) affect the main macroeconomic variables. The analysis highlights the role played by fiscal policy and uncovers its quantitative importance. Crucially, the approach is agnostic about the policy rules that gave rise to the *equilibrium paths* of the policy variables, and it is sufficiently general to accommodate any framework in which monetary policy has fiscal consequences. In particular, it nests the Taylor equilibrium and the FTPL as special cases.

We study the dynamic response of the economy to a monetary shock, which results in a deviation of the path of the nominal interest rate from its steady-state level and a simultaneous response of the fiscal authority. A novel finding emerges: when monetary policy has fiscal consequences,

[1] See, e.g., the definition in The New Palgrave Dictionary of Economics (Ireland, 2008).

[2] See Galí (2015). Woodford (2003) presents a limited analysis in Section 4 of Chapter 4.

[3] See Leeper and Leith (2016) for a review and Cochrane (2023) for a detailed analysis.

monetary variables affect the *timing* of aggregate output, while it is fiscal variables that shape its *present value*, a counterpart of the households' wealth effect. This result implies that contractionary monetary policy reduces inflation *only if* followed by contractionary fiscal policy.[4] Moreover, a stronger initial response of inflation to monetary policy requires a larger fiscal contraction. In particular, in the absence of a change in the fiscal policy stance, an increase in nominal interest rates has no effect on inflation on impact, and it actually raises inflation in future dates. Therefore, the ability of the central bank to control inflation in the New Keynesian model implicitly relies on the response of fiscal policy.

Underlying the analysis, there is a novel decomposition of the dynamic response of aggregate consumption into two terms that represent distinct economic forces. One term is uniquely determined by the equilibrium path of the nominal interest rate and captures the change in the timing of aggregate demand due to the monetary shock without affecting its present value. This term has an interpretation in terms of the substitution effect from microeconomic theory: it corresponds to the households' *Hicksian demand* extended to a general equilibrium setting. We call this term the *intertemporal substitution effect* (ISE).[5] The second term depends on the *wealth effect*. The wealth effect is defined as (minus) the compensation necessary for households to be able to consume their initial (pre-shock) consumption bundle. Therefore, a policy change creates a negative wealth effect if a positive compensation is necessary for households to afford their previous consumption bundle.[6] We show that the wealth effect corresponds to the revaluation of the households' financial and human wealth net of the change in the cost of households' initial consumption bundle. Under this definition, a positive wealth effect implies that households can afford higher consumption paths than at the steady-state equilibrium (and vice-versa). Even though monetary shocks generate only transitory changes in income and households conform to the permanent income hypothesis in the RANK model, the general equilibrium dynamics of inflation can significantly amplify the impact of the wealth effect on initial output. We present a numerical exercise in which, for a standard calibration, the wealth effect is amplified by a factor of 30 on impact, and it explains more than half of the initial response of consumption to a monetary shock in the Taylor equilibrium.

[4]Contractionary fiscal policy can take the form of an increase in lump-sum taxes or a reduction in the value of government bonds that is not followed by a reduction in taxes (or a combination of both).

[5]King (1991) and Leeper and Yun (2006) provide an analogous decomposition to study the effects of government spending and tax changes in DSGE models. An important distinction is that, in this paper, the inflation rate used to compute the substitution effect is consistent with the New Keynesian Phillips curve evaluated at the Hicksian demand. This is the sense in which it corresponds to a general equilibrium extension of the standard substitution effect.

[6]The wealth effect also corresponds to the equivalent variation of the policy change, that is, the amount that households would be indifferent to accept instead of the policy change (see Mas-Colell et al., 1995).

A significant feature of the decomposition is that the ISE is uniquely determined by the equilibrium path of the nominal interest rate, while the wealth effect is indeterminate under an interest rate peg. Indeed, it is possible to index *all* the bounded equilibria of the New Keynesian model by the wealth revaluation they generate. Moreover, as long as monetary policy has fiscal consequences, we show that the wealth effect can be expressed in terms of fiscal variables. This characterization underscores the main result of the paper: in the New Keynesian model, the magnitude of the wealth effect depends on the fiscal response to monetary policy rather than on the change in the path of the nominal interest rate *per se*. This result *does not* require that fiscal policy is set independently of monetary policy. Even in a monetary-active regime (see Leeper, 1991), fiscal policy needs to adjust to guarantee that the government's budget constraint is satisfied in equilibrium. The result states that it is this adjustment that shapes the wealth effect. In this sense, the Taylor equilibrium could be interpreted as acting through two separate channels: i) changing the path of the nominal interest rate, which affects the timing of output (i.e. the ISE), and ii) triggering a fiscal response that changes the present value of output (i.e. the wealth effect). Thus, combining the fiscal determination of the wealth effect and the general equilibrium amplification described before, we conclude that the fiscal response to monetary policy is not just an adjustment that happens in the background but a significant determinant of the monetary transmission mechanism.[7] In our baseline calibration, around 60% of the initial consumption response to a monetary shock can be attributed to the wealth effect and, therefore, fiscal policy.

The importance of the wealth effect and the fiscal response associated with monetary policy becomes even more apparent when considering the dynamics of inflation. We find that the initial response of inflation is entirely determined by the wealth effect rather than by the contemporaneous response of consumption. Moreover, absent a wealth effect, the inflation dynamics after the initial period has a *Neo-Fisherian* property: it moves in the same direction as the change in the nominal interest rate. These results shed new light on the channels through which the central bank controls inflation in these models. A contractionary monetary shock reduces initial inflation not because of a reduction in the *contemporaneous* level of consumption but because households are overall *poorer* after the shock. Put differently, initial inflation decreases after a contractionary monetary shock if and only if there is a simultaneous *contractionary fiscal response*.

[7] A noteworthy exception is when monetary policy does not have fiscal consequences. This is the only case that renders the households' budget constraint irrelevant: since output is demand determined, any level of the households' demand can be consistent with equilibrium. The monetary-active equilibrium selection solves this indeterminacy by making only one equilibrium to be bounded. However, this case is non-generic, in the sense that even a small fiscal effect leads to the fiscal characterization, and is also at odds with reality.

Naturally, the numerical results depend on the calibration. In a sensitivity analysis, we show that the degree of price stickiness is a crucial parameter determining the relevance of fiscal backing in the monetary transmission mechanism. The general equilibrium amplification of the wealth effect relies on an inflation channel: a reduction in households' wealth reduces aggregate demand, which puts downward pressure on inflation and, for a given path of the nominal interest rate, increases the real rate, generating a second-round reduction in aggregate demand. Thus, this amplification mechanism *increases* with the degree of price flexibility. Since the wealth effect depends on the fiscal response to monetary policy, it follows that fiscal policy has a stronger effect in economies with a high degree of price flexibility. This implies that for low degrees of price flexibility, the Taylor equilibrium and a version of the FTPL in which the present value of primary surpluses does not change generate virtually identical aggregate dynamics for the same given path of the nominal interest rate. This finding can prove relevant to assess the effectiveness of monetary policy in economies with different institutional arrangements, as the degree of monetary-fiscal coordination appears to be more important in economies with a steeper Phillips curve. Moreover, even if fiscal policy might not be crucial for macroeconomic stabilization in an economy with relatively rigid prices, an uncoordinated policy may eventually trigger a regime change.[8]

As a final exercise, we show that all the intuitions built in the simple RANK model extend to richer settings. First, we solve a two-agent New Keynesian (TANK) model analytically. Then, we extend the analysis to a medium-scale New Keynesian model based on Smets and Wouters (2007) and provide a simple recipe to compute the decomposition numerically.

Literature There is a long tradition that studies the role of monetary and fiscal policies as macroeconomic stabilizers (see Keynes, 1936; Friedman, 1948). One of the most famous quotes related to the origins of inflation is Friedman's "Inflation is always and everywhere a monetary phenomenon," (Friedman, 1963). This view is reflected in much of the modern analysis of the monetary transmission mechanism. However, careful inspection of the government's budget constraint highlights the tight connection between monetary and fiscal policy (see Sargent and Wallace, 1981, for an early formalization). We contribute to this literature by providing a novel characterization of the role that monetary and fiscal policy play in the monetary transmission mechanism.

The paper shares several features emphasized by the Fiscal Theory of the Price Level (FTPL)

[8] Alvarez et al. (2019) estimate the firms' price-setting behavior in Argentina for different inflation rates. They find that the frequency of price changes is relatively constant for low inflation levels but increases for higher rates, suggesting that the degree of price flexibility depends on the policy regime.

(see Leeper, 1991; Sims, 1994; Woodford, 1995, 2001, for early developments).[9] We make four important contributions. First, we formalize the interpretation of the monetary transmission mechanism in terms of substitution and wealth effects and show that the wealth effect is linked to fiscal policy. The connection between the wealth effect and fiscal policy is a recurrent narrative in this literature, but to the best of our knowledge, the formalization was missing.[10] Second, the paper expands on a recent approach that characterizes equilibria in terms of equilibrium paths for policy variables rather than on policy rules (see Werning, 2012; Cochrane, 2017, 2018a). A significant difference with these papers is that we explicitly consider the joint determination of monetary and fiscal variables and the economic forces involved in the transmission channel, i.e. the intertemporal substitution and wealth effects.[11] Third, we identify the importance of the slope of the Phillips curve in the results, noting that monetary-fiscal coordination is more relevant in economies with relatively flexible prices. Finally, we extend the analysis to two settings of independent interest: a TANK model and a medium-scale DSGE model.

The HANK literature has also recognized the importance of wealth effects and fiscal policy in heterogenous agents models in which Ricardian equivalence does not hold (see Kaplan et al., 2018). Our paper makes three contributions. First, it shows that the aggregate wealth effect is a crucial component of the monetary transmission mechanism even in RANK models. The decomposition allows us to identify a general equilibrium amplification of the wealth effect that operates through the inflation rate. Second, it emphasizes that fiscal policy matters even when Ricardian equivalence holds. Third, it shows the extent to which redistributive wealth effect (e.g., see Auclert, 2019) can affect the channels of transmission. Our analysis shows the robustness of the connection between the aggregate wealth effect, fiscal policy, and the inflation rate.

Finally, Caramp and Silva (2023) extend the decomposition in this paper to a setting with aggregate risk and private debt. They show that fiscal policy remains the main determinant of the wealth effect despite significant fluctuations in financial asset valuations and redistribution predicted by the model.

The rest of the paper is organized as follows. Section 2 describes the model and Section 3 presents the equilibrium decomposition. Section 4 shows that the wealth effect can be expressed

[9] A related literature emphasizes the importance of fiscal policy to understand several historical events such as the recovery from the Great Depression (Jacobson et al., 2023), the run-up and end of the Great Inflation (Bianchi and Ilut, 2017) and the missing inflation during the Great Recession (Bianchi and Melosi, 2017). Bassetto et al. (2022) highlight monetary-fiscal interactions and costly information acquisition to explain sudden inflationary events.

[10] Kaplan et al. (2018) present an alternative decomposition in terms of the *direct* and *indirect* effects of monetary policy. We compare the two approaches in Appendix B.3.

[11] This is related to the literature that tries to identify the policy regime in the data. See, e.g., Canzoneri et al. (2001).

in terms of the fiscal response to monetary policy. Section 5 presents the extensions to richer settings. Finally, Section 6 concludes.

2 The Model

We study a standard RANK model in discrete time augmented to incorporate fiscal variables and explicitly account for the households' budget constraint. We study the dynamic response of an economy hit by a monetary shock, resulting in a deviation of the path of the nominal interest rate from its steady-state level and a simultaneous response of the fiscal authority. We analyze the economy's reaction to the resulting *equilibrium paths* of the monetary and fiscal variables. By focusing on the equilibrium paths of policy variables, we obtain results that are robust to any monetary-fiscal regime that generated those paths. In particular, the Taylor equilibrium and the FTPL are special cases of the general approach.[12]

Environment Time is discrete and denoted by $t \in \{0, 1, 2, \dots\}$. The economy is populated by a large number of identical, infinitely-lived households and a government. There is also a continuum of firms that produce final and intermediate goods. Final-goods producers operate competitively and combine intermediate goods using a CES aggregator with elasticity $\epsilon > 1$. Intermediate-goods producers use labor as the only factor of production to produce a differentiated good that is traded in a monopolistically competitive market. As is standard, we assume that intermediate-goods firms face a pricing friction à la Calvo, so that only a fraction $1 - \theta$ of firms can set a new price each period. Finally, the government chooses monetary policy, which entails a path for the nominal interest rate, and fiscal policy, comprised of nominal debt and lump-sum taxes. We assume that government debt consists of perpetuities that pay coupons that decay exponentially at a rate $\rho \in [0, \beta^{-1})$. The case with $\rho = 0$ corresponds to one-period bonds, while $\rho = 1$ corresponds to consols. More generally, the duration of these bonds in a steady-state equilibrium is given by $\frac{1}{1-\beta\rho}$. This assumption allows us to study the effects of long-term debt with a minimal departure from the standard model (see Woodford, 2001). As is standard in the literature, we log-linearize the model around its zero inflation steady-state equilibrium and consider the first-order approximation of the response of the economy to exogenous shocks.[13]

[12] Our analysis will focus on a simple version of the FTPL where the present value of primary surpluses does not respond to monetary shocks, which we label "pure" FTPL. Extending the analysis to alternative specifications is straightforward.

[13] For the detailed derivation of the model, see A.

Given a path of interest rates $\{i_t\}_{t=0}^{\infty}$ and lump-sum taxes $\{\tau_t\}_{t=0}^{\infty}$, the log-linearized solution to the model can be characterized by four equations: the households' intertemporal Euler equation

$$c_t = \mathbb{E}_t[c_{t+1}] - \sigma(i_t - \mathbb{E}_t[\pi_{t+1}]), \tag{1}$$

the New Keynesian Phillips curve

$$\pi_t = \beta \mathbb{E}_t[\pi_{t+1}] + \kappa c_t, \tag{2}$$

the households' intertemporal budget constraint

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t c_t \right] = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left[y_t - \tau_t + (i_t - \pi_{t+1})\, Qb \right] - \left[\sum_{t=0}^{\infty} (\beta\rho)^t\, i_t \rho + \frac{1}{\beta}\pi_0 \right] Qb \right], \tag{3}$$

and the resource constraint

$$c_t = y_t, \tag{4}$$

where c_t and y_t denote, respectively, the percentage difference between actual consumption and output and their corresponding levels in the steady state; π_t denotes the inflation rate; i_t denotes the short-term, risk-free nominal interest rate; $\sigma > 0$ denotes the households' intertemporal elasticity of substitution; $\beta \in (0,1)$ denotes the households' subjective discount factor; $\kappa > 0$ is the slope of the Phillips curve; Qb denotes the steady-state *value* of government debt as a fraction of output, where $Q = \frac{\beta}{1-\beta\rho}$ is the steady-state price of a unit of the nominal bond; τ_t denotes the lump-sum tax as a fraction of steady-state output; and $\mathbb{E}_t$ denotes the expectation operator conditional on the information set in period t.

Since the analysis emphasizes the role of the households' budget constraint in the dynamic behavior of consumption, it is helpful to briefly describe its components. The left-hand side of equation (3) is the present value of consumption, discounted at the steady-state real interest rate. The right-hand side is the present value of the households' incomes and the revaluation of initial assets. This includes changes in the after-tax profits and wages, which combined equal $y_t - \tau_t$, the interest income from government bond holdings, and the revaluation of initial bond holdings. Note that there are three channels through which fiscal variables affect the households' budget constraint. First, they affect non-interest income through τ_t.[14] Second, the level of government

[14] Note that Ricardian equivalence holds in the model regardless of the monetary-fiscal regime, so only the present value of taxes, $\sum_{t=0}^{\infty} \beta^t \tau_t$, rather than the whole path, $\{\tau_t\}_{t=0}^{\infty}$, matters for the equilibrium.

debt determines the households' exposure to changes in the real interest rate. While changes in the real interest rate affect the present discounted value of both consumption and after-tax income, in a closed economy, the net impact depends only on the steady-state level of government debt.[15] Finally, the change in the path of the nominal interest rate, $\{i_t\}_{t=0}^{\infty}$, and initial inflation, π_0, affect the real return of initial nominal bond holdings. On the one hand, the change in nominal interest rates generates a revaluation of long-term bonds, given by $-\sum_{t=0}^{\infty}(\beta\rho)^t i_t \rho Q b$. This effect is absent when bonds are one-period, i.e. when $\rho = 0$. On the other hand, initial inflation affects the realized return on initial nominal bond holdings, summarized by $-\frac{1}{\beta}\pi_0 Q b$.

Finally, given the certainty-equivalence property of linearized models, we consider a perfect foresight dynamic response after a one-time, unexpected shock. Thus, in what follows, we drop the expectations operator $\mathbb{E}_t$.

Policy rules The exercise focuses on the paths of policy variables and studies the channels through which these paths affect equilibrium dynamics. This exercise differs from the standard approach, which typically assumes monetary and fiscal rules and then considers a monetary shock, i.e. a disturbance to the central bank's policy rule, that leads to the endogenous reaction of policy variables, namely the path of nominal interest rates and fiscal transfers. Under this approach, the response of output and inflation to a monetary shock captures not only the impact of changes in nominal interest rates but also the impact of changes in fiscal policy. Thus, disentangling the effect of nominal interest rates and fiscal backing is challenging. By considering equilibrium paths of policy variables directly, we can isolate the impact of each policy instrument while being able to accommodate *any* monetary-fiscal interactions that generate a particular path for monetary and fiscal variables.

A popular approach is to assume that monetary policy follows an interest rate rule of the form

$$i_t = \phi_\pi \pi_t + \phi_y y_t + \varepsilon_t, \tag{5}$$

where $\kappa(\phi_\pi - 1) + (1-\beta)\phi_y > 0$ and ε_t represents an innovation of the rule relative to its systematic response to inflation and output. Fiscal policy is assumed to be *passive* or *Ricardian*, and the exogenous monetary shock is represented by a path for $\{\varepsilon_t\}_{t=0}^{\infty}$ rather than a path for the nominal

[15] Formally, the impact of changes in the interest rate on the present discounted value of consumption as a fraction output is $-\frac{\beta}{1-\beta}\sum_{t=0}^{\infty}\beta^t(i_t - \pi_{t+1})$, and the corresponding impact on after-tax income is $-\frac{\beta}{1-\beta}\frac{Y-T}{Y}\sum_{t=0}^{\infty}\beta^t(i_t - \pi_{t+1})$, where T denotes the steady-state level of lump-sum taxes. Combining the two and using that $\frac{C}{Y} = 1 = \frac{1-\beta}{\beta}Qb + \frac{Y-T}{Y}$, we obtain $\sum_{t=0}^{\infty}\beta^t(i_t - \pi_{t+1})Qb$.

interest rate. Under these assumptions, equation (3) is often dropped when finding an equilibrium of the economy because transfers $\{\tau_t\}_{t=0}^{\infty}$ are assumed to automatically adjust so that the government's budget constraint is satisfied for any path of the endogenous and exogenous variables.[16] Since lump-sum taxes do not affect any of the other equations characterizing the equilibrium, they represent a free variable that adjusts to guarantee that any solution to the system given by (1), (2), (4) and (5) is an equilibrium of the economy. We call this case the *Taylor equilibrium*.

An alternative approach follows the *Fiscal Theory of the Price Level* (FTPL), which in its simplest specification assumes an exogenous path for the primary surplus, given by $\{\tau_t\}_{t=0}^{\infty}$, and an interest rate rule (5) with $\kappa(\phi_\pi - 1) + (1-\beta)\phi_y < 0$ and $\phi_\pi, \phi_y \geq 0$. Then, an equilibrium of the economy is a solution to the system (1)-(5) given $\{\tau_t\}_{t=0}^{\infty}$.[17]

Despite the stark differences between the two approaches, our formulation is consistent with both. The determination of the paths of policy variables, $\{i_t\}_{t=0}^{\infty}$ and $\{\tau_t\}_{t=0}^{\infty}$, depends on the specific monetary-fiscal regime in place. However, by analyzing the impact of the policy variables *directly* on consumption and inflation, we are able to bypass the debate on the monetary-fiscal policy regime and obtain results about the monetary transmission mechanism that are robust to any regime. Given equilibrium paths, we can always find rules that lead to these paths, although only certain paths will be consistent with specific rules.

The wealth effect The wealth effect is a key object in the analysis. Thus, it is useful to consider how to compute it. Let us start with the households' non-linear budget constraint. Let $\widetilde{\Omega}_t$ denote the households' wealth in period t, that is

$$\widetilde{\Omega}_t \equiv (1+\rho Q_t)\frac{B_t}{P_t} + \sum_{s=0}^{\infty}\prod_{h=0}^{s-1}\left(\frac{P_{t+1+h}/P_{t+h}}{1+i_{t+h}}\right)[Y_{t+s} - T_{t+s}],$$

where Y_t denotes the level of output in period t, B_t is the face value of outstanding government debt, P_t denotes the price level, and T_t is the level of the lump-sum tax. The households' intertemporal budget constraint can be written as

$$\sum_{s=0}^{\infty}\prod_{h=0}^{s-1}\left(\frac{P_{t+1+h}/P_{t+h}}{1+i_{t+h}}\right)C_{t+s} = \widetilde{\Omega}_t,$$

[16] See Woodford (2003) for a discussion.

[17] Richer versions of the FTPL assume rules for the primary surplus that can depend on the monetary shock as well as other endogenous and exogenous variables. Our analysis is robust to these specifications.

where C_t denotes the level of consumption in period t. Then, in the steady state,

$$\sum_{t=0}^{\infty} \beta^t C = \widetilde{\Omega} \implies \frac{\widetilde{\Omega}}{C} = \frac{1}{1-\beta}.$$

Let $c_t \equiv \log \frac{C_t}{C}, \hat{\widetilde{\Omega}}_t \equiv \log \frac{\widetilde{\Omega}_t}{\widetilde{\Omega}}$, $\tau_t \equiv \frac{T_t - T}{Y}$, and, with a slight abuse of notation, $i_t \equiv \log\left[\beta\left(1+i_t\right)\right]$. Then, the first-order approximation of the households' intertemporal budget constraint in period 0 around a zero-inflation steady state is given by

$$\sum_{t=0}^{\infty} \beta^t c_t - \frac{\beta}{1-\beta} \sum_{t=0}^{\infty} \beta^t \left(i_t - \pi_{t+1}\right) = \frac{1}{1-\beta} \hat{\widetilde{\Omega}}_0,$$

where

$$\hat{\widetilde{\Omega}}_0 = (1-\beta)\left[\sum_{t=0}^{\infty} \beta^t \left[y_t - \tau_t\right] + \left(\rho q_0 - \frac{1}{\beta}\pi_0\right) Q b_0\right] - \beta \sum_{t=0}^{\infty} \beta^t \left(i_t - \pi_{t+1}\right) \frac{Y-T}{Y},$$

with $q_0 = -\sum_{t=0}^{\infty} \left(\beta\rho\right)^t i_t$, and we used that $\frac{Y}{\widetilde{\Omega}} = 1-\beta$. Note that the first-order approximation of the households' intertemporal budget constraint has three components. First, there is the change in the consumption and after-tax income, discounted at the steady-state interest rate, β^t. Second, there is the change in the value of the households' initial portfolio, given by $\left(q_0\rho - \frac{1}{\beta}\pi_0\right) Q b_0$. Third, there is the change in the *discount factor*, summarized by the change in the interest rate path. Importantly, the change in the discount factor affects both the cost of the consumption bundle and the present value of after-tax income. For example, an increase in the discount factor reduces the present value of income but also the cost of the consumption bundle. Thus, to determine whether the change in the discount factor implies that the household can afford to consume more or less than in the steady state, we need to compute the *net* effect. Analogous to the standard definition of income effect, we define the *wealth effect* as (minus) the compensation required for households to be able to afford their initial (steady-state) consumption bundle, which corresponds to the change in the households' wealth, $\hat{\widetilde{\Omega}}_0$, net of the change in the cost of the consumption bundle:

$$\Omega_0 \equiv \hat{\widetilde{\Omega}}_0 + \beta \sum_{t=0}^{\infty} \beta^t \left(i_t - \pi_{t+1}\right).$$

Using that the government's budget constraint in the steady state satisfies $\frac{T}{Y} = \frac{1-\beta}{\beta} Qb$, and setting $b_0 = b$ for simplicity, we get[18]

$$\Omega_0 = (1-\beta)\left[\sum_{t=0}^{\infty} \beta^t \left[y_t - \tau_t + (i_t - \pi_{t+1})\, Qb\right] - \left(\sum_{t=0}^{\infty} (\beta\rho)^t\, i_t \rho + \frac{1}{\beta}\pi_0\right) Qb\right]. \tag{6}$$

Thus, the *wealth effect* is the revaluation of real and financial assets *net* of the change in the cost of the steady-state consumption bundle. The wealth effect takes into account that monetary policy affects households directly through changes in wealth and indirectly through changes in the cost of future consumption. Therefore, the wealth effect is positive if and only if the household can afford to consume more than the steady-state consumption at every period. In the analysis that follows, Ω_0 will be the key object of interest.

3 The Decomposition: Substitution and Wealth Effects

We are interested in disentangling the effects of changes in nominal interest rates and fiscal transfers in response to a monetary shock. To answer this question, we must express output and inflation in terms of policy variables. We proceed in two steps. First, in this section, we solve the system (1)-(2) for output and inflation taking as given the path of nominal interest rates and the wealth effect, Ω_0. Second, in the next section, we use (3)-(4) to solve for the wealth effect in terms of policy variables, $\{i_t, \tau_t\}_0^\infty$, and then study the role of fiscal policy in the monetary transmission mechanism.

Consider first the solution to the system of difference equations (1)-(2) given a path for the nominal interest rate, $\{i_t\}_{t=0}^\infty$. The eigenvalues of the system are given by

$$\overline{\lambda} = \frac{1+\beta+\sigma\kappa+\sqrt{(1+\beta+\sigma\kappa)^2-4\beta}}{2\beta} > 1, \tag{7}$$

$$\underline{\lambda} = \frac{1+\beta+\sigma\kappa-\sqrt{(1+\beta+\sigma\kappa)^2-4\beta}}{2\beta} \in (0,1). \tag{8}$$

Note that the system has one eigenvalue outside the unit circle and one inside the unit circle. Focusing on bounded paths, we need one additional condition to determine the solution. A standard approach is to index all solutions of the system by the response of consumption or inflation in pe-

[18] Note that the dependence of the wealth effect on b_0 implies that even though Ricardian equivalence holds in this model, the equilibrium path of government debt can affect the response of the economy to *new* shocks. Since we study the economy's response to a one-time shock, we can ignore this consideration.

riod 0, that is, by the value of c_0 or π_0 (see Cochrane, 2017). More generally, one can use the value of consumption or inflation at any point in time, or a combination of different periods, as the extra boundary condition of the system. Here, we choose to index the solutions by the wealth effect, which, using equations (3) and (6) implies

$$(1-\beta)\sum_{t=0}^{\infty}\beta^t c_t = \Omega_0.$$

As we will see, choosing Ω_0 as the boundary condition allows us to uncover new properties of the New Keynesian model.

Consumption We are ready to present the main result of this section. The following proposition provides a characterization of the equilibrium path of consumption in *any* solution to the New Keynesian model for a given path of the nominal interest rate, $\{i_t\}_{t=0}^{\infty}$, and the wealth effect, Ω_0. It shows that consumption can be decomposed into the sum of a term that is *uniquely* determined by the path of the nominal interest rate and only affects the *timing* of consumption and a term that depends on the households' wealth effect.

Proposition 1 (Consumption Decomposition in General Equilibrium)**.** *Given an equilibrium path for the nominal interest rate, $\{i_t\}_{t=0}^{\infty}$, all bounded solutions of the system (1)-(2) generate a path of consumption that is given by*

$$c_t = \underbrace{c_t^S}_{ISE} + \underbrace{\frac{1-\beta\underline{\lambda}}{1-\beta}\underline{\lambda}^t}_{GE\ factor} \times \underbrace{\Omega_0}_{WE},$$

where

$$c_t^S \equiv \sigma\frac{1-\beta\underline{\lambda}}{\overline{\lambda}-\underline{\lambda}}\underline{\lambda}^t\left[\sum_{s=0}^{t-1}\left(\frac{\overline{\lambda}}{\underline{\lambda}^s}-\frac{\underline{\lambda}}{\overline{\lambda}^s}\right)i_s + \sum_{s=t}^{\infty}\left(\frac{1-\beta\overline{\lambda}}{1-\beta\underline{\lambda}}\left(\frac{\overline{\lambda}}{\underline{\lambda}}\right)^t - 1\right)\frac{\underline{\lambda}}{\overline{\lambda}^s}i_s\right]$$

is uniquely determined by the path of the nominal interest rate, $\{i_t\}_{t=0}^{\infty}$, and satisfies $\sum_{t=0}^{\infty}\beta^t c_t^S = 0$, and Ω_0 is given by (6).

Proof. Using the lag operator, we can rewrite equation (2) as $\pi_{t+1} = \frac{\kappa}{L-\beta}c_t$. Replacing into (1), we get $\left(1-L^{-1}\right)c_t = -\sigma i_t + \frac{\sigma\kappa}{L-\beta}c_t$, or $\left[1-(1+\beta+\sigma\kappa)L^{-1}+\beta L^{-2}\right]Lc_t = -\sigma(L-\beta)i_t$. Note that $1-(1+\beta+\sigma\kappa)L^{-1}+\beta L^{-2} = \beta\left(L^{-1}-\underline{\lambda}\right)\left(L^{-1}-\overline{\lambda}\right)$, where $\overline{\lambda}$ and $\underline{\lambda}$ are given by (7) and (8), respectively. Then, the general solution to this difference equation is

$$c_t = -\frac{1}{\beta}\frac{\sigma L(L-\beta)}{(1-\underline{\lambda}L)\left(1-\overline{\lambda}L\right)}i_t + a_1\underline{\lambda}^t + a_2\overline{\lambda}^t, \tag{9}$$

where a_1 and a_2 are two constants. Focusing on bounded solutions, we have that $a_2 = 0$. Note that since $\overline{\lambda} \neq \underline{\lambda}$, $\frac{1}{(1-\underline{\lambda}L)(1-\overline{\lambda}L)} = \frac{1}{\overline{\lambda}-\underline{\lambda}}\left(\frac{\overline{\lambda}}{1-\overline{\lambda}L} - \frac{\underline{\lambda}}{1-\underline{\lambda}L}\right)$. Hence, we can rewrite equation (9) as $c_t = -\frac{1}{\beta}\frac{\sigma L(L-\beta)}{\overline{\lambda}-\underline{\lambda}}\left(\frac{\overline{\lambda}}{1-\overline{\lambda}L} - \frac{\underline{\lambda}}{1-\underline{\lambda}L}\right) i_t + a_1\underline{\lambda}^t$. After some simple algebra, we get

$$c_t = \sigma\frac{1-\beta\underline{\lambda}}{\overline{\lambda}-\underline{\lambda}}\overline{\lambda}\underline{\lambda}^t\left(\sum_{s=0}^{t-1}\frac{1}{\underline{\lambda}^s}i_s - \frac{1-\underline{\lambda}}{\overline{\lambda}-1}\left(\frac{\overline{\lambda}}{\underline{\lambda}}\right)^t\sum_{s=t}^{\infty}\frac{1}{\overline{\lambda}^s}i_s\right) + a_1\underline{\lambda}^t. \tag{10}$$

Multiplying (10) by β^t and summing across time, we get $\sum_{t=0}^{\infty}\beta^t c_t = \sigma\frac{1}{\overline{\lambda}-\underline{\lambda}}\underline{\lambda}\sum_{s=0}^{\infty}\frac{1}{\overline{\lambda}^s}i_s + \frac{1}{1-\beta\underline{\lambda}}a_1$. Note that $\sum_{t=0}^{\infty}\beta^t c_t = \frac{1}{1-\beta}\Omega_0$. Then, $a_1 = \frac{1-\beta\underline{\lambda}}{1-\beta}\Omega_0 - \sigma\frac{1-\beta\underline{\lambda}}{\overline{\lambda}-\underline{\lambda}}\underline{\lambda}\sum_{s=0}^{\infty}\frac{1}{\overline{\lambda}^s}i_s$. Replacing into equation (10), we get the desired result. Moreover, note that $\sum_{t=0}^{\infty}\beta^t\frac{1-\beta\underline{\lambda}}{1-\beta}\underline{\lambda}^t\Omega_0 = \frac{1}{1-\beta}\Omega_0$, hence $\sum_{t=0}^{\infty}\beta^t c_t^S = 0$. $\square$

Proposition 1 shows that the equilibrium response of consumption to a monetary shock can be decomposed into two terms.[19] The first term corresponds to an *intertemporal substitution effect* (ISE). Because prices are sticky, a change in the nominal interest rate represents a change in the relative price of present versus future consumption. The households' response to this change in relative prices corresponds to a substitution effect: they change the *timing* of consumption while keeping the total cost of the bundle constant. Note that the ISE has a backward-looking and a forward-looking term. In particular, we have that $\frac{\partial c_0^S}{\partial i_0} = -\frac{\sigma}{\overline{\lambda}} < 0$ and $\frac{\partial c_t^S}{\partial i_0} = \sigma\left(1-\beta\underline{\lambda}\right)\underline{\lambda}^t > 0$ for all $t > 0$. Intuitively, an increase in the interest rate in period 0 always reduces the ISE in period 0 and increases it afterward. In B, we show that c_t^S corresponds to the Hicksian demand from microeconomic theory evaluated at the inflation rate *consistent* with the consumption plan $\{c_t^S\}_{t=0}^{\infty}$ according to the New Keynesian Phillips curve (2).[20] This result formalizes the sense in which the ISE can be interpreted as an intertemporal substitution channel. Moreover, given a path for the nominal interest rate, the ISE is unique.

The second term has two components: the *wealth effect* (WE) and a *general equilibrium (GE) factor*. The wealth effect captures the revaluation of the households' after-tax financial and human wealth after a change in the path of the nominal interest rate. In Section 4 we show that the wealth effect is tightly connected to the fiscal response associated with a monetary shock. As is common in representative-agent models, the permanent income hypothesis implies that households try to smooth any changes in their transitory income, which generates small changes in each period's consumption for standard-sized shocks. However, Proposition 1 shows that when prices

[19] In B.3, we compare the decomposition in Proposition 1 with the one found in Kaplan et al. (2018).

[20] The corresponding inflation rate is $\{\pi_t^S\}_{t=0}^{\infty}$ defined in Proposition 2 below.

are sticky, the impact of the wealth effect on consumption in period t is mediated by the GE factor. The intuition is as follows. When their wealth decreases, households reduce their consumption, which puts downward pressure on inflation. For a given equilibrium path of the nominal interest rate, the reduction in inflation increases the real interest rate, further contracting the economy in the initial periods. Since $\underline{\lambda} < 1$, the GE factor at $t = 0$ is greater than one. In fact, as we show below, the GE factor can be very large on impact. Naturally, since the households' budget constraint needs to be satisfied, this implies that the GE factor needs to be less than one in some periods. In particular, we have that the GE factor goes to zero as t goes to infinity. Thus, the GE factor shifts the wealth effect over time.

Inflation Proposition 1 presented a decomposition of consumption. There is a similar decomposition of inflation.

Proposition 2 (Inflation Decomposition)**.** *In the bounded solutions of the system* (1)-(2)*, inflation is given by*

$$\pi_t = \pi_t^S + \frac{\kappa}{1-\beta}\underline{\lambda}^t \times \Omega_0,$$

where

$$\pi_t^S \equiv \frac{\sigma\kappa}{\overline{\lambda} - \underline{\lambda}}\underline{\lambda}^t \left[\sum_{s=0}^{t-1}\left(\frac{\overline{\lambda}}{\underline{\lambda}^s} - \frac{\underline{\lambda}}{\overline{\lambda}^s}\right) i_s + \sum_{s=t}^{\infty}\left(\left(\frac{\overline{\lambda}}{\underline{\lambda}}\right)^t - 1\right)\frac{\underline{\lambda}}{\overline{\lambda}^s} i_s\right]$$

is uniquely determined by the path of the nominal interest rate, $\{i_t\}_{t=0}^{\infty}$, and satisfies $\frac{\partial \pi_t^S}{\partial i_s} > 0$ for all $t > 0$ and $s \geq 0$, and Ω_0 is given by (6)*. In $t = 0$,*

$$\pi_0 = \frac{\kappa}{1-\beta}\Omega_0.$$

Proof. Iterating the Phillips curve forward, we get $\pi_t = \kappa \sum_{s=0}^{\infty} \beta^s c_{t+s}$ Plugging in the expression for c_{t+s}, and after some algebra, we get the desired result. Note that $\frac{\partial \pi_t^S}{\partial i_s} = \frac{\sigma\kappa}{\overline{\lambda}-\underline{\lambda}}\underline{\lambda}^t\left(\frac{\overline{\lambda}}{\underline{\lambda}^s} - \frac{\underline{\lambda}}{\overline{\lambda}^s}\right) > 0$ if $s < t$, and $\frac{\partial \pi_t^S}{\partial i_s} = \frac{\sigma\kappa}{\overline{\lambda}-\underline{\lambda}}\underline{\lambda}^t\left(\left(\frac{\overline{\lambda}}{\underline{\lambda}}\right)^t - 1\right)\frac{\underline{\lambda}}{\overline{\lambda}^s} > 0$ if $s \geq t$. Finally, since $\sum_{s=0}^{\infty}\beta^s c_{t+s} = \frac{1}{1-\beta}\Omega_0$, it is immediate that $\pi_0 = \frac{\kappa}{1-\beta}\Omega_0$. □

Proposition 2 presents a decomposition of inflation analogous to the decomposition of consumption in Proposition 1. The first term, π_t^S, is the inflation rate consistent with the ISE (i.e., $\{c_t^S\}_{t=0}^{\infty}$), that is, the inflation rate one would obtain according to the NK Phillips Curve if $c_t = c_t^S$. The second term is proportional to the wealth effect by a factor $\frac{\kappa}{1-\beta}\underline{\lambda}^t$, which we label *GE factor (π)*

to distinguish it from the *GE factor* associated with consumption. The decomposition in Proposition 2 uncovers a novel result: inflation in period 0 is entirely determined by the wealth effect. In particular, initial inflation does not depend on the change in *initial* consumption but on whether the households' *lifetime* consumption is on average higher or lower after the shock. That is, initial inflation depends on whether households are richer or poorer rather than on the specific timing of the consumption path. To understand this result, it is helpful to note the forward-looking nature of the New Keynesian Phillips curve, which depends only on the present value of *future* consumption. Since the present value of the ISE is zero, initial inflation is determined solely by the wealth effect. In particular, the old-Keynesian idea that lowering consumption in a period is sufficient to lower inflation contemporaneously does not apply to this New Keynesian environment. Hence, in the absence of wealth effects, the monetary authority is unable to control initial inflation.

Moreover, inflation has *Neo-Fisherian* forces according to $\{\pi_t^S\}_{t=0}^{\infty}$, as an increase in nominal interest rates actually *raises* future inflation, $\frac{\partial \pi_t^S}{\partial i_s} > 0$, for $t > 0$ and any $s \geq 0$. Therefore, the inverse relationship between the nominal interest rate and inflation under the Taylor equilibrium is driven entirely by a negative wealth effect. In the absence of such wealth effects, not only does the monetary authority lose control of initial inflation, but the effect on future inflation has the opposite sign than in the standard result.

Notably, the decomposition in Propositions 1 and 2 provide new insights about the source of multiplicity in the New Keynesian model.

Corollary 1 (Multiplicity in the New Keynesian model)**.** *Given a path for the nominal interest rate, $\{i_t\}_{t=0}^{\infty}$, all bounded solutions of the system* (1)-(2) *generate the same ISE and GE factor. Thus, all bounded solutions to the New Keynesian model for a given path of the nominal interest rate can be indexed by Ω_0.*

Proof. Immediate from the fact that, given $\{i_t\}_{t=0}^{\infty}$, $\{c_t^S, \pi_t^S\}_{t=0}^{\infty}$ is unique, and $\frac{1-\beta\lambda}{1-\beta}\underline{\lambda}^t$ and $\frac{\kappa}{1-\beta}\underline{\lambda}^t$ depend only on the parameters of the model. $\square$

The decomposition in Propositions 1 and 2 characterize *all* the bounded solutions of the system (1)-(2) for a given path of the nominal interest rate. This result provides a new perspective on the multiplicity of equilibria of the New Keynesian model under an interest rate peg. The solutions of the model can be indexed by the level of wealth effect they generate, i.e. the extent of revaluation of households' financial assets and human wealth. In this sense, the standard Taylor rule equilibrium and the FTPL are ways of selecting a particular level of the wealth effect.[21]

[21] This interpretation is valid conditional on these regimes generating the same equilibrium path for the nominal

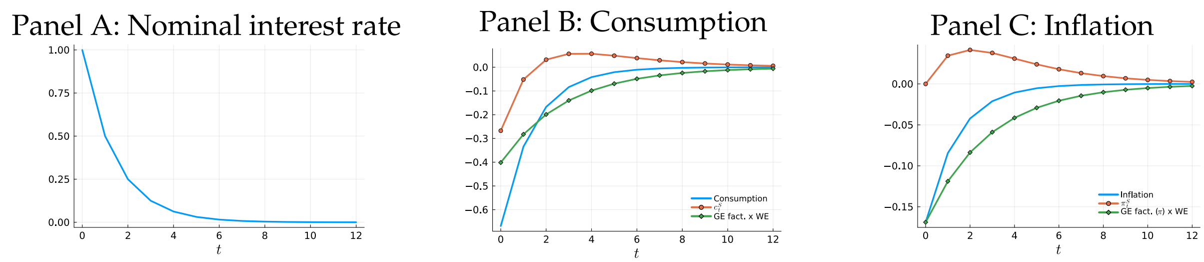


Figure 1: Decomposition of the consumption and inflation response to a monetary shock in the Taylor Equilibrium

Calibration: quarterly time period, $\beta = 0.99$, $\sigma = 1$, $\kappa = 0.1275$. The nominal interest rate follows $i_t = \rho_r^t i_0$, with $\rho_r = 0.5$ (which implies a half-life of the monetary shock of three months). We set i_0 to 25bps (100bps annualized).

To illustrate the relationship between the decompositions of Propositions 1 and 2 and the standard analysis with policy rules, we consider the Taylor equilibrium next.

The Taylor equilibrium. Consider an economy characterized by equations (1)-(2) and the interest rate rule (5) with $\kappa(\phi_\pi - 1) + (1-\beta)\phi_y > 0$. Moreover, suppose that $\{\varepsilon_t\}_{t=0}^{\infty}$ follows an AR(1) process $\varepsilon_t = \rho_r \varepsilon_{t-1} + u_t$ with $\rho_r \in (0, \underline{\lambda})$.[22] Consider an unexpected monetary shock in period 0, i.e. $u_0 \neq 0$ and $u_t = 0$ for all $t > 0$. We can guess and verify that the equilibrium takes the form $c_t^{Taylor} = -(1-\beta\rho_r)\Lambda i_t^{Taylor}$, $\pi_t^{Taylor} = -\kappa\Lambda i_t^{Taylor}$, and $i_t^{Taylor} = \rho_r^t \nu u_0$, where $\Lambda \equiv \frac{\sigma}{\beta(\overline{\lambda}-\rho_r)(\underline{\lambda}-\rho_r)} > 0$ and $\nu \equiv \frac{\beta(\overline{\lambda}-\rho_r)(\underline{\lambda}-\rho_r)}{(1-\beta\rho_r)\left[(1-\rho_r)+\sigma\phi_y\right]+\sigma\kappa(\phi_\pi-\rho_r)} > 0$. Then, by introducing the equilibrium path for the nominal interest rate, $\{i_t^{Taylor}\}_{t=0}^{\infty}$, into the expressions in Propositions 1 and 2, we get that

$$c_t^{S,Taylor} = \sigma \frac{(1-\beta\underline{\lambda})\underline{\lambda}^t - (1-\beta\rho_r)\rho_r^t}{\beta(\overline{\lambda}-\rho_r)(\underline{\lambda}-\rho_r)} i_0^{Taylor},$$

$$\pi_t^{S,Taylor} = \sigma\kappa \frac{\underline{\lambda}^t - \rho_r^t}{\beta(\overline{\lambda}-\rho_r)(\underline{\lambda}-\rho_r)} i_0^{Taylor},$$

and

$$\Omega_0^{Taylor} = -(1-\beta)\Lambda i_0^{Taylor}. \tag{11}$$

Note that in the Taylor equilibrium, an increase in nominal interest rates leads to a decline in consumption and inflation at all dates. Thus, the wealth effect has to be sufficiently negative to offset the increase in consumption embedded in the ISE.

interest rate. In more general exercises, the two regimes could potentially have different implications for the equilibrium path of the interest rate and, therefore, for the decomposition.

[22] Assuming that $\rho_r < \underline{\lambda}$ implies that a positive monetary shock generates an increase in the nominal interest rate.

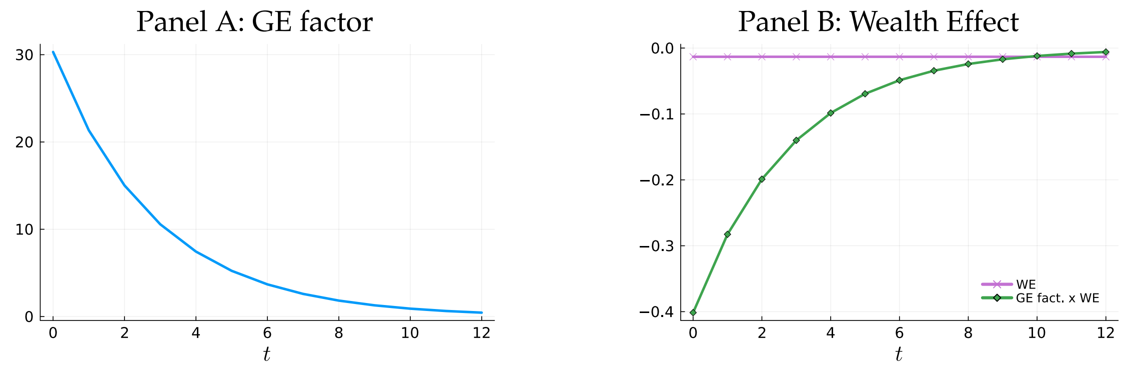


Figure 2: GE factor and the wealth effect in the Taylor Equilibrium

Calibration: quarterly time period, $\beta = 0.99$, $\sigma = 1$, $\kappa = 0.1275$. The nominal interest rate follows $i_t = \rho_r^t i_0$, with $\rho_r = 0.5$ (which implies a half-life of the monetary shock of three months). We set i_0 to 25bps (100bps annualized).

In order to get a sense of the quantitative importance of each component, we present a numerical example in Figure 1. In this section, we use a standard calibration found, for example, in Galí (2015). A crucial parameter is κ, the slope of the Phillips curve. We study the sensitivity of the results to alternative calibrations in Section 4. The solid lines represent the equilibrium paths of the nominal interest rate (Panel A), the households' consumption (Panel B), and inflation (Panel C). The interest rate follows an AR(1) process with an autoregressive coefficient of 0.5, implying a half-life of the monetary shock of 3 months. Panel B decomposes the equilibrium response of consumption into the components defined in Proposition 1. Both components of consumption are negative on impact. Regarding their contribution to the total response, the ISE accounts for 40% of the initial response, while the GE amplified wealth effect (i.e. the GE factor times the wealth effect) accounts for 60%. That is, even in the Taylor equilibrium of a RANK model, more than half of the economy's initial response to a monetary shock is explained by a term that depends on the wealth effect rather than the ISE. The role of the wealth effect becomes even starker when considering the inflation dynamics. Panel C shows that while the inflation rate decreases at all dates in the Taylor equilibrium, π_t^S is (weakly) positive at all horizons. It is then the wealth effect that generates the negative equilibrium response.

Thus, the wealth effect plays a crucial role in shaping the economy's response to monetary policy. However, this apparent importance may look contrary to the lessons of the permanent income hypothesis. After all, a short-lived monetary shock should have only a small effect on the households' wealth and, therefore, on their consumption. This logic is correct in partial equilibrium but not in general equilibrium. To see this, let us consider the GE amplified wealth effect in more detail. Figure 2 plots the dynamics of the GE factor (Panel A), and the wealth effect together with the GE amplified wealth effect (Panel B). The wealth effect alone only explains 2% of the consumption response in period 0. The small impact of the wealth effect on equilibrium consumption is consistent with the fact that households in the model conform to the permanent income hypothesis and the shock is transitory. However, the GE factor magnifies the wealth effect on impact to the point that the GE amplified wealth effect accounts for more than half of the total initial response of consumption. Notably, the baseline calibration generates a GE factor in period 0 equal to 30. These results show that the wealth effect can play a substantial role in the RANK model, though indirectly, through powerful endogenous amplification mechanisms. In the next section, we show that this observation has important implications for the role of fiscal policy in the monetary transmission mechanism.

4 The Fiscal Determination of the Wealth Effect

Recall that the wealth effect is given by

$$\Omega_0 = (1-\beta)\left[\sum_{t=0}^{\infty}\beta^t\left[y_t - \tau_t + (i_t - \pi_{t+1})\,Qb\right] - \left[\sum_{t=0}^{\infty}(\beta\rho)^t\, i_t\rho + \frac{1}{\beta}\pi_0\right]Qb\right]. \tag{12}$$

Note that market clearing implies that $\Omega_0 = (1-\beta)\sum_{t=0}^{\infty}\beta^t y_t$ and from Proposition 2 we have that $\pi_t = \pi_t^S + \frac{\kappa}{1-\beta\underline{\lambda}}\underline{\lambda}^t\Omega_0$, where $\left\{\pi_t^S\right\}_{t=0}^{\infty}$ is uniquely determined by the path of the nominal interest rate. Plugging these two relations into (12), we get

$$\Omega_0 = \left[1 - \left(\frac{1}{\beta} + \frac{\underline{\lambda}}{1-\beta\underline{\lambda}}\right)\kappa Qb\right]\Omega_0 + (1-\beta)\sum_{t=0}^{\infty}\beta^t\left[\left(i_t - \pi_{t+1}^S\right)Qb - \rho^{t+1}i_tQb - \tau_t\right]. \tag{13}$$

To determine the equilibrium value of Ω_0, thus, we need to consider two separate cases: *i*) monetary policy has no fiscal consequences, that is, $b = 0$; and *ii*) monetary policy has fiscal consequences, that is, $b > 0$. The equilibrium implications of the model are very different in these two cases.

Consider first the case $b = 0$. This is a knife-edge case and not the empirically relevant one, but it is still important to study as it is commonly assumed in the literature. Evaluating equation (13) at $b = 0$, we get

$$\Omega_0 = \Omega_0 - (1-\beta)\sum_{t=0}^{\infty}\beta^t\tau_t \implies \sum_{t=0}^{\infty}\beta^t\tau_t = 0.$$

In this case, the only restriction we get from the households' intertemporal budget constraint is that the present value of transfers must be zero. But beyond that, the households' budget constraint imposes no restrictions on the present value of consumption. In particular, consumption and the wealth effect have a self-fulfilling nature: if agents expect to receive a higher income, they increase their consumption accordingly, and since output is demand-determined, output increases to satisfy that demand. But since households' income equals the value of output, the increase in consumption becomes self-fulfilling. In the standard equilibrium selection, the Taylor rule pins down Ω_0 by imposing that only a specific path of inflation and output is consistent with a bounded equilibrium.

In contrast, the indeterminacy of the wealth effect disappears when monetary policy has fiscal consequences. As we move away from $b = 0$, the wealth effect can be characterized by the observed paths of policy variables.

Proposition 3 (Fiscal Determination of the Wealth Effect)**.** *Suppose $b > 0$. The wealth effect, Ω_0, is given by*

$$\Omega_0 = \frac{1-\beta}{\left(\frac{1}{\beta} + \frac{\underline{\lambda}}{1-\beta\underline{\lambda}}\right)\kappa Q b} \left[\sum_{t=0}^{\infty} \beta^t \left(\underline{\lambda}^{t+1} - \rho^{t+1} \right) i_t Q b - \sum_{t=0}^{\infty} \beta^t \tau_t \right]. \tag{14}$$

Thus, given $\{i_t, \tau_t\}_{t=0}^{\infty}$ and if $b > 0$, the equilibrium of the economy is unique.

Proof. Using the expression for π_t^S from Proposition 2, we get that $\sum_{t=0}^{\infty} \beta^t \pi_{t+1}^S = \sum_{t=0}^{\infty} \beta^t \left(1 - \underline{\lambda}^{t+1}\right) i_t$. Replacing this into equation (13) and using that $b > 0$, we get the desired result. Uniqueness follows from the fact that $\{c_t^S, \pi_t^S\}_{t=0}^{\infty}$ is unique given $\{i_t\}_{t=0}^{\infty}$, and Ω_0 is unique given $\{i_t, \tau_t\}_{t=0}^{\infty}$ if $b > 0$. □

Proposition 3 completes the characterization of equilibrium with an important insight. It establishes that we can express the wealth effect (i.e. the present value of the households' human and financial wealth net of the changes in the cost of consumption) as a function of *fiscal variables only*: government debt and lump-sum taxes.[23] Therefore, a contractionary monetary policy leads to a negative wealth effect only if followed by contractionary fiscal policy. In the absence of a fiscally induced wealth effect, so that $\Omega_0 = 0$, monetary policy can only shift demand over time. This has important implications for the determination of inflation, as Proposition 2 established that inflation drops after a contractionary monetary shock only if the wealth effect is negative.

[23] Note that while the nominal interest rate enters in the expression for Ω_0, it does so as the cost of servicing debt. Since this channel operates through the government's budget constraint, we label it as part of the fiscal policy channel.

Proposition 3 allows us to sharpen this result and conclude that inflation drops after a contractionary monetary shock only if *fiscal policy* is contractionary. This conclusion is often overlooked in standard analysis. Crucially, the characterization in Proposition 3 is relevant independently of whether fiscal policy is active or passive, and it helps understand the role of fiscal policy in the monetary transmission mechanism.

It may sound surprising that the wealth effect can be expressed in terms of fiscal variables in the Taylor equilibrium. After all, it is well-known that, in monetary-active regimes, fiscal policy is irrelevant to determine the economy's response to monetary policy as long as it is guaranteed that the government's intertemporal budget constraint is satisfied. The analysis here, however, does not contradict conventional wisdom. In the Taylor equilibrium, there exists a unique value of the wealth effect that is consistent with a bounded equilibrium. In fact, equation (11) expressed the wealth effect as a function of the nominal interest rate rather than fiscal variables. However, this analysis hides the tight connection between monetary and fiscal variables implied by the Taylor equilibrium. In particular, equation (14) allows us to recover the fiscal backing necessary to *sustain* such equilibrium. Rewriting (14), we obtain an expression for the fiscal transfers that are necessary to sustain a particular level of the wealth effect:

$$\sum_{t=0}^{\infty} \beta^t \tau_t = \sum_{t=0}^{\infty} \beta^t \left(\underline{\lambda}^{t+1} - \rho^{t+1} \right) i_t Q b - \frac{\left(\frac{1}{\beta} + \frac{\lambda}{1-\beta\underline{\lambda}} \right) \kappa Q b}{1-\beta} \Omega_0. \tag{15}$$

For example, the transfers in the Taylor equilibrium can be recovered by evaluating this expression at $\Omega_0 = \Omega_0^{Taylor}$ (see equation (11)), which gives us

$$\sum_{t=0}^{\infty} \beta^t \tau_t = \left[\frac{1}{\beta} \frac{1}{\overline{\lambda} - \rho_r} \left(1 + \frac{1}{\beta} \frac{\overline{\lambda}}{\overline{\lambda} - 1} \frac{\kappa\sigma}{\underline{\lambda} - \rho_r} \right) - \frac{\rho}{1 - \beta\rho\rho_r} \right] Q b i_0^{Taylor}.$$

That is, the Taylor equilibrium comprises a path for the nominal interest rate *and* a fiscal response associated with it. The fiscal determination of the wealth effect shows that it is the fiscal policy associated with this equilibrium selection rather than the path of the nominal rate that delivers the standard results of the monetary transmission mechanism.

To get a quantitative sense of the importance of fiscal policy in the Taylor equilibrium, Figure 3 Panel A plots $\sum_{t=0}^{\infty} \beta^t \tau_t$ as a function of the duration of government debt, while Panel B plots $\sum_{t=0}^{\infty} \beta^t \tau_t$ as a function of Qb.[24] For our calibration of the duration of government bonds, the

[24] Recall that the duration of a perpetuity in the steady state is given by $D = \frac{1}{1-\beta\rho}$. Thus, we can replace ρ by

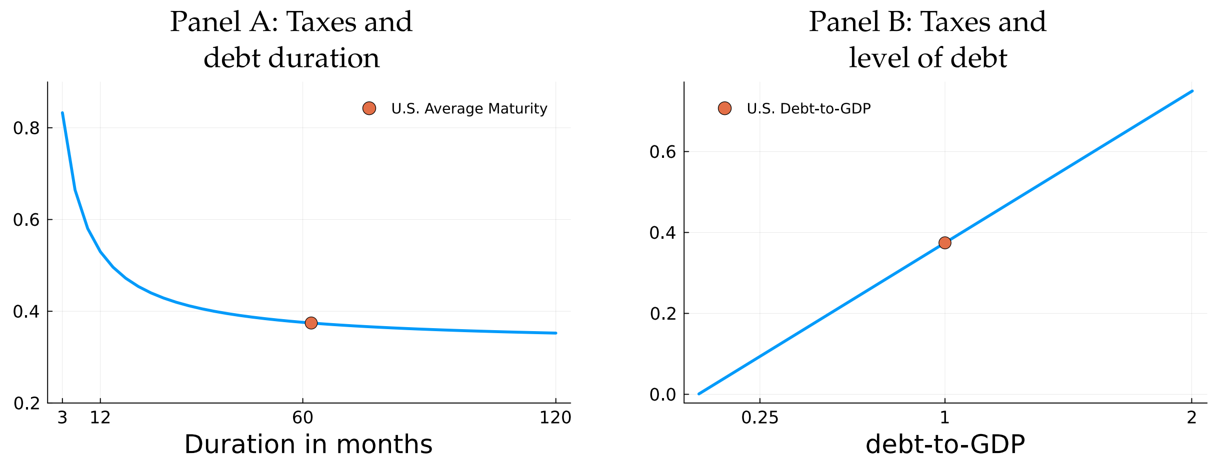


Figure 3: Fiscal backing in the Taylor Equilibrium

Calibration: quarterly time period, $\beta = 0.99$, $\sigma = 1$, $\kappa = 0.1275$. The nominal interest rate follows $i_t = \rho_r^t i_0$, with $\rho_r = 0.5$ (which implies a half-life of the monetary shock of three months). We set i_0 to 25bps (100bps annualized). The duration of government debt is set to 62 months (20.67 quarters), and debt-to-GDP (annual) is 1. Taxes are in percentage of annual steady-state level of output.

fiscal backing in the Taylor equilibrium is 0.37% of steady-state annual output, a considerable adjustment.[25] Panel A shows that taxes in the Taylor equilibrium decrease with the duration of government bonds. In particular, if government debt had a duration of one quarter, the transfers would need to be more than 120% larger to sustain the Taylor equilibrium. Finally, Panel B shows that the fiscal backing also depends on the level of government debt. If government debt were 25% of GDP (like at the beginning of the Volcker era), the fiscal backing necessary to sustain the Taylor equilibrium would be cut by 75%, to 0.09% of annual output. In contrast, it would increase by 100%, to 0.74% of annual output, if the debt-to-GDP ratio increased to 2, as projected by the CBO for 2051 (see Congressional Budget Office, 2021). These observations can prove helpful for the design of debt maturity management, and to understand potential tensions between the monetary and fiscal authorities as the debt-to-GDP ratio increases.

Next, we consider the final policy response of independent interest: the FTPL case.

Wealth effects in the "pure" FTPL equilibrium In the spirit of the canonical formulation of the FTPL, we consider the case in which the change in the path of the nominal interest rate does not affect the present value of the government's primary surpluses, i.e. $\sum_{t=0}^{\infty} \beta^t \tau_t = 0$. We label this

$\frac{1}{\beta}\left(1 - \frac{1}{D}\right)$ in equation (15) to get the fiscal backing as a function of the bond duration.

[25] The duration of government debt is set to the average maturity of the U.S. debt, which is 62 months. The value of ρ is relatively insensitive to changes in duration in the neighborhood of 62 months.

case the "pure" FTPL equilibrium. Then,

$$\Omega_0^{FTPL} = \frac{1-\beta}{\left(\frac{1}{\beta} + \frac{\underline{\lambda}}{1-\beta\underline{\lambda}}\right)\kappa} \sum_{t=0}^{\infty} \beta^t \left(\underline{\lambda}^{t+1} - \rho^{t+1}\right) i_t.$$

Thus, only government bonds generate wealth effects in this economy. Interestingly, the determination of Ω_0^{FTPL} features two opposing forces. An increase in the nominal interest rate leads to an increase in real rates, so households can reinvest their savings at higher rates after the monetary shock, generating a positive wealth effect. However, an increase in nominal interest rates also reduces the value of long-term government bonds, negatively affecting households' wealth. Which effect dominates depends on the duration of public debt. In particular, for a sufficiently long duration, the second effect prevails, and an increase in interest rates generates a *negative* wealth effect.[26]

Proposition 4 (FTPL and Long-Term Bonds)**.** *Suppose $b > 0$ and $\sum_{t=0}^{\infty} \beta^t \tau_t = 0$. Then,*

$$\frac{\partial \Omega_0}{\partial i_t} < 0 \iff \rho > \underline{\lambda}.$$

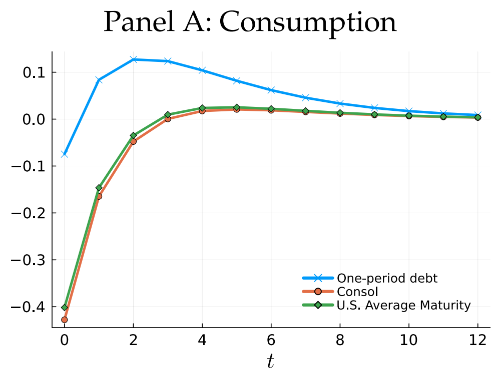
Proof. We have

$$\frac{\partial \Omega_0^{FTPL}}{\partial i_t} = \frac{1-\beta}{\left(\frac{1}{\beta} + \frac{\underline{\lambda}}{1-\beta\underline{\lambda}}\right)\kappa} \beta^t \left(\underline{\lambda}^{t+1} - \rho^{t+1}\right) < 0 \iff \rho > \underline{\lambda}.$$

□

To understand the relevance of Proposition 4, Figure 4 plots the response of consumption and inflation in the FTPL equilibrium for different durations of government bonds. Consider first one-period bonds. The wealth effect in response to an increase in the interest rate is positive. This positive wealth effect explains why consumption decreases only in the first quarter and increases afterward (Panel A). The result is even starker for inflation. A contractionary monetary policy shock uniformly *increases* inflation. Recall that, absent wealth effects, inflation has a strong Neo-Fisherian component. A positive wealth effect exacerbates this force to the extreme that a contractionary monetary shock that increases the nominal interest rate by 100 bps in $t = 0$ generates an increase in inflation of 32 bps. This phenomenon is related to the one emphasized by Loyo

[26]Notably, the *quantity* of government debt does not matter in this case. This result holds only when $\sum_{t=0}^{\infty} \beta^t \tau_t$ scales with Qb (including when $\sum_{t=0}^{\infty} \beta^t \tau_t = 0$).



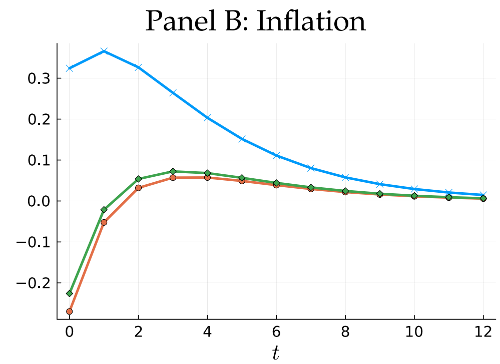


Figure 4: Consumption and inflation response to a monetary shock for various debt durations in the "pure" FTPL equilibrium.

Calibration: quarterly time period, $\beta = 0.99$, $\sigma = 1$, $\kappa = 0.1275$. The nominal interest rate follows $i_t = \rho_r^t i_0$, with $\rho_r = 0.5$ (which implies a half-life of the monetary shock of three months). We set i_0 to 25bps (100bps annualized). The duration of government debt is set to 62 months (20.67 quarters), and debt-to-GDP (annual) is 1.

(1999) as an explanation for hyperinflation spirals, where tight monetary policy generates higher inflation which triggers an even tighter monetary response, leading to a vicious cycle of higher nominal rates and higher inflation.[27]

The results change with long-term bonds. An increase in nominal interest rates reduces the value of government bonds, reducing households' wealth. If this effect is sufficiently strong, an increase in interest rates generates a *negative* wealth effect. This happens when the duration of government debt satisfies $\rho > \underline{\lambda}$, which in the baseline calibration corresponds to a duration longer than 10 months (recall that U.S. debt average maturity is 62 months). Figure 4 shows that consumption and inflation drop on impact in the calibrated duration of government debt. However, the negative wealth effect generated by government bonds does not overturn the Neo-Fisherian predictions after the first quarter, and inflation becomes positive until it converges back to zero in the limit. Note that combining the results in Proposition 2 and Proposition 3, it is immediate to see that, in the FTPL, $\pi_0 < 0$ if and only if the duration of government bonds is sufficiently long. That is, in the context of the standard New Keynesian model and absent any change in the present value of primary surpluses, only the maturity of debt can generate a negative co-movement between nominal rates and the inflation rate in period 0. Finally, Figure 4 shows that setting $\rho = 1$ (i.e. a consol) has only a marginal effect relative to the baseline calibration.

The idea that government liabilities are the relevant assets for assessing wealth effects is not

[27] An important distinction is that Loyo (1999) focused on an active fiscal regime with $\phi_\pi > 1$, which triggers the mentioned hyperinflation spirals, while we focus on a given equilibrium path of the nominal interest rate.

new. This observation was central to Pigou's argument in his response to Keynesian economics. For instance, Patinkin describes Pigou's argument as follows:[28]

> (...) the private sector considered in isolation is, on balance, neither debtor nor creditor, when in its relationship to the government, it *must be* a net "creditor." (...) If we assume that government activity is not affected by the movements of the price level, then the net effect of a price decline must always be stimulatory.

Two aspects of this quote are important. First, the idea that private assets cancel out in the aggregate, but households are on net creditors of the government. Second, the fact that it is assumed that "government activity is not affected" by the shock. The Pigou effect, as described here, is remarkably similar to the formulation of the "pure" FTPL equilibrium.

Moreover, the result in Proposition 4 echoes some of the findings in Sims (2011) and Cochrane (2018b) (see, also, Woodford, 2001). These papers highlight the difficulties of the standard RANK model in generating a negative co-movement between inflation and the nominal interest rate when primary surpluses do not react to the monetary shock. Sims (2011) builds a model with long-term government debt that can generate an initial drop in inflation after a contractionary monetary shock, but absent the appropriate contractionary fiscal backing, inflation eventually increases. This effect is present in our analysis, as reflected in Figure 4 Panel B. Cochrane (2018b) builds over Sim's model and shows that the result is robust to several modeling choices, but long-term debt is a necessary ingredient. Our findings sharpen these results in two dimensions. First, we show that absent a change in primary surpluses, *only a sufficiently long* bond maturity can generate a negative wealth effect, where the threshold is determined by the lowest eigenvalue of the system. Second, our results formalize that when monetary policy has fiscal consequences, a necessary condition for a drop of initial inflation to a contractionary monetary shock is either the presence of (sufficiently long) government debt or contractionary fiscal policy as summarized by the present value of primary surpluses. In particular, the RANK model has no alternative channels to generate a drop in inflation.

Wealth Effect, the GE factor, and price stickiness The previous analysis highlights two novel features of the New Keynesian model. First, it shows that while the direct impact of the wealth effect is small in RANK (consistent with the permanent income hypothesis), its general equilibrium effects can be significantly amplified, as reflected by a potentially large GE factor. Second,

[28] See Patinkin (1948).

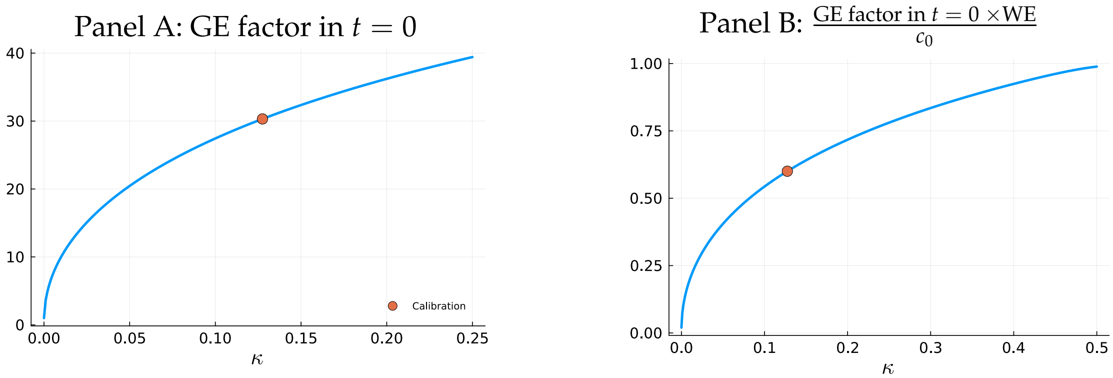


Figure 5: GE factor and the degree of price flexibility.

Calibration: quarterly time period, $\beta = 0.99$, $\sigma = 1$. The nominal interest rate follows $i_t = \rho_r^t i_0$, with $\rho_r = 0.5$ (which implies a half-life of the monetary shock of three months). We set i_0 to 25bps (100bps annualized). The solution in Panel B corresponds to the Taylor equilibrium.

the wealth effect can be characterized in terms of the fiscal response to the monetary shock. Thus, put together, these results imply that fiscal policy can play an essential role in the monetary transmission mechanism.

Here, we consider in more detail the properties of the GE factor. Recall that the GE factor captures the effect on consumption (and output) of changes in households' wealth that is mediated through inflation. When their wealth decreases, households reduce consumption putting downward pressure on prices and increasing the real interest rate, further reducing initial consumption. Under the baseline calibration of Section 2, we find a strong effect arising from this channel. We now show that this result is highly sensitive to the degree of price flexibility in the economy.

Figure 5 Panel A plots the GE factor in $t = 0$ as a function of κ, indicating the calibrated value from Section 2 as a reference. The GE factor is strictly increasing in the degree of price flexibility, achieving a minimum of 1 when prices are perfectly rigid and a maximum of $\frac{1}{1-\beta}$ in the flexible price limit (which is equal to 100 in our calibration). Panel B shows how the value of κ affects the fraction of the consumption response due to the GE amplified wealth effect in the Taylor equilibrium. For the calibration in Section 2, 60% of the total response of output is explained by the wealth effect. In contrast, with rigid prices, the GE amplified wealth effect explains only 2% of the consumption response, consistent with only the permanent income hypothesis being operative. More generally, the fraction increases with κ.

These results suggest that the importance of the fiscal backing in the RANK model depends significantly on the degree of price flexibility. Figure 6 plots the path of consumption after a mon-

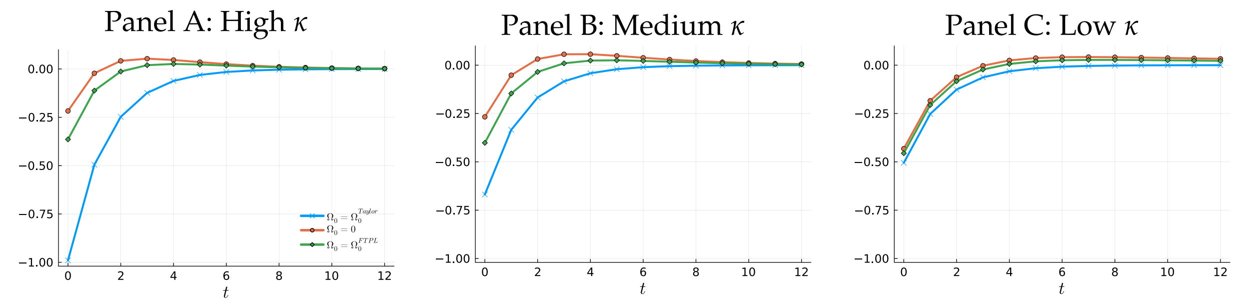


Figure 6: Consumption response to a monetary shock for various values of κ and Ω_0

Calibration: quarterly time period, $\beta = 0.99$, $\sigma = 1$, $\kappa \in \{0.25, 0.1275, 0.005\}$. The nominal interest rate follows $i_t = \rho_r^t i_0$, with $\rho_r = 0.5$ (which implies a half-life of the monetary shock of three months). We set i_0 to 25bps (100bps annualized). The duration of government debt is set to 62 months (20.67 quarters), and debt-to-GDP (annual) to 1.

etary shock and different values of κ and Ω_0.[29] Panel A plots the consumption path for different values of Ω_0 when κ is set to 0.25, which is approximately double the value in the calibration of Section 2. Panel B sets κ to the baseline calibration. Finally, Panel C presents the consumption response in a fairly rigid-price environment.[30]

A striking result emerges. While the wealth effect has a substantial impact on the consumption path when prices are relatively flexible, the effect is marginal for lower degrees of price flexibility. In Panel A ($\kappa = 0.25$), the fiscal backing represents 78% of the consumption response in period 0 (taking into account the GE amplification). In contrast, in Panel C ($\kappa = 0.005$), the fiscal backing represents less than 15% of the response. Thus, in the RANK model, monetary-fiscal interactions are particularly relevant in economies with a relatively high degree of price flexibility, while coordination is less relevant when prices are more rigid. Figure 7 shows analogous plots for inflation.

This finding may have important implications for the design of policies in economies that differ in their degree of price flexibility. In economies with a high degree of price flexibility, monetary-fiscal coordination might be a crucial element of an effective stabilization policy, and the monetary authority by itself might have limited power to affect the equilibrium. In contrast, in economies with relatively fixed prices, monetary-fiscal coordination might be secondary, and the monetary authority might be very effective in affecting aggregate variables. Of course, the degree of price flexibility is likely endogenous to the monetary-fiscal institutions. Still, this result suggests that

[29] To interpret the results for the Taylor equilibrium, note that the path of the nominal interest rate is set the same for all values of κ. We can achieve this result by adjusting the size of the initial shock, u_0. Thus, while the Taylor equilibrium features a consumption response to a *monetary shock*, u_0, that is decreasing in κ, it features a response to a *path of the nominal interest rate*, $\{i_t\}_{t=0}^{\infty}$, that is increasing in κ.

[30] We set $\kappa = 0.005$. Lower values of κ drastically change the properties of the FTPL equilibrium, as a monetary shock generates a positive wealth effect even with long-term debt (recall that we obtain $\frac{\partial \Omega_0^{FTPL}}{\partial i_t} < 0$ if and only if $\rho > \underline{\lambda}$, and $\underline{\lambda}$ is strictly increasing in κ, with $\underline{\lambda} = 1$ when $\kappa = 0$).

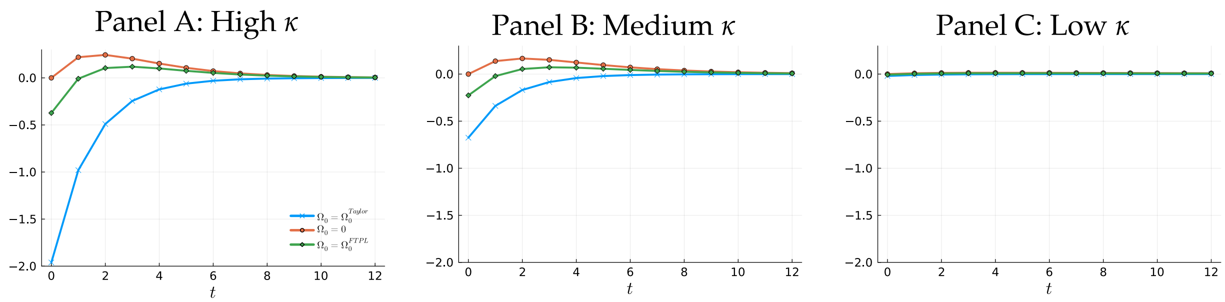


Figure 7: Inflation response to a monetary shock for various values of κ and Ω_0

Calibration: quarterly time period, $\beta = 0.99$, $\sigma = 1$, $\kappa \in \{0.005, 0.1275, 0.25\}$. The nominal interest rate follows $i_t = \rho_r^t i_0$, with $\rho_r = 0.5$ (which implies a half-life of the monetary shock of three months). We set i_0 to 25bps (100bps annualized). The duration of government debt is set to 62 months (20.67 quarters), and debt-to-GDP (annual) to 1.

when prices are more rigid, the monetary authority can approximate the outcomes of the Taylor equilibrium without the full backing of the fiscal authority.

The flexible price limit Before concluding this section, we consider the flexible price limit. This case is an important benchmark to evaluate the properties of the model and the implications of our analysis.

Proposition 5 (Flexible price limit)**.** *Given a path for policy variables, $\{i_t, \tau_t\}_{t=0}^{\infty}$, as $\kappa \to \infty$ all bounded solutions of the New Keynesian model converge to*

$$c_t = 0 \; \forall t \geq 0 \quad \textit{and} \quad \pi_t = \begin{cases} -\frac{1}{\frac{1}{\beta}Qb}\left[\sum_{t=0}^{\infty} (\beta\rho)^t \rho i_t Qb + \sum_{t=0}^{\infty} \beta^t \tau_t\right] & \textit{if } t = 0 \\ i_{t-1} & \textit{if } t > 0 \end{cases},$$

with $c_t^s = 0 \; \forall t \geq 0$*,* $\Omega_0 = 0$*,* $\pi_t^S = i_{t-1} \; \forall t \geq 0$*, and* $\kappa\Omega_0 = -\frac{1-\beta}{\frac{1}{\beta}Qb}\left[\sum_{t=0}^{\infty} (\beta\rho)^t \rho i_t Qb + \sum_{t=0}^{\infty} \beta^t \tau_t\right]$.

Proof. First, note that as $\kappa \to \infty$, $\overline{\lambda} \to \infty$, $\underline{\lambda} \to 0$, $\overline{\lambda}\underline{\lambda} = \frac{1}{\beta}$, $\kappa\underline{\lambda} = \frac{\kappa}{\beta\overline{\lambda}} \to \frac{1}{\sigma}$, and $\frac{\overline{\lambda}-\underline{\lambda}}{\kappa} \to \frac{\sigma}{\beta}$. Then, it is immediate to see that, given $\sum_{t=0}^{\infty} \beta^t \tau_t$, $\Omega_0 = \frac{(1-\beta)}{\left(\frac{1}{\beta}+\frac{\underline{\lambda}}{1-\beta\underline{\lambda}}\right)\kappa Qb}\left[\sum_{t=0}^{\infty} \beta^t \left(\underline{\lambda}^{t+1} - \rho^{t+1}\right) i_t Qb - \sum_{t=0}^{\infty} \beta^t \tau_t\right] \to 0$, and $\kappa\Omega_0 \to -\frac{1-\beta}{\frac{1}{\beta}Qb}\left[\sum_{t=0}^{\infty} (\beta\rho)^t \rho i_t Qb + \sum_{t=0}^{\infty} \beta^t \tau_t\right]$. For consumption, we have

$$c_t^S = \underbrace{\sigma \frac{\overline{\lambda} - 1}{\overline{\lambda} - \underline{\lambda}}}_{\to \sigma} \left[\underbrace{\sum_{s=0}^{t-1} \left(\underline{\lambda}^{t-s} - \frac{\underline{\lambda}^{t+1}}{\overline{\lambda}^{s+1}} \right) i_s}_{\to 0} + \underbrace{\sum_{s=t}^{\infty} \left(\frac{\underline{\lambda} - 1}{\overline{\lambda} - 1} \frac{1}{\overline{\lambda}^{s-t}} - \frac{\underline{\lambda}^{t+1}}{\overline{\lambda}^{s+1}} \right) i_s}_{\to 0} \right] \to 0,$$

and $\frac{1-\beta\lambda}{1-\beta}\underline{\lambda}^t\Omega_0 \to 0$, hence $c_t \to 0$. For inflation, we have

$$\pi_t^S = \underbrace{\frac{\sigma\kappa}{\overline{\lambda}-\underline{\lambda}}}_{\to\beta}\left[\underbrace{\left(\underline{\lambda}\overline{\lambda}-\frac{\underline{\lambda}^{t+1}}{\overline{\lambda}^{t-1}}\right)i_{t-1}}_{\frac{1}{\beta}i_{t-1}}+\underbrace{\sum_{s=0}^{t-2}\left(\underline{\lambda}^{t-s}\overline{\lambda}-\frac{\underline{\lambda}^{t+1}}{\overline{\lambda}^{s}}\right)i_s}_{\to 0}+\underbrace{\sum_{s=t}^{\infty}\left(\frac{\underline{\lambda}}{\overline{\lambda}^{s-t}}-\frac{\underline{\lambda}^{t+1}}{\overline{\lambda}^{s}}\right)i_s}_{\to 0}\right]\to i_{t-1}$$

and $\frac{\kappa}{1-\beta}\underline{\lambda}^t\Omega_0 \to \begin{cases} -\frac{1}{\frac{1}{\beta}Qb}\left[\sum_{t=0}^{\infty}\left(\beta\rho\right)^t\rho i_t Qb+\sum_{t=0}^{\infty}\beta^t T_t\right] & \text{if } t=0 \\ 0 & \text{if } t>0 \end{cases}$. Using that $i_{-1}=0$, we get the desired result. $\square$

As prices become more flexible, π_t^S tracks the interest rate more closely, so that $i_t - \pi_{t+1}^S \to 0$. Consequently, the ISE converges to zero, i.e. $c_t^S \to 0$. As stated above, the GE factor in period 0 increases with κ. As $\kappa \to \infty$, the GE factor reaches its maximum value of $\frac{1}{1-\beta}$. However, fixing fiscal policy, the wealth effect converges to zero, so $c_t \to 0$. Finally, note that π_0 is not pinned down by π_t^S, since we assume that $i_{-1}=0$. Instead, $\kappa\Omega_0$ converges to a finite value. That is, even though the wealth effect converges to zero as $\kappa \to \infty$, it still has an impact on initial inflation. Moreover, Proposition 5 shows that any equilibrium that does not converge to its flexible price counterpart must involve an associated fiscal response that diverges. This suggests that the behavior of fiscal policy may be relevant to understand some of the puzzles in New Keynesian models, such as the *paradox of flexibility* (see, e.g., Eggertsson and Krugman, 2012).

5 Extensions

Sections 3 and 4 considered in detail the equilibria in a simple RANK model. While useful, this model presents several limitations relative to the richer models currently used for policy analysis. This section extends the analysis in two dimensions. First, we solve a simple TANK model in the spirit of Bilbiie (2008). In this model, agents have different MPCs out of changes in their transitory income and monetary policy may have redistributive consequences. As a consequence, the individual-level wealth effects may differ. We consider the implications of these features on aggregate dynamics. Then, we extend our insights to a medium-scale DSGE New Keynesian model. In particular, we present some general analytical results and then show how to compute numerically the decomposition of the equilibrium described in the previous sections.

5.1 Household Heterogeneity: A TANK Model

We now extend the RANK model from Section 2 to incorporate household heterogeneity, in the spirit of Bilbiie (2008, 2019). The economy is populated by a continuum of measure one of households. A measure $1-\omega$ of households are *savers* (denoted by s): they are forward-looking and can trade in asset markets. The complementary fraction ω corresponds to households that are *hand-to-mouth* (HtM) (denoted by h): they have no access to financial markets and consume their labor income each period. We log-linearize the model around a symmetric zero-inflation steady state. We provide the details of the model in A.

Let c_t denote aggregate consumption, τ_t aggregate government taxes, and $\tau_{h,t}$ the taxes to HtM households. It turns out that the equilibrium of this model can be characterized by the same equations as the RANK model of Section 2, except that the Euler equation is replaced by the following *generalized* version:

$$c_{t+1} = c_t + \tilde{\sigma}(i_t - \pi_t) - v_t,$$

where $\tilde{\sigma} \equiv \frac{1-\omega}{1-\omega\chi_y}\sigma$ and $v_t \equiv \frac{\omega\chi_T}{1-\omega\chi_y}(\tau_{h,t+1} - \tau_{h,t})$, with $\chi_y, \chi_T > 0$. This equation differs from the standard Euler equation in two dimensions. First, the macro-EIS $\tilde{\sigma}$ can differ from the micro-EIS σ. The difference between the two is determined by χ_y, which denotes the cyclicality of HtM households' income. In particular, the macro-EIS is larger than the micro-EIS if and only if $\chi_y > 1$, echoing the result in Bilbiie (2019). Second, the Euler equation includes an additional term, v_t, which depends on the transfers to the HtM households. Note that v_t does not depend on the contemporaneous level of the transfer but on future changes. This feature will be important when we describe the channels of transmission below.

Notably, the equilibrium can be characterized by an *aggregate* intertemporal budget constraint, which is given by the sum of all the households' budget constraints and coincides with that of the representative household in the RANK model. In particular, let $\{c_{j,t}, n_{j,t}\}_{t=0}^{\infty}$ denote the consumption and labor, respectively, of an agent type $j \in \{s, h\}$, and $\{w_t, p_t\}_{t=0}^{\infty}$ denote the nominal wage and price level, respectively. Then, the flow budget constraint of the HtM agents is given by

$$c_{h,t} \leq \frac{WN}{PY}(w_t - p_t + n_{h,t}) - \tau_{h,t}, \tag{16}$$

while the flow budget constraint of the savers is given by

$$c_{s,t} + Qb_s\left(q_t + b_{s,t+1}\right) \leq \frac{1}{\beta}Qb\left(b_{s,t} + \beta\rho q_t - \pi_t\right) + \frac{WN}{PY}\left(w_t - p_t + n_{s,t}\right) + \frac{y_t - \frac{WN}{PY}\left(w_t - p_t + n_t\right)}{1-\omega} - \tau_{s,t}. \tag{17}$$

Multiplying (16) by ω and (17) by $1-\omega$, and adding up, we get

$$c_t + Qb\left(q_t + b_{t+1}\right) \leq \frac{1}{\beta}Qb\left(b_t + \beta\rho q_t - \pi_t\right) + y_t - \tau_t.$$

Multiplying by β^t, summing over time, and using the government's No-Ponzi condition and the savers' transversality condition, we get

$$\sum_{t=0}^{\infty}\beta^t c_t \leq \sum_{t=0}^{\infty}\beta^t\left[y_t - \tau_t + (i_t - \pi_{t+1})Qb\right] - \left[\sum_{t=0}^{\infty}(\beta\rho)^t i_t\rho + \frac{1}{\beta}\pi_0\right]Qb, \tag{18}$$

where we used that $\beta\rho q_{t+1} - q_t = i_t$ and $q_0 = -\sum_{t=0}^{\infty}(\beta\rho)^t i_t$. Note that (18) coincides with (3).

The following proposition extends the decomposition of Proposition 1 to this TANK model.

Proposition 6 (Consumption Decomposition in TANK)**.** *Given an equilibrium path for the policy variables, $\{i_t, \tau_{h,t}, \tau_{s,t}\}_{t=0}^{\infty}$, all bounded solutions of the TANK model generate a path of consumption that is given by*

$$c_t = \underbrace{c_t^S}_{\textit{ISE}} + \underbrace{c_t^T}_{\substack{\textit{Fiscal}\\ \textit{redistribution}}} + \underbrace{\frac{1-\beta\underline{\lambda}}{1-\beta}\underline{\lambda}^t}_{\textit{GE factor}} \times \underbrace{\Omega_0}_{\textit{WE}}$$

where

$$c_t^S = \tilde{\sigma}\frac{1-\beta\underline{\lambda}}{\overline{\lambda}-\underline{\lambda}}\underline{\lambda}^t\left[\sum_{s=0}^{t-1}\left(\frac{\overline{\lambda}}{\underline{\lambda}^s} - \frac{\underline{\lambda}}{\overline{\lambda}^s}\right)i_s + \sum_{s=t}^{\infty}\left(\frac{1-\beta\overline{\lambda}}{1-\beta\underline{\lambda}}\left(\frac{\overline{\lambda}}{\underline{\lambda}}\right)^t - 1\right)\frac{\underline{\lambda}}{\overline{\lambda}^s}i_s\right],$$

$$c_t^T \equiv -\frac{\omega\chi_T}{1-\omega\chi_y}\frac{1-\beta\underline{\lambda}}{\overline{\lambda}-\underline{\lambda}}\underline{\lambda}^t\left[\sum_{s=0}^{t-1}\left(\frac{\overline{\lambda}}{\underline{\lambda}^s} - \frac{\underline{\lambda}}{\overline{\lambda}^s}\right)(\tau_{h,s+1}-\tau_{h,s}) + \sum_{s=t}^{\infty}\left(\frac{1-\beta\overline{\lambda}}{1-\beta\underline{\lambda}}\left(\frac{\overline{\lambda}}{\underline{\lambda}}\right)^t - 1\right)\frac{\underline{\lambda}}{\overline{\lambda}^s}(\tau_{h,s+1}-\tau_{h,s})\right].$$

with $\sum_{t=0}^{\infty}\beta^t c_t^S = \sum_{t=0}^{\infty}\beta^t c_t^T = 0$, and

$$\Omega_0 = \frac{1-\beta}{\left(\frac{1}{\beta} + \frac{\lambda}{1-\beta\underline{\lambda}}\right)\kappa Qb}\sum_{t=0}^{\infty}\beta^t\left[\left(i_t - \pi_{t+1}^S - \pi_{t+1}^T\right)Qb - \rho^{t+1}i_tQb - \tau_t\right],$$

where $\pi_t^h \equiv \kappa\sum_{s=0}^{\infty}\beta^s c_{t+s}^h$ for $h \in \{S, T\}$. Moreover, $\pi_0 = \frac{\kappa}{1-\beta}\Omega_0$.

Proof. Let $\iota_t \equiv i_t - \tilde{\sigma}^{-1}\frac{\omega\chi_T}{1-\omega\chi_y}(\tau_{h,t+1} - \tau_{h,t})$. Then, the solution of the system is analogous to that of Propositions 1 and 2, with $\tilde{\sigma}$ replacing σ, and $\{\iota_t\}_{t=0}^{\infty}$ replacing $\{i_t\}_{t=0}^{\infty}$. Using the definition of ι_t and distributing terms leads to the desired result. □

Proposition 6 presents a channel of transmission absent in the RANK model: a fiscal redistribution channel. The formula clarifies how the redistribution channel operates. First, it shows that fiscal redistribution does not affect the present value of aggregate consumption but only its timing. Second, it shows that only the *growth rate* of the transfers to HtM households affects aggregate demand, not their levels. The reason for these results is that both types of agents have an MPC of 1 to changes in their *permanent* income. Thus, any redistribution that is perfectly smooth over time will only affect the distribution of consumption but not the aggregate level.

Moreover, Proposition 6 also shows the robustness of the results in Sections 3 and 4. Absent any fiscal redistribution effect (i.e. $c_t^T = 0$ for all t), the TANK model can be represented as a RANK model with a different EIS. Put differently, in the TANK model, the EIS and the cyclicality of HtM income contribute to the same *macro* channel of transmission. This also implies that while heterogeneous agents models have the ability to amplify the response of the economy to a monetary shock, the tight connection between the wealth effect and fiscal policy is not affected. Even if at the microeconomic level the channels of transmission are different than in RANK, at the macroeconomic level the economic forces are similar. The difference is that the TANK model can rely less on a counterfactually large calibration of the micro-EIS and more on household heterogeneity to generate a meaningful output response to monetary policy. We conjecture that this result is robust to richer sources of heterogeneity, as in quantitative HANK models. What matters is the aggregate intertemporal budget constraint, which takes the private sector as a whole in relation to the government.[31]

Our fiscal determination of the wealth effect bears some similarities to the results of Auclert et al. (2018), who study fiscal policy in a HANK model. They find that the output response to fiscal policy depends on so-called intertemporal marginal propensities to consume, or *iMPCs*, the derivative of aggregate consumption with respect to disposable income in a given date. Crucially, they focus on the standard equilibrium selection in a regime with a *constant real interest rate*. Their framework allows them to use the iMPCs to provide a simple formula that captures the role of the fiscal redistribution and wealth effects on aggregate consumption in a large class of models. In

[31] Caramp and Silva (2023) extend these results to a setting with private debt and aggregate risk and show that a version of Proposition 3 holds.

contrast, we focus on a setting with limited heterogeneity, but we allow for more flexible dynamics of policy variables and the inflation rate. Our setting allows us to characterize the response of aggregate consumption in terms of the aggregate wealth effect, which has a tight connection to fiscal variables, and the GE factor, which replaces the iMPCs as the key object mediating the impact of fiscal policy on aggregate output, a dimension that is absent when the real interest rate is constant.

5.2 A Medium-Scale DSGE Model

As a final exercise, we extend our decomposition to a medium-scale DSGE New Keynesian model.

The Smets & Wouters model We augment the model in Smets and Wouters (2007) to explicitly account for fiscal variables. Time is discrete and denoted by $t = 0, 1, 2, \ldots, \infty$. The economy is populated by a continuum of mass one of infinitely-lived households. Households derive utility from the consumption of a final good and leisure. Their preference for consumption exhibits an external habit variable. Labor supply is differentiated across households. Wages for each type of labor are negotiated by a union, which chooses the wage but is subject to nominal rigidities à la Calvo. Households are the owners of the capital of the economy. They rent capital services to the firms, which are a function of the capital stock they hold and the utilization level they choose. A higher utilization level comes at the cost of higher depreciation. Households also decide how much capital to accumulate given the adjustment costs they face.

There are two types of firms in the economy. There is a continuum of intermediate goods producers, which transform labor and capital services into differentiated goods and set prices subject to a Calvo friction. Those wages and prices that cannot be re-optimized in a given period are indexed to past inflation. The second type of firm is a representative firm that produces the final consumption good using the intermediate goods as inputs and sells the output in competitive markets. Finally, there is a government that chooses a path for the nominal interest rate, lump-sum taxes, and debt. The reader can refer to C for the complete set of equations characterizing the equilibrium of the model.

Following the steps in Sections 3 and 4, we can uncover several properties of the economy's equilibrium, including the decomposition of consumption into a substitution and wealth effects. Let Z_t denote the vector of the (log-linearized) endogenous variables excluding the nominal interest rate. The dynamics of Z_t can be written in recursive form as

$$Z_{t+1} = AZ_t + bi_t, \tag{19}$$

where A is an N-dimensional matrix of coefficients and b is a $N \times 1$ vector.[32] Then, the characterization of the equilibrium given a path for the policy variables, $\{i_t, \tau_t\}_{t=0}^{\infty}$, is given by the system (19), and an intertemporal budget constraint

$$\sum_{t=0}^{\infty} \beta^t c_y c_t \leq \sum_{t=0}^{\infty} \beta^t \left[y_t - x_y x_t - \frac{r^k k_y}{\gamma} u_t - \tau_t + (i_t - \pi_{t+1}) Qb \right] - \left[\sum_{t=0}^{\infty} (\beta\rho)^t i_t \rho + \frac{1}{\beta}\pi_0 \right] Qb, \tag{20}$$

where x_t and u_t denote, respectively, the log-linear deviations of investment and capital utilization relative to their steady-state levels; c_y, x_y, and k_y denote, respectively, consumption, investment, and capital over output in the steady state; r^k denotes the steady-state rental rate of capital; and γ denotes the trend growth rate of the economy. We define the wealth effect in this economy as

$$\Omega_0 \equiv (1-\beta) \left[\sum_{t=0}^{\infty} \beta^t \left[y_t - x_y x_t - \frac{r^k k_y}{\gamma} u_t - \tau_t + (i_t - \pi_{t+1}) Qb \right] - \left[\sum_{t=0}^{\infty} (\beta\rho)^t i_t \rho + \frac{1}{\beta}\pi_0 \right] Qb \right]. \tag{21}$$

Note that this expression is similar to (6), with the difference that we now subtract the expenditures on investment and capital utilization.

Analytical decomposition We now decompose the equilibrium dynamics of consumption into substitution and wealth effects. In the analysis of Section 3, we were able to establish analytically that the system of difference equations characterizing the equilibrium was missing exactly one boundary condition, and we selected Ω_0 as the condition. While we cannot get general analytical results characterizing the eigenvalues of the matrix of coefficients A in the system (19), standard calibrations imply that there is one missing boundary condition. Thus, in what follows we make the following assumption.

Assumption 1 (Generalized Blanchard-Kahn condition)**.** *Let A denote the matrix of coefficients de-*

[32] Note that the system is represented as a system of first-order difference equations. If any of the equilibrium conditions are of higher order, they can be transformed into first-order difference equations by adding the appropriate auxiliary variables.

fined in (19). *Suppose A is diagonalizable and has the following associated eigendecomposition:*

$$A = \underbrace{\begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix}}_{V} \underbrace{\begin{bmatrix} \Lambda_{11} & \mathbf{0} \\ \mathbf{0} & \Lambda_{22} \end{bmatrix}}_{\Lambda} \underbrace{\begin{bmatrix} V^{11} & V^{12} \\ V^{21} & V^{22} \end{bmatrix}}_{V^{-1}} \tag{22}$$

where V is the matrix of eigenvectors of A, Λ is a diagonal matrix of eigenvalues, the elements of Λ_{11} are greater than 1*, and the elements of Λ_{22} are less than* 1*. Then,*

i. The number of eigenvalues of A with absolute value greater than one equals $j-1$, where j is the number of jump variables in the system.

ii. The $(j-1)\times(j-1)$ matrix V^{11} is invertible.

Assumption 1 coincides with the standard Blanchard-Kahn conditions if we treat consumption as a predetermined variable and the path of the nominal interest rate as exogenous. Given this assumption, we get the extension of Propositions 1, 2, and 3 to the DSGE model.

Proposition 7 (Decomposition in General Equilibrium (DSGE))**.** *Suppose Assumption 1 holds. Then, for all $z_t \in Z_t$,*

$$z_t = z_t^S + \sum_{k=1}^{p+1} \nu_{z,k}\lambda_k^t \times \Omega_0,$$

where z_t^S is a function of $\{i_t\}_{t=0}^{\infty}$ and is independent of Ω_0, p is the number of predetermined variables in the system, $\{\nu_{z,k}\}_{k=1}^{p+1}$ are fixed coefficients, and $\lambda_k < 1$, for $k = 1, \dots, p+1$, and

$$\Omega_0 = \frac{1}{\left(\frac{1}{\beta}\sum_{k=1}^{p+1}\nu_{\pi,k} + \frac{\sum_{k=1}^{p+1}\nu_{\pi,k}\lambda_k}{1-\beta\lambda_k}\right)Qb}\left[\sum_{t=0}^{\infty}\beta^t\left[(i_t - \pi_{t+1}^S)Qb - \sum_{t=0}^{\infty}\rho^{t+1}i_tQb - \tau_t\right] - \frac{1}{\beta}\pi_0^S Qb\right].$$

For consumption, $\sum_{t=0}^{\infty}\beta^t c_t^S = 0$.

Proof. Assume that there are j jump variables and p predetermined variables, such that $j + p = N$. We will denote by $Z_{J,t}$ the vector of jump variables **except** for consumption, and $Z_{P,t}$ the vector of predetermined variables **plus** consumption. Thus, $Z_{J,t}$ has dimension $j-1$ and $Z_{P,t}$ has dimension $p+1$, and $Z_t = \left[Z'_{J,t}, Z'_{P,t}\right]'$.

Adopting the change of coordinates $\tilde{Z}_t = V^{-1}Z_t$ and $\tilde{b} = V^{-1}b$, we can write the system in decoupled form $\tilde{Z}_{t+1} = \Lambda\tilde{Z}_t + \tilde{b}i_t$. Given the decomposition (22) and Assumption 1, we can write

$\tilde{Z}_{J,t} = -\Lambda_{11}^{-1}\tilde{b}_J i_t + \Lambda_{11}^{-1}\tilde{Z}_{J,t+1} \Rightarrow \tilde{Z}_{J,t} = -\sum_{k=1}^{\infty}\Lambda_{11}^{-k}\tilde{b}_J i_{t+k-1}$. Note that $\tilde{Z}_{J,t} = V^{11}Z_{J,t} + V^{12}Z_{P,t}$. Then,

$$Z_{J,t} = -(V^{11})^{-1}V^{12}Z_{P,t} - (V^{11})^{-1}\sum_{k=1}^{\infty}\Lambda_{11}^{-k}\tilde{b}_J i_{t+k-1}. \tag{23}$$

Similarly, we can write $\tilde{Z}_{P,t+1} = \Lambda_{22}\tilde{Z}_{P,t} + \tilde{b}_J i_t \Rightarrow \tilde{Z}_{P,t} = \Lambda_{22}^{t}\tilde{Z}_{P,0} + \sum_{k=0}^{t}\Lambda_{22}^{k}\tilde{b}_P i_{t-k-1}$. Note that $\tilde{Z}_{P,t} = V^{21}Z_{J,t} + V^{22}Z_{P,t}$. Then,

$$Z_{P,t} = V_{22}\Lambda_{22}^{t}V_{22}^{-1}Z_{P,0} + W_{P,t}, \tag{24}$$

where we used that $\left[V^{22} - V^{21}(V^{11})^{-1}V^{12}\right]^{-1} = V_{22}$ by the inverse of a partitioned matrix, and $W_{P,t}$ is a function of the sequence of nominal interest rates only:

$$W_{P,t} \equiv V_{22}V^{21}(V^{11})^{-1}\sum_{k=1}^{\infty}\Lambda_{11}^{-k}\tilde{b}_J i_{t+k-1} + V_{22}\sum_{k=0}^{t}\Lambda_{22}^{k}\tilde{b}_P i_{t-k-1} - V_{22}\Lambda_{22}^{t}V^{21}(V^{11})^{-1}\sum_{k=1}^{\infty}\Lambda_{11}^{-k}\tilde{b}_J i_{k-1}.$$

Using the fact that the initial condition for all predetermined variables is equal to zero, we can write $c_t = \sum_{k=1}^{p+1}\tilde{v}_k\lambda_k^t c_0 + W_{c,t}$, where λ_k is the $k-$th element in the diagonal of Λ_{22}, and $\tilde{v}_k$ are fixed coefficients. Averaging the previous equation over all t and writing c_0 in terms of Ω_0, we obtain $c_t = c_t^S + \sum_{k=1}^{p+1}v_k\lambda_k^t\Omega_0$, where c_t^S is a function of the sequence of nominal interest rates only, and v_k are fixed coefficients. Finally, using equations (23) and (24), we can express $Z_{J,t}$ as $Z_{J,t} = Z_{J,t}^S + V_{12}\Lambda_{22}^t V_{22}^{-1}Z_{P,0}$, where $Z_{J,t}^S \equiv -(V^{11})^{-1}\sum_{k=1}^{\infty}\Lambda_{11}^{-k}\tilde{b}_J i_{t+k-1} - (V^{11})^{-1}V^{12}W_{P,t}$ depends only on the path of the nominal interest rate. Noting that the elements of $Z_{P,0}$ are equal to zero except for the one corresponding to c_0, we can express all variables as the sum of a term that depends only on the path of the nominal interest rate and a term that depends on Ω_0. Finally, plugging these expressions into (21), we get the desired expression for Ω_0.

□

Proposition 7 extends the results in Propositions 1, 2, and 3 to the context of the richer DSGE model. It decomposes the equilibrium consumption into the same three components: the *ISE*, the *GE factor*, and the *wealth effect*. The ISE has properties analogous to those derived in the context of the simple New Keynesian model of Section 2. First, the present value of the ISE is equal to zero when evaluated using steady-state prices. Second, the ISE and the GE factor are independent of the wealth effect Ω_0, so the ISE is uniquely determined by the sequence of nominal interest rates. The GE factor in the DSGE model has richer dynamics than the factor obtained in the simple model. In particular, it can have hump-shaped dynamics, an important feature for matching the sluggish response of consumption observed in the data. Finally, the proposition establishes that despite the presence of capital and investment, the equilibrium determination of the wealth effect can be expressed in terms of fiscal policy. Note that π_0^S appears in the expression for Ω_0 because in the DSGE model it may be different than zero. In the numerical exercise below, we find that, after a contractionary monetary shock, π_0^S is positive and quantitatively small.

Numerical computation Proposition 7 shows that the analytical results in Sections 3 and 4 extend to a medium-scale NK model. Here, we present a numerical example. In what follows, we assume that monetary policy follows a standard Taylor rule, but extending the analysis to alternative policy rules is straightforward. In particular, we assume that monetary policy is set according to the rule

$$i_t = \rho_r i_{t-1} + (1-\rho_r)(\phi_\pi \pi_t + \phi_y y_t) + \phi_{\Delta y}(y_t - y_{t-1}) + \varepsilon_t,$$

where $\rho_r \in [0,1)$ and $\{\phi_\pi, \phi_y, \phi_{\Delta y}\}$ are chosen to guarantee the uniqueness of a bounded equilibrium

There are two options to compute the decomposition. One option extends the steps followed in Sections 3 and 4 to the DSGE setting. First, we compute the full equilibrium of the economy and extract the equilibrium path of the policy variables, $\{i_t\}_{t=0}$ and $\sum_{t=0}^{\infty} \beta^t \tau_t$ (note that Ricardian equivalence implies that the path $\{\tau_t\}_{t=0}^{\infty}$ is indeterminate). Then, we obtain (often numerically) the eigendecomposition of the matrix A. Finally, we plug these variables into the components of Proposition 7.

There is also a second option that bypasses the computation of the eigendecomposition and is straightforward to implement using standard software like Dynare. There are three steps:

i. Compute the full equilibrium of the economy (including the policy rules). Extract the equilibrium paths for the policy variables. In our example, $\{i_t, \tau_t\}_{t=0}^{\infty}$. Calculate $\Omega_0 = (1-\beta)\sum_{t=0}^{\infty} \beta^t c_y c_t$.

ii. Given $\{i_t\}_{t=0}^{\infty}$, compute the solution to the system (19) and the additional equation

$$c_t^S + \beta b_{t+1} = b_{t-1}.$$

This equation guarantees that $(1-\beta)\sum_{t=0}^{\infty} \beta^t c_y c_t^S = \Omega_0 = 0$. Denote the solution by $\{z_t^S\}$.

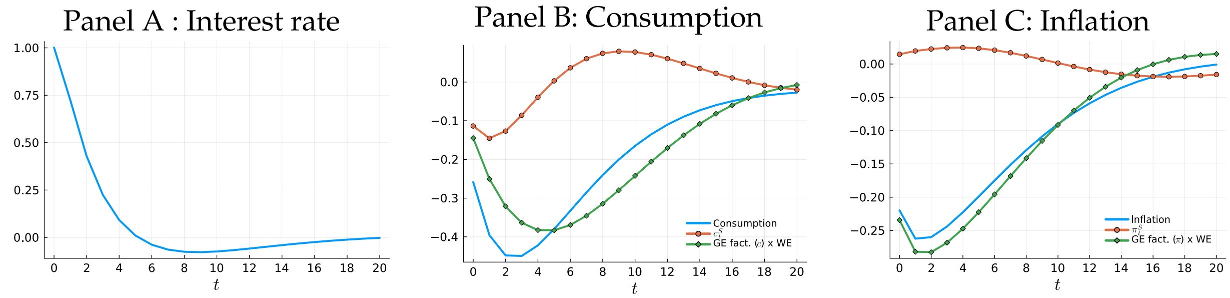


Figure 8: Inflation and consumption decomposition in the DSGE model

The model corresponds to the medium-scale NK model in Smets and Wouters (2007). Calibration: See C.

iii. Compute the GE amplification for each variable $z_t \in Z_t$ as

$$\text{GE fact.}(z) \times \Omega_0 = z_t - z_t^S.$$

Figures 8 and 9 depict the results. For the calibration of the model, we choose the mode of the estimation in Smets and Wouters (2007). Figure 8 Panel A shows the equilibrium path of the nominal interest rate after an exogenous monetary shock. We calibrated the monetary shock to generate a 100 bps (annualized) increase in the nominal interest in period 0. Panel B presents the consumption decomposition. The solid line represents the equilibrium response of consumption, which features the standard hump-shaped dynamics. A contractionary monetary shock generates an initial drop in consumption of 26 bps, and a peak response of 45 bps after 3 quarters. In terms of its components, we find that the ISE represents 45% of the consumption response in period 0, but less than 20% in the third quarter. Moreover, Panel C shows the decomposition for inflation. We find that $\pi_0^S > 0$, though small, while the wealth effect generates a strong negative response. These results reinforce our previous analysis: the economy reacts mildly to the change in the path of the nominal interest rate but substantially to the resulting change in the households' wealth.

Figure 9 presents the decomposition for additional variables as well as the implied fiscal response to the monetary shock. Consistent with the finding for consumption, the equilibrium response of output and investment are mainly driven by the wealth effect. Panel C presents the path of the primary surplus under the assumption of no change in the stock of debt in any period. The fiscal adjustments required to sustain the Taylor equilibrium are not small. On impact, the primary surplus needs to increase by 9 bps of steady-state output, and it remains positive for almost 20 quarters. Overall, the Taylor equilibrium in this model requires an increase in the present value

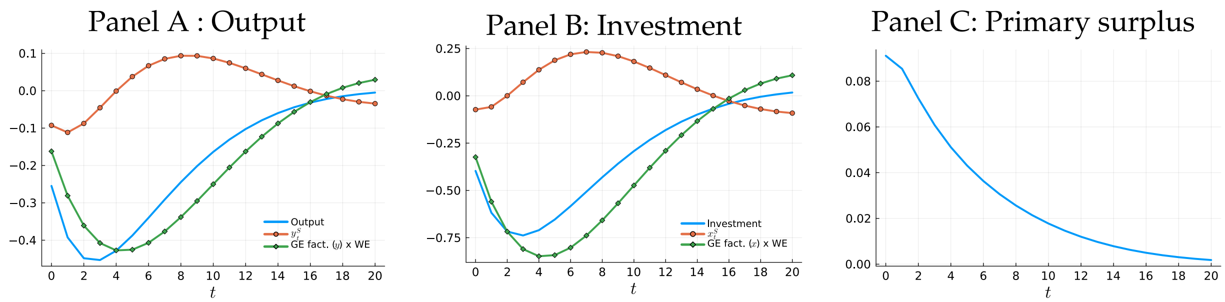


Figure 9: Inflation and consumption decomposition in the DSGE model

The model corresponds to the medium-scale NK model in Smets and Wouters (2007). Calibration: See C.

of the primary surplus of 61 bps of steady-state (annual) output.

6 Conclusion

Despite being often overlooked, the fiscal response to monetary policy is central to how the economy responds to monetary shocks. In this paper, we provided novel analytical tools to understand the role of fiscal policy and wealth effects in the monetary transmission mechanism. We presented a decomposition of the equilibrium response of consumption into an intertemporal substitution effect and a wealth effect. General equilibrium forces resulting from inflation dynamics can significantly amplify the impact of the wealth effect on households' consumption, even in a RANK model. Moreover, initial inflation is entirely determined by the wealth effect and not by the initial response of consumption. Crucially, when monetary policy has fiscal consequences, contractionary monetary policy reduces inflation only if followed by contractionary fiscal policy. These results highlight the importance of fiscal policy in the monetary transmission mechanism.

The analysis in the paper provides a comprehensive analysis of the role of fiscal policy in the monetary transmission mechanism. Future work should focus on applying these insights to identify and test the channels empirically. This task will require building models that incorporate realistic features absent in the models studied here. Caramp and Silva (2023) take a step in this direction by extending the analysis to a setting with aggregate risk and richer household heterogeneity.

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A Derivation of the New Keynesian model

This section derives a TANK model that nests the standard RANK model as a special case. Time is discrete and runs forever. The economy is populated by households, firms, and a government. Given our focus on the response of the linearized economy to an unexpected shock, we consider a perfect foresight equilibrium.

Households The economy is populated by a continuum of measure one of households. A measure $1-\omega$ of households are savers, indexed by s: they are forward-looking and can trade in asset markets. The complementary fraction ω corresponds to hand-to-mouth households (HtM), indexed by h: they have no access to financial markets and consume their labor income each period. The RANK model is a particular case in which $\omega = 0$.

Households receive labor income $W_t N_{j,t}$, profits from corporate holdings $\Pi_{j,t}$, and government transfers $P_t T_{j,t}$, for $j \in \{s, h\}$. We assume that corporations are owned by savers, so $\Pi_{h,t} = 0$ for all $t \geq 0$.

The problem of a saver is given by

$$\max_{\{C_{s,t},N_{s,t}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t U(C_{s,t}, N_{s,t})$$

subject to the flow budget constraint

$$P_t C_{s,t} + Q_t B_{s,t+1} \leq (1+\rho Q_t) B_{s,t} + W_t N_{s,t} + \Pi_{s,t} - P_t T_{s,t},$$

where the price of long-term bonds satisfies $Q_t = \frac{1+\rho Q_{t+1}}{1+i_t}$.

The saver's optimality conditions are given by

$$-\frac{U^n_{s,t}}{U^c_{s,t}} = \frac{W_t}{P_t}$$
$$1 = (1+i_t)\beta \frac{U^c_{s,t+1}}{U^c_{s,t}} \frac{P_t}{P_{t+1}},$$

where $U^c_{j,t} \equiv \frac{\partial U(C_{j,t},N_{j,t})}{\partial C_{j,t}}$ and $U^n_{j,t} \equiv \frac{\partial U(C_{j,t},N_{j,t})}{\partial N_{j,t}}$, and we used that $1+i_t = \frac{1+\rho Q_{t+1}}{Q_t}$.

The problem of a HtM household is

$$\max_{\{C_{h,t},N_{h,t}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t U(C_{h,t}, N_{h,t})$$

subject to

$$P_t C_{h,t} \leq W_t N_{h,t} - P_t T_{h,t}.$$

The HtM household's optimality condition is given by

$$-\frac{U^n_{h,t}}{U^c_{h,t}} = \frac{W_t}{P_t}.$$

In what follows, we assume that $U(C_{j,t}, N_{j,t}) = \frac{C_{j,t}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \frac{N_{j,t}^{1+\varphi}}{1+\varphi}$.

Firms There are two types of firms in the economy: final-goods producers and intermediate-goods producers. Final-goods producers operate in a perfectly competitive market and combine a unit mass of intermediate goods $Y_t(i)$, for $i \in [0,1]$, using the production function

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}}. \tag{25}$$

The problem of the final-good producer is given by

$$\max_{[Y_t(i)]_{i\in[0,1]}} P_t Y_t - \int_0^1 P_t(i) Y_t(i) di$$

subject to (25). The solution to this problem gives the standard CES demand

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} Y_t, \tag{26}$$

where $P_t \equiv \left(\int_0^1 P_t(i)^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}$.

Intermediate goods are produced using the following technology:

$$Y_t(i) = N_t(i)^{1-\gamma},$$

with $\gamma \in [0,1)$. Firms choose the price for their good, $P_t(i)$, subject to the demand for their good, given by (26), taking the aggregate price level P_t and aggregate output, Y_t, as given. As is standard in New Keynesian models, we assume that firms are subject to a pricing friction à la Calvo: each firm may set a new price with probability $1-\theta$ in each period. Let P_t^* denote the price chosen by a firm that is able to set the price in period t. Then, P_t^* is the solution to the following problem:

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} [P_t^* Y_{t+k|t} - \Psi_{t+k}(Y_{t+k|t})]$$

subject to

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon} Y_{t+k},$$

where $Q_{t,t+k} \equiv \beta^k U^c_{s,t+k}/U^c_{s,t} P_t/P_{t+k}$ is the savers' stochastic discount factor for nominal payoffs, $\Psi_t(Y_{t+k|t}) = W_{t+k} Y_{t+k|t}^{\frac{1}{1-\gamma}}$ is the cost function, and $Y_{t+k|t}$ denotes output in period $t+k$ for a firm that last set price in period t. The first-order condition associated with this problem is given by

$$\sum_{k=0}^{\infty} \theta^k Q_{t,t+k} Y_{t+k|t} \left[P_t^* - \frac{\epsilon}{\epsilon - 1} \Psi_t'(Y_{t+k|t}) \right] = 0.$$

Dividing this expression by P_t, we get

$$\sum_{k=0}^{\infty} \theta^k Q_{t,t+k} Y_{t+k|t} \left[\frac{P_t^*}{P_t} - \frac{\epsilon}{\epsilon - 1} MC_{t+k|t} \frac{P_{t+k}}{P_t} \right] = 0,$$

where $MC_{t+k|t} \equiv \Psi_t'(Y_{t+k|t})/P_{t+k}$ is the real marginal cost in period $t+k$ for a firm whose price was last set in period t.

Government We assume that the monetary authority follows an interest rate rule of the form

$$\log(1+i_t) = r_n + \phi_\pi \pi_t + \phi_y \log\left(\frac{Y_t}{Y}\right) + \varepsilon_{m,t},$$

where $r_n \equiv -\log\beta$, $\pi_t \equiv \log\left(\frac{P_t}{P_{t-1}}\right)$, Y is the zero-inflation steady-state level of output, and $\varepsilon_{m,t}$ denotes a monetary policy shock.

Moreover, the government chooses transfers to savers and HtM households, $\{T_{s,t}, T_{h,t}\}_{t=0}^{\infty}$ to satisfy the flow budget constraint

$$Q_t B_{t+1} = (1 + \rho Q_t) B_t - P_t(\omega T_{h,t} + (1-\omega) T_{s,t})$$

and the No-Ponzi condition $\lim_{t\to\infty} Q_t B_{t+1} = 0$.

Market clearing The market clearing conditions for goods, labor, and bonds are given by

$$\begin{aligned} \omega C_{h,t} + (1-\omega) C_{s,t} &= Y_t, \\ \omega N_{h,t} + (1-\omega) N_{s,t} &= N_t, \\ (1-\omega) B_{s,t} &= B_t, \end{aligned}$$

where $N_t = \int_0^1 N_t(i) di$ denotes the aggregate labor demand in period t.

Because of the Calvo friction, the price level can be written as

$$P_t = \left[(1-\theta)(P_t^*)^{1-\epsilon} + \int_{S(t)} (P_{t-1}(i))^{1-\epsilon} di \right]^{\frac{1}{1-\epsilon}},$$

where $S(t) \subset [0,1]$ is the set of firms that do not set a new price in period t. Since a random set of firms is able to change prices every period (independent of any firm characteristic), we have that

$$\int_{S(t)} (P_{t-1}(i))^{1-\epsilon} di = \theta P_{t-1}^{1-\epsilon}.$$

Hence, we can write the price level as

$$P_t = \left[(1-\theta)(P_t^*)^{1-\epsilon} + \theta P_{t-1}^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}.$$

Steady state Let the variables without subscript denote the value of the variables in a zero-inflation steady state.

Consumption of the HtM households is given by

$$C_h = \frac{W}{P} N_h - T_h.$$

Consumption of savers is given by

$$C_s = \frac{W}{P} N_s + \frac{Y - \frac{W}{P} N}{1-\omega} - T_s + \frac{1-\beta}{\beta} \frac{QB_s}{P},$$

where $B_s = \frac{B}{1-\omega}$, and $Q = \frac{\beta}{1-\beta\rho}$. Combining these two conditions, we obtain the government's budget constraint

$$\omega T_h + (1-\omega) T_s = \frac{1-\beta}{\beta} \frac{QB}{P}.$$

Optimal labor implies

$$\frac{W}{P} = N_j^{\varphi} C_j^{\frac{1}{\sigma}}$$

From the optimal pricing equation, we obtain

$$P = \frac{\epsilon}{\epsilon - 1} W \frac{Y^{\frac{\gamma}{1-\gamma}}}{1-\gamma},$$

with $Y = N^{1-\gamma}$. Note that

$$\frac{WN}{PY} = (1-\gamma) \frac{\epsilon - 1}{\epsilon}.$$

The distribution of consumption in steady state will depend on fiscal policy. Fix a steady state with a given value for (C_h, C_s) and government debt B. The required value of transfers that implement the given level of consumption are

$$T_h = \left(\frac{W}{P}\right)^{\frac{1+\varphi}{\varphi}} C_h^{-\frac{1}{\varphi\sigma}} - C_h,$$
$$T_s = \left(\frac{W}{P}\right)^{\frac{1+\varphi}{\varphi}} C_s^{-\frac{1}{\varphi\sigma}} + \frac{1+(\epsilon-1)\gamma}{\epsilon}\frac{1}{1-\omega}Y + \frac{1-\beta}{\beta}\frac{QB}{1-\omega} - C_s,$$

where $Y = \omega C_h + (1-\omega)C_s$.

Log-linearization As is standard, we study the dynamics of the economy around a steady-state equilibrium with zero inflation. For a variable X_t, let $x_t \equiv \log\left(\frac{X_t}{X}\right)$, where X denotes the zero-inflation steady-state value. We derive the equilibrium conditions for the general case where C_h may differ from C_s, and then specialize to the $C_h = C_s$ case considered in Section 5.1.

The log-linearized version of the savers' Euler equation is given by

$$c_{s,t+1} = c_{s,t} + \sigma^{-1}\left(i_t - \pi_{t+1}\right),$$

where we used that $\log(\beta(1+i_t)) \approx i_t$.

The labor supply condition can be written as

$$w_t - p_t = \varphi n_{j,t} + \sigma^{-1} c_{j,t}.$$

Log-linearizing the market clearing conditions for consumption and labor, we obtain

$$\omega_c c_{h,t} + (1-\omega_c)c_{s,t} = y_t, \quad \omega_n n_{h,t} + (1-\omega_n)n_{s,t} = n_t,$$

where $\omega_c \equiv \frac{\omega C_h}{Y}$ and $\omega_n \equiv \frac{\omega N_h}{N}$.

From the labor-supply condition, we obtain

$$\begin{aligned} n_{s,t} &= n_{h,t} + (\varphi\sigma)^{-1}(c_{h,t} - c_{s,t}) \\ &= n_{h,t} + (\varphi\sigma)^{-1}(1-\omega_c)^{-1}(c_{h,t} - y_t), \end{aligned}$$

using the market-clearing condition for goods to eliminate $c_{s,t}$. Plugging this expression into the market-clearing condition for labor, we obtain

$$n_{h,t} = \left(\frac{1}{1-\gamma} + (\varphi\sigma)^{-1}\right) y_t - (\varphi\sigma)^{-1} c_{h,t} + (\varphi\sigma)^{-1}\frac{\omega_c - \omega_n}{1-\omega_c}(y_t - c_{h,t}),$$

where we used that $n_t = \frac{1}{1-\gamma} y_t$. The real wage is then given by

$$w_t - p_t = \left(\frac{\varphi}{1-\gamma} + \sigma^{-1}\right) y_t + \sigma^{-1}\frac{\omega_c - \omega_n}{1-\omega_c}(y_t - c_{h,t}).$$

Linearizing the borrowers' budget constraint, we obtain

$$c_{h,t} = \frac{WN_b}{PC_b}(w_t - p_t + n_{h,t}) - \tau_{h,t},$$

where $\tau_{h,t} \equiv \frac{T_{h,t}-T_h}{C_h}$. Plugging the expressions for the real wage and labor supply into this expression, we obtain

$$c_{h,t} = \frac{WN_h}{PC_h}\left[\left(1+\varphi^{-1}\right)\left(\frac{\varphi}{1-\gamma}+\sigma^{-1}\right)y_t - (\varphi\sigma)^{-1}c_{h,t} + \left(1+\varphi^{-1}\right)\sigma^{-1}\frac{\omega_c-\omega_n}{1-\omega_c}\left(y_t - c_{h,t}\right)\right] - \tau_{h,t}.$$

Then,

$$c_{h,t} = \chi_y y_t - \chi_T \tau_{h,t},$$

where

$$\chi_y \equiv \frac{\frac{WN_h}{PC_h}\left[\left(1+\varphi^{-1}\right)\left(\frac{\varphi}{1-\gamma}+\sigma^{-1}\right) + \left(1+\varphi^{-1}\right)\sigma^{-1}\frac{\omega_c-\omega_n}{1-\omega_c}\right]}{1+\frac{WN_h}{PC_h}\left[(\varphi\sigma)^{-1} + \left(1+\varphi^{-1}\right)\sigma^{-1}\frac{\omega_c-\omega_n}{1-\omega_c}\right]}$$

$$\chi_T \equiv \frac{1}{1+\frac{WN_h}{PC_h}\left[(\varphi\sigma)^{-1} + \left(1+\varphi^{-1}\right)\sigma^{-1}\frac{\omega_c-\omega_n}{1-\omega_c}\right]}$$

The symmetric steady state case is obtained by imposing $C_h = C_s = Y$, so $\omega_c = \omega_n = \omega$, and $1-\alpha \equiv \frac{WN}{PY} = (1-\gamma)\frac{\epsilon-1}{\epsilon}$.

From the borrower's consumption and market clearing, we obtain

$$c_{s,t} = \frac{1-\omega_c\chi_y}{1-\omega_c}y_t + \frac{\omega_c\chi_T}{1-\omega_c}\tau_{h,t}.$$

Introducing this expression into the saver's Euler equation, we get

$$c_{t+1} = c_t + \tilde{\sigma}\left(i_t - \pi_{t+1}\right) - v_t,$$

where

$$\tilde{\sigma} \equiv \frac{1-\omega_c}{1-\omega_c\chi_y}\sigma, \quad v_t \equiv \frac{\omega_c\chi_T}{1-\omega_c\chi_y}\left(\tau_{h,t+1} - \tau_{h,t}\right).$$

The flow budget constraint for savers can be written as

$$c_{s,t} + \frac{QB_s}{PC_s}\left(q_t + b_{s,t+1}\right) \leq \frac{1}{\beta}\frac{QB_s}{PC_s}\left(b_{s,t} + \beta\rho q_t - \pi_t\right) + \frac{WN_s}{PC_s}\left(w_t - p_t + n_{s,t}\right) + \frac{y_t - \frac{WN}{PY}\left(w_t - p_t + n_t\right)}{1-\omega_c} - \tau_{s,t}.$$

By summing the flow budget constraint of savers multiplied by $1-\omega_c$ and HtM households multiplied by ω_c, we get

$$c_t + \frac{QB}{PY}\left(q_t + b_{t+1}\right) \leq \frac{1}{\beta}\frac{QB}{PY}\left(b_t + \beta\rho q_t - \pi_t\right) + y_t - \tau_t,$$

where $\tau_t = (1-\omega_c)\,\tau_{s,t} + \omega_c \tau_{h,t}$ and $b_t = b_{s,t}$. Thus, we obtain the intertemporal aggregate budget constraint of the households,

$$\sum_{t=0}^{\infty} \beta^t c_t \leq \sum_{t=0}^{\infty} \beta^t \left[y_t + \left(i_t - \pi_{t+1}\right)\frac{QB}{PY} - \tau_t\right] - \left[\sum_{t=0}^{\infty} (\beta\rho)^t\, i_t \rho + \frac{1}{\beta}\pi_0\right]\frac{QB}{PY},$$

which coincides with equation (3) in Section 2.

Now, consider the firms. The log-linear approximation of the intermediate-goods producers' first-order condition around the zero-inflation steady state yields

$$p_t^* - p_t = (1-\theta\beta)\sum_{k=0}^{\infty} (\theta\beta)^k \left(mc_{t+k|t} + p_{t+k} - p_t\right).$$

Approximating the expression for the marginal cost, we get

$$mc_{t+k|t} = w_{t+k} - p_{t+k} + \frac{\gamma}{1-\gamma} y_{t+k|t},$$

where

$$y_{t+k|t} = -\epsilon\left(p_t^* - p_{t+k}\right) + y_{t+k}.$$

Let mc_{t+k} denote the average marginal cost in the economy, which is given by

$$mc_{t+k} = w_{t+k} - p_{t+k} + \frac{\gamma}{1-\gamma} y_{t+k}.$$

Introducing the labor supply optimality condition, and using that $n_t = \frac{1}{1-\gamma} y_t$, we get

$$mc_{t+k} = \left(\sigma + \frac{\varphi+\gamma}{1-\gamma}\right) y_{t+k} + \sigma\frac{\omega_c - \omega_n}{1-\omega_c}\left(y_{t+k} - c_{h,t+k}\right).$$

Moreover, by approximating the price level equation, we get

$$p_t^* - p_t = \frac{\theta}{1-\theta}\pi_t.$$

Hence, we can write the firm's optimality condition as

$$\pi_t = \beta\pi_{t+1} + \left[\kappa y_t + \kappa_h (y_t - c_{h,t})\right],$$

where $\kappa \equiv \frac{(1-\theta)(1-\theta\beta)}{\theta}\frac{1-\gamma}{1-\gamma+\gamma\epsilon}\left(\sigma + \frac{\varphi+\gamma}{1-\gamma}\right)$ and $\kappa_h \equiv \frac{(1-\theta)(1-\theta\beta)}{\theta}\frac{1-\gamma}{1-\gamma+\gamma\epsilon}\sigma\frac{\omega_c-\omega_n}{1-\omega_c}$. Imposing a symmetric steady state, $\kappa_h = 0$.

B Hicksian Demand

This appendix presents an extension of the Slutsky equation of microeconomic theory to a general equilibrium setting. We begin by computing the Hicksian demand, i.e. the solution to the expenditure minimization problem subject to delivering a minimum level of utility. In this setting, the different goods are consumption at different dates, and the price of one unit of consumption at date t is $\prod_{s=0}^{t-1}\left(\frac{P_{s+1}/P_s}{1+i_s}\right)$. After that, we show that $\{c_t^S\}_{t=0}^{\infty}$ in the decomposition of Section 3 (see Proposition 1) can be reinterpreted as the (log-linearized) Hicksian demand evaluated at the inflation rate consistent with the Hicksian demand according to the New Keynesian Phillips curve. Finally, we compare the decomposition in this paper with one that looks at the direct and indirect effects of monetary policy, as in Kaplan, Moll, and Violante (2018).

B.1 Derivation of the Hicksian demand

The Hicksian demand of the non-linear model is obtained as the solution to the following problem:

$$\min_{\{C_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty}\prod_{s=0}^{t-1}\left(\frac{P_{s+1}/P_s}{1+i_s}\right)C_t$$

subject to

$$\sum_{t=0}^{\infty}\beta^t\frac{C_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \geq \overline{U},$$

for some $\overline{U} \in \mathbb{R}$. The FOCs of this problem are given by

$$\prod_{s=0}^{t-1}\left(\frac{P_{s+1}/P_s}{1+i_s}\right) = \lambda\beta^t C_t^{-\frac{1}{\sigma}},$$

where λ is the Lagrange multiplier associated to the constraint. This implies that

$$C_t = \prod_{s=0}^{t-1}\left(\frac{1+i_s}{P_{s+1}/P_s}\right)^{\sigma}\lambda^{\sigma}\beta^{\sigma t} \implies \sum_{t=0}^{\infty}\beta^t\frac{C_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} = \sum_{t=0}^{\infty}\beta^{\sigma t}\frac{\prod_{s=0}^{t-1}\left(\frac{1+i_s}{P_{s+1}/P_s}\right)^{\sigma-1}\lambda^{\sigma-1}}{1-\frac{1}{\sigma}} = \overline{U},$$

and hence

$$\lambda = \left[\frac{(1-\frac{1}{\sigma})\overline{U}}{\sum_{t=0}^{\infty}\beta^{\sigma t}\prod_{s=0}^{t-1}\left(\frac{1+i_s}{P_{s+1}/P_s}\right)^{\sigma-1}}\right]^{\frac{1}{\sigma-1}}.$$

Replacing in the FOC for C_t, we get

$$C_t^H = \frac{\beta^{\sigma t} \prod_{s=0}^{t-1} \left(\frac{1+i_s}{P_{s+1}/P_s} \right)^{\sigma}}{\left[\sum_{t=0}^{\infty} \beta^{\sigma t} \prod_{s=0}^{t-1} \left(\frac{1+i_s}{P_{s+1}/P_s} \right)^{\sigma-1} \right]^{\frac{\sigma}{\sigma-1}}} [(1-\frac{1}{\sigma})\overline{U}]^{\frac{\sigma}{\sigma-1}}.$$

Log-linearizing around the zero-inflation steady state, we get

$$c_t^H = \sigma \sum_{s=0}^{t-1} (i_s - \pi_{s+1}) - \sigma \sum_{s=0}^{\infty} \beta^{s+1} (i_s - \pi_{s+1}),$$

where we used that, in steady state, $C = [(1-\beta)(1-\frac{1}{\sigma})\overline{U}]^{\frac{\sigma}{\sigma-1}}$. The present value of the Hicksian demand is given by

$$\begin{aligned}
\sum_{t=0}^{\infty} \beta^t c_t^H &= \sum_{t=0}^{\infty} \beta^t \sigma \sum_{s=0}^{t-1} (i_s - \pi_{s+1}) - \sum_{t=0}^{\infty} \beta^t \sigma \sum_{s=0}^{\infty} \beta^{s+1} (i_s - \pi_{s+1}) \\
&= \sigma \sum_{s=0}^{\infty} \sum_{t=s+1}^{\infty} \beta^t (i_s - \pi_{s+1}) - \frac{1}{1-\beta} \sigma \sum_{s=0}^{\infty} \beta^{s+1} (i_s - \pi_{s+1}) \\
&= 0.
\end{aligned}$$

Moreover, note that

$$c_{t+1}^H = c_t^H + \sigma (i_t - \pi_{t+1}),$$

that is, the Hicksian demand satisfies the households' Euler equation.

B.2 The Intertemporal Substitution Effect in General Equilibrium

To find the inflation rate consistent with the Hicksian demand, we need to solve the following system of difference equations:

$$\begin{aligned}
c_{t+1}^H &= c_t^H + \sigma \left(i_t - \pi_{t+1}^H \right) \\
\pi_t^H &= \beta \pi_{t+1}^H + \kappa c_t^H,
\end{aligned}$$

with terminal condition

$$\sum_{t=0}^{\infty} \beta^t c_t^H = 0.$$

It should be straightforward that this system is equivalent to the system in Section 2 with the terminal condition $\Omega_0 = 0$. Thus, the solution is

$$c_t^H = c_t^S, \quad \pi_t^H = \pi_t^S.$$

B.3 An alternative consumption decomposition: direct and indirect effects

An alternative decomposition separates the response of equilibrium consumption into a direct effect of the real interest rate, keeping output and fiscal policy fixed, and an indirect effect that incorporates the changes in output and fiscal policy. Let $\tilde{c}_t^H$ denote the Hicksian demand in period t evaluated at the *equilibrium* path of the inflation rate.[33] Recall that Proposition 2 states that the equilibrium inflation rate satisfies

$$\pi_t = \pi_t^S + \frac{\kappa}{1-\beta}\underline{\lambda}^t \Omega_0.$$

Introducing this expression into the Hicksian demand, we get

$$\tilde{c}_t^H = \sigma \sum_{s=0}^{t-1} \left(i_s - \pi_{s+1}^S - \frac{\kappa}{1-\beta}\underline{\lambda}^{s+1}\Omega_0 \right) - \sigma \sum_{s=0}^{\infty} \beta^{s+1} \left(i_s - \pi_{s+1}^S - \frac{\kappa}{1-\beta}\underline{\lambda}^{s+1}\Omega_0 \right),$$

and after some algebra,

$$\tilde{c}_t^H = \underbrace{\sigma \sum_{s=0}^{t-1} \left(i_s - \pi_{s+1}^S \right) - \sigma \sum_{s=0}^{\infty} \beta^{s+1} \left(i_s - \pi_{s+1}^S \right)}_{=c_t^S} + \left(\frac{1-\underline{\lambda}\beta}{1-\beta}\underline{\lambda}^t - 1 \right) \Omega_0.$$

Hence,

$$c_t = \tilde{c}_t^H + \Omega_0.$$

Introducing the definition of Ω_0, we get

$$c_t = \underbrace{\tilde{c}_t^H + (1-\beta) \left[\sum_{t=0}^{\infty} \beta^t \left(i_t - \pi_{t+1} \right) - \sum_{t=0}^{\infty} (\beta\rho)^t \, i_t \rho - \frac{1}{\beta}\pi_0 \right] Qb}_{\text{direct effect}} + \underbrace{(1-\beta) \sum_{t=0}^{\infty} \beta^t \left(y_t - \tau_t \right)}_{\text{indirect effect}}.$$

This decomposition appears in, for example, Kaplan, Moll, and Violante (2018).

There are two main differences between this decomposition and the one proposed in Proposition 1. On the one hand, the direct effect includes the wealth effect arising from the holdings of government bonds (interest payments, revaluation of long-term bonds, and surprise inflation). On the other hand, and more importantly, the direct effect and the ISE can both be interpreted as a Hicksian demand but evaluated at different paths of the inflation rate. While the ISE is evaluated at the Hicksian-consistent inflation rate, the direct effect is evaluated at the equilibrium rate. This is the main distinction between the two approaches, and it reflects the different objectives pursued in both papers. Kaplan, Moll, and Violante (2018) are interested in understanding the *micro* channels of transmission, which justifies evaluating the households' Hicksian demand at

[33] In contrast, c_t^S is the Hicksian demand evaluated at the Hicksian inflation rate π_t^S.

the equilibrium inflation rate. Our focus is on *macro* channels, so distinguishing between the inflation rate arising from the ISE and the wealth effect is crucial. It is this feature that allows us to identify the importance of the wealth effect in the equilibrium dynamics of the economy and later connect it to the fiscal response to monetary policy. Notably, while neither the direct nor the indirect effect is uniquely determined by the path of the nominal interest rate, the ISE is. This feature uncovers new insights about the source of multiplicity in the New Keynesian model. Finally, note that both decompositions coincide when $b = 0$ and prices are fully rigid. In this case, there is no wealth effect arising from government bonds, and the general equilibrium factor is equal to 1 every period.

C Smets & Wouters (2007) Model

This Appendix presents the log-linearized system of equations of the model in Smets and Wouters (2007) augmented to incorporate fiscal variables. We focus on monetary shocks only. All variables are log-linearized around their steady-state balanced growth path. Starred variables denote steady-state variables.

The system of equations characterizing the equilibrium is given by

- the aggregate resource constraint

$$y_t = c_y c_t + x_y x_t + u_y u_t \tag{27}$$

where $c_y = \frac{c^*}{y^*}$, $x_y = \frac{x^*}{y^*}$, $u_y = \frac{r^{k*}k^*}{\gamma y^*}$, and γ is the growth rate of the economy. Output is denoted by y_t, consumption by c_t, investment by x_t, capital utilization by u_t, and the rental rate of capital by r_t^k.

- the consumption Euler equation

$$c_t = c_1 c_{t-1} + (1 - c_1) E_t c_{t+1} + c_2 (l_t - E_t l_{t+1}) - c_3 (i_t - E_t \pi_{t+1}) \tag{28}$$

where $c_1 = \frac{h/\gamma}{1+h/\gamma}$, $c_2 = \frac{\sigma_c - 1}{\sigma_c} \frac{W^{h*} L^*}{(1+h/\gamma)c^*}$, and $c_3 = \frac{1-h/\gamma}{(1+h/\gamma)\sigma_c}$, h is the external habit formation parameter, and σ_c is the (inverse) of the elasticity of intertemporal substitution. Labor is denoted by l_t, the nominal interest rate by i_t, and the inflation rate by π_t.

- the investment Euler equation

$$x_t = x_1 x_{t-1} + (1 - x_1) E_t x_{t+1} + x_2 q_t \tag{29}$$

where $x_1 = \frac{1}{1+\beta\gamma^{(1-\sigma_c)}}$, $x_2 = \frac{1}{(1+\beta\gamma^{(1-\sigma_c)})\gamma^2\varphi}$, φ is the steady-state elasticity of the capital adjustment cost function, and β is the discount factor applied by households.

- the Tobin's q

$$q_t = q_1 E_t q_{t+1} + (1 - q_1) E_t r^k_{t+1} - (i_t - E_t \pi_{t+1}) \tag{30}$$

where $q_1 = \beta\gamma^{-\sigma_c}(1-\delta) = \frac{1-\delta}{r^{k*}+(1-\delta)}$

- the aggregate production function

$$y_t = \phi_p \left(\alpha k^s_t + (1-\alpha) l_t\right) \tag{31}$$

where α is the share of capital in production and ϕ_p is one plus the share of fixed costs in production. k^s_t denotes capital services.

- current capital services used in production

$$k^s_t = k_{t-1} + u_t \tag{32}$$

where k_t is the stock of capital.

- optimal capital utilization

$$u_t = u_1 r^k_t \tag{33}$$

where $u_1 = \frac{1-\psi}{\psi}$ and ψ is a positive function of the elasticity of capital utilization adjustment function and normalized to be between zero and one

- the accumulation of installed capital

$$k_t = k_1 k_{t-1} + (1 - k_1) x_t \tag{34}$$

where $k_1 = \frac{1-\delta}{\gamma}$

- the price mark-up

$$\mu^p_t = -mc_t = \alpha\left(k^s_t - l_t\right) - w_t \tag{35}$$

- the NK Phillips curve

$$\pi_t = \pi_1 \pi_{t-1} + \pi_2 E_t \pi_{t+1} - \pi_3 \mu^p_t \tag{36}$$

where $\pi_1 = \frac{\iota_p}{1+\beta\gamma^{1-\sigma_c}\iota_p}$, $\pi_2 = \frac{\beta\gamma^{1-\sigma_c}}{1+\beta\gamma^{1-\sigma_c}\iota_p}$, $\pi_3 = \frac{1-\zeta_p}{\zeta_p}\frac{1-\zeta_p\beta\gamma^{1-\sigma_c}}{1+\iota_p\beta\gamma^{1-\sigma_c}}[1+(\phi_p-1)\varepsilon_p]$, ι_p is the degree of price indexation, and ζ_p is the degree of price stickiness

- the firms' cost minimization equation

$$r^k_t = -\left(k_t - l_t\right) + w_t \tag{37}$$

- the wage mark-up

$$\mu_t^w = w_t - mrs_t$$
$$= w_t - \left(\frac{c_t - \frac{h}{\gamma} c_{t-1}}{1 - \frac{h}{\gamma}} + \sigma_l l_t \right) \tag{38}$$

where σ_l is the elasticity of labor supply with respect to the real wage

- aggregate wage index

$$w_t = w_1 w_{t-1} + (1 - w_1)\left(E_t w_{t+1} + E_t \pi_{t+1}\right) - w_2 \pi_t + w_3 \pi_{t-1} - w_4 \mu_t^w \tag{39}$$

where $w_1 = \frac{1}{1+\beta\gamma^{1-\sigma_c}}$, $w_2 = \frac{1+\beta\gamma^{1-\sigma_c}\iota_w}{1+\beta\gamma^{1-\sigma_c}}$, $w_3 = \frac{\iota_w}{1+\beta\gamma^{1-\sigma_c}}$, and $w_4 = \frac{1-\beta\gamma^{1-\sigma_c}\xi_w}{1+\beta\gamma^{1-\sigma_c}} \frac{1-\xi_w}{\xi_w}[1 + (\lambda_w - 1)\varepsilon_w]$.

- the monetary policy reaction function

$$r_t = \rho r_{t-1} + (1 - \rho)\left\{ r_\pi \pi_t + r_Y y_t \right\} + r_{\Delta y}\left[y_t - y_{t-1} \right] + \varepsilon_t^r \tag{40}$$

$$\varepsilon_t^r = \rho_r \varepsilon_{t-1}^r + \eta_{r,t} \tag{41}$$

- the households' budget constraint

$$c_y c_t + i_y i_t + b_y \left(Q_t^L + b_t \right) = \frac{\rho_L}{1+\pi} \frac{1}{\gamma} b_y Q_t^L + \frac{R}{1+\pi} \frac{1}{\gamma} b_y \left(b_{t-1} - \pi_t \right) + y_t - \frac{1}{\gamma} \frac{r^k k}{y} u_t - s_t^y \tag{42}$$

where

$$q_t^L = \frac{\rho_L}{1+i} q_{t+1}^L - r_t \tag{43}$$

s_t^y is fiscal surplus over steady-state GDP, and $s^y = \frac{(1+i)-(1+\pi)\gamma}{(1+\pi)\gamma} b_y$.

We calibrate the model using the mode of the estimation in Smets and Wouters (2007). See Table 1. As in the paper, four parameters are fixed in the estimation: $\delta = 0.025$, $\lambda_w = 1.5$, $\varepsilon_p = \varepsilon_w = 10$.

Variables	Value
φ	5.48
σ_c	1.39
h	0.71
ξ_w	0.73
σ_l	1.92
ξ_p	0.65
ι_w	0.59
ι_p	0.22
ψ	0.54
ϕ_p	1.61
ρ	0.81
r_π	2.03
r_y	0.08
$r_{\Delta y}$	0.22
π	0.81
$100(\beta^{-1} - 1)$	0.16
γ	0.43
α	0.19
ρ_r	0.12

Table 1: Model calibration. Mode of the posterior distribution in Smets and Wouters (2007).