Bond Premium Cyclicality and Liquidity Traps*

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Abstract

Safe asset shortages can expose an economy to liquidity traps. The nature of these traps is determined by the cyclicality of the bond premium. A counter-cyclical bond premium opens the possibility of expectations-driven liquidity traps in which small issuances of government debt crowd out private debt and reduce output. In contrast, when the bond premium is pro-cyclical and the economy is in a liquidity trap, government debt is expansionary. In the data, we find evidence of a counter-cyclical bond premium. Large interventions can prevent the emergence of self-fulfilling traps, but they require sufficient fiscal capacity. In a quantitative model calibrated to the Great Recession, a promise to increase the government debt-to-GDP ratio by 20 percentage points precludes the possibility of self-fulfilling traps.

Keywords: bond premium, safe assets, liquidity trap

JEL Classification: E0, E1, E5, E32, E52

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1 Introduction

A prominent narrative about the origins of the Great Recession is that the crisis was triggered by a collapse of the supply of safe assets. A combination of factors, such as the sudden realization that mortgage-related assets (e.g., agency and private label MBSs and CDOs) could no longer be considered safe, the deterioration of the creditworthiness of several European countries, and a global flight-to-quality, generated a scarcity of safe assets that put downward pressure on short-term nominal rates and marked the beginning of the deepest recession in the post-war era.

With conventional monetary policy constrained by the zero lower bound (ZLB) on the short-term nominal interest rate, an important body of literature advocated for an increase in the supply of (safe) U.S. government bonds in order to compensate for the private safe asset shortage.[1] The idea is simple but powerful: if the crisis was caused by a drop in the supply of private safe assets, an increase in the supply of safe government bonds can at least partially offset the decline, stimulating the economy relative to *laissez-faire*. In this paper, we show that this policy prescription is not robust. Whether issuances of public safe assets are expansionary or not depends on the nature of the shock that hit the economy and the cyclical properties of asset prices.

We develop a theory of the macroeconomic consequences of safe asset scarcity. Our theory puts at the forefront the *bond premium*; that is, the premium households pay to hold assets that provide non-pecuniary benefits. When the bond premium is counter-cyclical, that is, there is a negative correlation between the bond premium and changes in aggregate output, the economy admits two steady-state equilibria.[2] One equilibrium features a positive nominal interest rate, output at potential, and full employment. The second equilibrium is a liquidity trap, with a zero nominal interest rate, high bond premium, and below-potential output. With pessimistic expectations about employment or safe-asset production, an economy can find itself transitioning to the liquidity trap equilibrium without any change in the fundamentals. We label this liquidity trap equilibrium a *self-fulfilling* liquidity trap (SFLT), following the seminal work of Benhabib, Schmitt-Grohé and Uribe (2001*b*). Importantly, we show that public provisions of safe assets may reduce welfare in such economies.

Our model has three main ingredients. First, households are willing to pay a premium for assets that provide safety or liquidity services. In the model, this willingness arises from workers' retirement concerns. Following the terminology in Gorton, Lewellen and Metrick

[1] See, e.g., Caballero and Farhi (2017); Caballero, Farhi and Gourinchas (2021, 2016); Kiyotaki and Moore (2019); Del Negro, Eggertsson, Ferrero and Kiyotaki (2017*b*).

[2] As we will formalize, it is the cyclicality of the bond premium *conditional* on safe asset demand and supply shifters, as well as the policy rate, that governs the nature of a liquidity trap.

(2012), we define assets with these properties as "safe assets." Second, the supply of safe assets is endogenous and varies with the state of the economy. Production is undertaken by firms that face a constraint on their capacity to issue safe debt. We assume that debt is safe if and only if it is free from roll-over risk, that is, if it can be fully repaid using internal funds. This safety constraint gives rise to a pro-cyclical supply of safe assets. Crucial for our results is that the stock of safe assets in the economy is insufficient to satisfy the households' demand. This scarcity gives rise to the premium. Third, the economy features nominal rigidities and a monetary policy that follows a Taylor-type interest rate rule subject to a ZLB constraint.[3] The rich set of interactions between the demand and the supply of assets and the non-linear interest rate rule allows us to obtain novel results about the nature of recessions and their implications for policy.

When the bond premium is counter-cyclical, expectations of low output imply a higher bond premium and a lower short-term nominal interest rate. If the bond premium is sufficiently high, the presence of the ZLB constrains the central bank in its ability to stabilize the economy, leading to a drop in employment and output, which justifies agents' pessimism. In contrast, with a pro-cyclical bond premium, expectations of low output imply a low bond premium and, hence, a high interest rate. However, according to the Taylor rule, high interest rates are not consistent with low levels of output, so SFLTs are not possible in this case. When the bond premium is pro-cyclical, only exogenous reductions in the natural rate of interest may lead the economy to a liquidity trap equilibrium. These liquidity trap equilibria that correspond to an exogenous reduction in the natural rate are often labeled as *fundamental liquidity traps* (FLTs), following Eggertsson and Woodford (2003). While both types of liquidity traps lead to similar outcomes in terms of output gap, unemployment, and bond premia, assessing the nature of the trap affecting the economy is crucial for a successful policy recommendation.

To determine the plausibility of SFLTs, we turn to the data and estimate the response of the bond premium to cyclical movements in GDP. We find strong evidence of a counter-cyclical bond premium for various measures of the premium and the cycle. Our baseline specification uses monthly data from 1973 to 2007. We use the Baa-Aaa corporate bond spread as the measure of the bond premium and the annual growth rate of the industrial production index to proxy for the cycle. Following recent work by Krishnamurthy and Vissing-Jorgensen (2012) and Nagel (2016), we control for the federal funds rate, the supply of Treasury bills, a measure of default risk, as well as for the VIX as a measure of uncertainty. Importantly, our measure of bond premium cyclicality is *conditional* on demand and supply

[3] Our results do not rely on the bound on the nominal interest rate to be at zero. As long as there is a high enough *effective lower bound* (ELB) that constrains the central bank, our results would go through.

shifters, as well as the policy rate, rather than an unconditional correlation between the bond premium and output. In the Online Appendix, we show that our results are robust to using various measures of the output gap and the bond premium. We also find that our results are robust to using an instrumental variables estimation strategy.

Our theoretical results also show that the cyclical properties of the bond premium depend on the relative elasticity of the demand and the supply of safe assets to aggregate output. In particular, we show that when the supply of safe assets is more (less) elastic to changes in output than the demand, the bond premium is counter-cyclical (pro-cyclical). Thus, we check the data to ascertain whether the counter-cyclicality of the bond premium is (at least in part) driven by a pro-cyclical safe asset supply. We measure total safe assets (public and private) following Gorton et al. (2012)'s classification of the U.S. financial accounts data. Cross-correlations of total safe assets and private safe assets and its sub-categories with various measures of the cycle provide suggestive evidence of a pro-cyclical supply of safe assets, justifying our modeling assumptions.

Finally, we analyze the effects of policy intervention. We show that the issuance of (safe) government debt in small quantities can be contractionary in SFLTs.[4] However, a sufficiently large increase in government debt can eliminate the SFLT. We interpret this result as a variation of Krugman (2014)'s *timidity trap*. Krugman coined the term "timidity trap" in the context of policy discussions around the Great Recession. He defined it as "(...) the consistent tendency of policymakers who have the right ideas in principle to go for half-measures in practice, and the way this timidity ends up backfiring, politically and even economically." In our setup, a small increase in government debt is contractionary in an SFLT. Instead, if the government were to *credibly* commit to implementing a sufficiently large-scale intervention, an SFLT scenario would cease to exist. Moreover, such a commitment would also improve the outcomes under an FLT. Thus, it is the implementation of *discrete* rather than *incremental* policies that can robustly lift the economy out of a slump. This finding might be of particular relevance for policymakers who might have difficulties in identifying in real time the exact nature of the liquidity trap they are facing, as the two types have similar observable dynamics of output, inflation, and private assets. We also show that increases in government spending can have similar effects but that they are dominated by bond issuances in terms of the fiscal costs they entail.

An important aspect of these policy interventions is that they need to be *credible*. Credibility requires sufficient *fiscal capacity* to take the necessary measures. As in He, Krishna-

[4] The intervention is contractionary if agents' expectations remain pessimistic. In contrast, it would be expansionary if it coordinates their expectations towards the full-employment equilibrium. Note, however, that the expansionary outcome is the result of the change in expectations rather than the intervention *per-se*.

murthy and Milbradt (2019), we constrain the government's ability to provide safe assets with a roll-over risk constraint. If the government cannot guarantee the safety of the bonds it is issuing, then the intervention will not have the desired effects. In a quantitative model calibrated to replicate a Great Recession scenario, we find that a commitment to increase government debt-to-GDP ratio by 20 percentage points eliminates the possibility of SFLTs.

Literature review. This paper is related to several strands of the literature. First, it is related to the literature on safe-asset shortages and credit market disruptions that lead to liquidity traps.[5] Caballero and Farhi (2017) and Caballero, Farhi and Gourinchas (2016) build models in which liquidity trap equilibria arise as the result of a shock that reduces the supply of safe assets and increases the bond premium. Similarly, Eggertsson and Krugman (2012) and Guerrieri and Lorenzoni (2017) focus on the role of debt-deleveraging in generating a liquidity trap; Curdia and Woodford (2011) and Del Negro et al. (2017*b*) formalize liquidity traps with financial market disruptions that cause credit spreads to rise. Fornaro and Romei (2019) formalize an international aggregate demand externality whereby macro-prudential policy interventions in some countries depress global aggregate demand when there is a global scarcity of safe assets. All these papers study FLTs, and the public provision of debt is expansionary and welfare improving (see also Gourinchas and Jeanne 2013). We contribute to this literature by connecting the type of a liquidity trap to the cyclical properties of the bond premium, and formalize a scenario in which a small increase in the provision of public debt reduces welfare.

In recent work, Acharya and Dogra (2021) study a liquidity trap episode in which increases in public debt crowd out capital investment but improve welfare. Mian, Straub and Sufi (2021) formalize a debt trap due to excessive debt in the economy and show that increases in public debt can reduce the level of output. In our setting, large enough provisions of debt can be expansionary even when small doses reduce welfare. See also Bacchetta, Benhima and Kalantzis (2020).

Our paper is also related to the literature examining expectations-driven liquidity traps. These papers show that the non-linearity of the Taylor rule can give rise to multiple steady states. One set of papers builds on the seminal work by Benhabib et al. (2001*b*) and characterizes the properties of expectations-driven liquidity traps that feature below-target inflation and below-potential output. Our paper is closest to Schmitt-Grohé and Uribe (2017), Nakata and Schmidt (forthcoming), and Bilbiie (forthcoming). Schmitt-Grohé and Uribe (2017) present a model in which a pessimistic confidence shock can generate a liquidity trap

[5]Bernanke (2005) and Caballero (2006) identified the role of a shortage of safe assets in global imbalances and capital flows.

featuring an initial fall in growth and a subsequent jobless recovery in which the output growth rate goes back to trend, but employment is permanently lower. Nakata and Schmidt (forthcoming) characterize temporary expectations-driven liquidity traps and study optimal policies. Bilbiie (forthcoming) considers FLTs and SFLTs in a unified framework. Our paper is complementary to the mechanisms emphasized in this literature. First, our focus is on economies that feature a state-contingent bond premium. In our framework, an SFLT can emerge even if wages are perfectly rigid. Second, in our setting, an SFLT is associated with a low (safe) real interest rate, consistent with the empirical evidence. In contrast, SFLTs driven by below-target inflation expectations are not associated with a drop in the real interest rate. Third, we find that the set of parameters consistent with an SFLT is increasing in the degree of wage flexibility, and that the policy interventions necessary to preclude the existence of SFLTs are increasing in the degree of wage flexibility.

A second set of expectations-driven liquidity trap papers considers endogenous productivity growth or unemployment risk instead of inflation pessimism. Benigno and Fornaro (2018) study a New Keynesian model featuring endogenous growth in which an expectations shock can permanently reduce the growth rate of the economy. Benigno and Fornaro (2018) share our finding that sufficiently large interventions can rule out self-fulfilling traps. Heathcote and Perri (2018) formalize the susceptibility of an economy to an expectations-driven trap due to a counter-cyclical demand for liquidity.[6] Our contribution is to identify the cyclicality of the bond premium as a crucial variable determining the type of recession an economy is exposed to, and consider the consequences for policy intervention. In particular, we show that discrete interventions can be robust to the type of trap and analyze the implications for the government's fiscal capacity.[7]

Finally, our paper builds on the recent literature documenting the properties of the convenience yield of various safe assets (Krishnamurthy and Vissing-Jorgensen, 2015; Nagel, 2016; Del Negro, Giannone, Giannoni and Tambalotti, 2017*a*). Like us, Jiang, Krishnamurthy and Lustig (2019) model pro-cyclical asset supply to analyze the global implications of U.S. dollar-denominated assets. We show that bond premium, controlling for policy rate and shifters to safe asset demand and supply, is counter-cyclical. The cyclicality of the bond-premium, we show, has important implications for theories of safe asset scarcity.

The paper proceeds as follows. Section 2 presents a bare-bones model formalizing our

[6] Other papers that study the possibility of self-fulfilling recessions that lead to increases in the demand for safe assets are: Chamley (2014), Ravn and Sterk (2021) and Asriyan, Fornaro, Martin and Ventura (2020).

[7] See Obstfeld (2013) for a discussion of the importance of fiscal capacity for attenuating the aftermaths of financial crises, and Jordà, Schularick and Taylor (2016) and Romer and Romer (2018) for empirical support.

key insights. We build on the analytical device of Caballero et al. (2016) to identify the role of bond-premium cyclicality in generating contrasting policy prescriptions. We also show empirical evidence consistent with the conditions for self-fulfilling liquidity traps arising in equilibrium. Section 3 shows the mechanisms through the lens of a microfounded infinite horizon model. Section 4 studies the policy implications and presents some extensions of the basic model. Section 5 develops a quantitative model, which allows us to show the transitional dynamics of the economy to a liquidity trap as well as to quantify the magnitudes of robust policies. Section 6 concludes.

2 Safe Asset Scarcity: A Simple Theory and Empirics

This section outlines our theory's main ingredients in a simple linear model in the spirit of Caballero et al. (2016). The main takeaway of the model is that the cyclicality of the bond premium, which depends on the output elasticity of safe asset supply relative to demand, is a key statistic determining the equilibrium properties of the economy and its policy implications. The empirical evidence suggests the plausibility of an expectations-driven scarcity of safe assets as an equilibrium outcome.

2.1 Log-linearized model

Consider a stationary equilibrium of an economy with permanently fixed prices. Agents prefer to hold certain financial assets because of their non-pecuniary benefits (e.g., liquidity or safety).[8] We label them safe assets. A representative firm produces the consumption good and issues safe assets. The central bank sets the nominal interest rate on safe assets following a Taylor-type rule to keep output at its natural level. We consider a cashless limit of the economy but include a zero lower bound (ZLB) constraint on the nominal interest rate. The model can be characterized by the following equations:

$$i^s = \max\{0, r^s + \phi(y - \bar{y})\} \tag{TR}$$

$$s^d = \psi_i i^s + \psi_y (y - \bar{y}) - \psi_\Delta (i - i^s) + \lambda \tag{D-SA}$$

$$s^s = b^g + \eta_y (y - \bar{y}) \tag{S-SA}$$

$$s^d = s^s \tag{SA*}$$

[8] Geromichalos, Herrenbrueck and Lee (forthcoming) provide a detailed analysis of the differences between safety and liquidity.

Equation (TR) is the monetary rule of a central bank that sets the nominal interest rate on safe assets, i^s, in order to stabilize the output gap, $y - \bar{y}$, subject to the ZLB constraint.[9] We denote the natural rate of interest (that is, the interest rate consistent with a zero output gap) by r^s. Moreover, we assume that $\phi > \max\left\{0, \frac{\eta_y - \psi_y}{\psi_i + \psi_\Delta}\right\}$, which guarantees the existence of a full-employment equilibrium when $r^s > 0$. Equation (D-SA) is the safe asset demand. The demand for safe assets, s^d, is increasing in the return on safe assets ($\psi_i > 0$). The parameter ψ_y captures the cyclicality of the safe asset demand. Intertemporal consumption smoothing may imply a pro-cyclical demand for safe assets $\psi_y > 0$, while a precautionary savings motive can generate a counter-cyclical demand $\psi_y < 0$. The demand for safe assets also depends on the *bond premium*, $i - i^s$, where i is the rate of return of an asset that does not provide safety services, and $\psi_\Delta > 0$ is the sensitivity of the demand to the premium. The variable λ represents an exogenous demand shifter. Equation (S-SA) is the supply of safe assets. We assume that the supply is pro-cyclical, i.e., $\eta_y > 0$. Borrowing constraints tied to the firms' profits can generate a pro-cyclical supply.[10] Finally, b^g denotes the provision of safe assets by the government. Equation (SA*) clears the market for safe assets.

Combining equations (D-SA), (S-SA) and (SA*), we can write a system of two equations (SA) - (TR) and two endogenous variables (y, i^s), given r^s, i, b^g and λ.[11] The safe asset (SA) equilibrium is given by

$$(\eta_y - \psi_y)(y - \bar{y}) = \psi_i i^s + (\lambda - b^g) - \psi_\Delta(i - i^s) \tag{SA}$$

In Caballero et al. (2016), $\eta_y = 0$ and $\psi_y > 0$, so that $\eta_y - \psi_y < 0$. Relative to Caballero et al. (2016), equation (SA) allows for more flexibility in the sign of $\eta_y - \psi_y$, which represents the difference between the elasticity with respect to output of the supply and the demand of safe assets. This extension allows us to identify a new theory of liquidity traps.

2.2 TR-SA representation and the bond premium

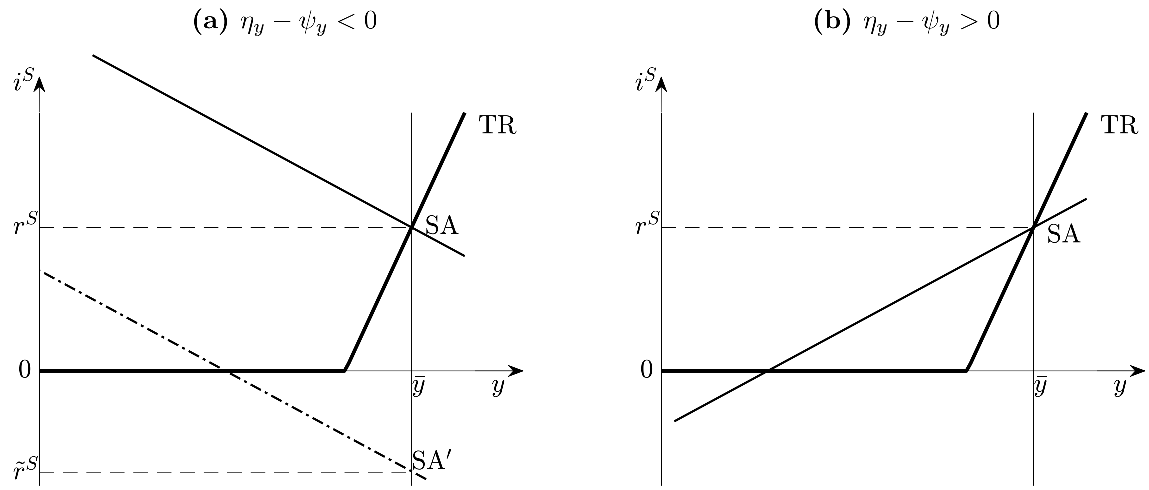
Figure 1 plots the system (TR)-(SA). In Panel (a), we plot the equation (SA) when $\eta_y - \psi_y < 0$, which implies a negative relation between the return on safe assets and the output gap. We assume that the economy starts with a positive natural interest rate, and the central bank keeps output at its potential. The economy is then hit by a shock that increases the

[9] We can extend the model to accommodate a more general ELB constraint of the form $i^s = \max\{\underline{i}^s, r^s + \phi(y - \bar{y})\}$, with $\underline{i}^s < 0$, as long as $\underline{i}^s$ is not "too negative."

[10] In the model of Section 3, we assume that in order for a financial asset to be safe, it has to be free of roll-over risk, providing a microfoundation for $\eta_y > 0$.

[11] We assume that i is independent of y and i^s in the stationary equilibrium of the economy. This is a typical result in standard models, where $i = \rho$ and ρ is the households' subjective discount rate.

Figure 1: TR-SA representation



demand for safe assets (e.g., an increase in λ or a reduction in b^g or η_y), which pushes the SA line down such that $r^s < 0$ and the economy finds itself in a liquidity trap. Output drops below its natural level, and the economy features a higher bond premium. We call this equilibrium a *Fundamental Liquidity Trap (FLT)*. In this equilibrium, an increase in the supply of safe assets by the government ($b^g \uparrow$) shifts the SA line upward, generating an increase in output.

In Panel (b), we plot the equation (SA) when $\eta_y - \psi_y > 0$, which implies a positive relationship between the return on safe assets and the output gap. We assume that the natural interest rate of the economy is positive. This implies that, given monetary policy, a zero output gap is an equilibrium of the economy. However, there exists a second equilibrium in which the economy is at the ZLB, with a relative scarcity of safe assets and a higher bond premium. Starting from a full-employment steady state, agents' pessimism about the availability of safe assets or the level of output can push the economy into this liquidity trap equilibrium. We call this equilibrium a *Self-Fulfilling Liquidity Trap (SFLT)*. In this equilibrium, a small increase in the provision of safe assets by the government can further reduce output and drive up the bond premium.[12]

It is useful to restate the previous results in terms of the cyclicality of the bond premium. Rearranging equation (SA), we get the following expression for the bond premium:

$$i - i^s = \beta_s i^s + \beta_g b^g + \beta_y (y - \bar{y}) + \beta_\lambda \lambda \tag{BP}$$

where β_s denotes the sensitivity of the convenience yield to the central bank rate (see Nagel,

[12]Since the equilibrium of the economy is not unique, policy changes can also generate a change in expectations that lifts the economy from the liquidity trap.

2016), β_g captures the sensitivity to the quantity of safe government debt, β_λ captures the sensitivity of the bond premium to the demand shifter λ, and β_y denotes the cyclicality of the bond premium. It is immediately apparent that the bond premium is pro-cyclical (i.e., $\beta_y > 0$) if and only if $\eta_y - \psi_y < 0$. Thus, this analysis allows us to connect the properties of the economy and the nature of liquidity traps to the bond premium's cyclicality. In particular, the previous results imply that SFLTs are associated with a counter-cyclical bond premium.

In Section 3, we present a microfounded model that clarifies the structural forces behind the cyclicality of the bond premium and the nature of liquidity traps. But first, we present some empirical evidence on two ingredients of our setup to assess the plausibility of SFLTs: the cyclicality of the bond premium and the cyclicality of the supply of safe assets.

2.3 The cyclicality of the bond premium

2.3.1 Data and measurement

The estimating equation of interest is (BP) evaluated at each period t

$$i_t - i_t^s = \beta_s i_t^s + \beta_g b_t^g + \beta_y (y_t - \bar{y}_t) + \beta_\lambda \lambda_t + \varepsilon_t, \qquad \text{(BP-est)}$$

where ε_t denotes the error term. Our main coefficient of interest is β_y, which measures the cyclicality of the bond premium *conditional* on the short-term policy rate, the public provision of safe assets, and the demand shifter.

In our baseline specification, we use monthly data from January 1973 until November 2007. This sample excludes the Global Financial Crisis (GFC), which started in December of 2007 according to the NBER Business Cycle Dating Committee. However, the results are robust to including the GFC. Following the seminal contribution of Krishnamurthy and Vissing-Jorgensen (2012) (henceforth KVJ), we consider the Baa-Aaa corporate bond spread as our measure of the bond premium. This spread is measured as the percentage difference between Moody's Baa-rated long-maturity corporate bond yield and Moody's Aaa-rated long-maturity corporate bond yield. The Moody's Aaa and Baa indices are constructed from a sample of long-maturity ($\geq$ 20 years) industrial and utility bonds. Because default risk is an important component of the corporate bond spread, we control for the median "distance to default" measure constructed by Gilchrist and Zakrajšek (2012), which is based on the framework developed in the seminal work of Merton (1974) (see also Bharath and Shumway, 2008).[13] Our results are robust to using other measures of the bond premium,

[13] The key insight behind this variable is that the equity of the firm can be viewed as a call option on the underlying value of the firm with a strike price equal to the face value of the firm's debt. Under the assumptions of the Merton model, one can then use the observed value of the firm's equity and its volatility

such as long-term Aaa-Treasury, three-month banker's acceptance rates and Treasury spread (Ba-Tbill), three-month high-grade commercial paper and Treasury spread (AACP-Tbill), three-month certificate of deposit (CD) rates and Treasury spread (CD-Tbill), and the spread between lower-grade commercial paper and Treasury (CPP2-Tbill). We borrow the Ba-Tbill and CD-Tbill series from Nagel (2016) and follow KVJ's data construction in extending Baa, Aaa, long-term and short-term Treasury, AACP and CPP2 yields to a monthly frequency. We describe the data sources in Appendix B.

To proxy for the output gap, we use a variety of methods to estimate potential output. Our main specifications measure output gap using the year-on-year change in the log of industrial production index (Stock and Watson, 2003, 2019). A major advantage of using year-on-year growth as a filter to proxy for output gap is that it is one-sided and does not suffer from end-point problems, nor does it induce revisions, as argued by Stock and Watson (2019). At the same time, there are disadvantages of using this one-sided filter: it passes more noise and has a phase shift relative to two-sided filters such as the band-pass filter. We conduct robustness to the following alternative measures of the output gap: the Hamilton (2018) filter, the band-pass filter of Baxter and King (1999), the polynomial sixth-degree time trend from Ramey and Zubairy (2018), the month-on-month growth rate of industrial production, the civilian unemployment rate, and the Chicago Fed National Activity Index (Brave, 2009). In the Online Appendix, we show robustness exercises that consider various combinations of output gap measures and financial spreads.

To be consistent across all specifications, we follow Nagel (2016) and use the overnight federal funds rate as our measure of the short-term safe rate, i^s_t, on the right-hand side of equation (BP-est). We use the log of the ratio of the outstanding stock of T-bills and GDP as our measure of the public safe asset supply.[14]

Following KVJ, we also control for the slope of the Treasury yield curve in our regressions. This slope is measured as the spread between the 10-year Treasury yield and the 3-month Treasury yield.[15] We expect the coefficient on the slope to be positive. Even though the slope variable is likely to attenuate the effect of output gap, we include it in our main specifications to capture unmodeled confounding factors that may bias our estimation. We measure shifts

to infer the underlying value of the firm and its volatility.

[14]The quarterly GDP series is interpolated to a monthly series for constructing the Tbill/GDP ratio. We find similar results using the (year-on-year) growth rate of Tbill supply instead of the Tbill/GDP ratio. The results are also robust to using debt-to-GDP ratio as in KVJ. However, debt-to-GDP ratio is only available at an annual frequency.

[15]KVJ write, "The slope of the yield curve is a measure of the state of the business cycle. It is known to predict the excess returns on stocks and may also pick up time-varying risk premia on corporate bonds. [...] We also note that to the extent that corporate default risk is likely to vary with the business cycle, the slope variable can furthermore help control for the expected default in the yield spread."

TABLE 1: Baa-Aaa spread on output gap (y-o-y $\Delta \log$ IP)

	(1)	(2)	(3)	(4)	(5)	(6)
Output gap	-5.28***		-5.06***	-5.19***	-4.90***	-5.00***
	(1.37)		(1.12)	(1.12)	(0.67)	(0.64)
Fed funds rate		4.63**	3.98***	4.92***	9.44***	9.87***
		(1.99)	(1.52)	(1.78)	(1.52)	(1.45)
log(T-Bill/GDP)				50.45	-25.55	-8.90
				(33.69)	(22.87)	(20.76)
Distance to Default				-0.12	2.98	4.87***
				(2.14)	(1.90)	(1.85)
Slope					18.71***	18.83***
					(2.55)	(2.44)
VIX						1.06**
						(0.53)
Intercept	246.68***	136.88**	157.40***	267.44***	-82.99	-94.01
	(36.92)	(60.97)	(42.73)	(74.35)	(69.70)	(64.32)
# Obs.	419	419	419	419	419	419
Adj R^2	0.45	0.27	0.52	0.54	0.68	0.69

Note: Newey-West standard errors (12 lags) in parentheses. $***p < 0.01, **p < 0.05, *p < 0.1$. Includes a linear time-trend. Baa-Aaa spread measures the percentage difference between Moody's Baa-rated long-maturity corporate bond yield and Moody's Aaa-rated long-maturity corporate bond yield. output gap is computed with year on year change in log of (monthly) industrial production index. Sample: Jan 1973 – Nov 2007 (monthly).

in the demand for safety using the VIX index. In addition, we include a linear time-trend to proxy for linear secular movements in the bond premium. Our results are robust to excluding the slope and the linear time trend from the empirical specifications. We report Newey-West standard errors with twelve lags.

2.3.2 Results

Table 1 reports the coefficients from estimating equation (BP-est) using the long-term Baa-Aaa corporate bond spread as our measure of the bond premium and the year-on-year change in (log) industrial production index as our measure of the output gap. We find that the bond premium is counter-cyclical across all columns, i.e., $\beta_y < 0$. The coefficients on the output gap in columns (4)-(5) imply reductions of 5.19 and 4.90 basis points (bps) in the bond premium, respectively, with a one percent increase in output growth above trend. Furthermore, the coefficients on the federal funds rate in row 2 are consistent with Nagel (2016), who finds that the convenience yield on liquid assets is positively related to the federal funds rate.

Table 2 reports the results for the baseline specification using different measures of the output gap. Columns (1-4) use standard methods of extracting trends from monthly in-

TABLE 2: Baa-Aaa spread on various measures of output gap

	Hamilton Filter	Band-Pass Filter	Δ_m log(IP$_t$)	Polynomial Filter	Unemployment Rate
	(1)	(2)	(3)	(4)	(5)
	1973-2007	1973-2007	1973-2007	1973-2007	1973-2007
Output gap	-4.02***	-17.54***	-11.18***	-4.59***	27.96***
	(0.43)	(2.03)	(3.34)	(1.23)	(3.17)
Fed funds rate	8.16***	11.01***	9.67***	5.99***	3.35**
	(1.34)	(1.65)	(2.48)	(2.18)	(1.46)
log(T-Bill/GDP)	-35.55**	-63.48**	-31.48	-138.46***	-102.68***
	(15.00)	(24.95)	(28.00)	(30.52)	(20.03)
Distance to Default	2.20	1.78	1.70	1.01	-1.11
	(1.53)	(1.83)	(2.33)	(2.12)	(1.53)
Slope	12.48***	18.14***	20.44***	10.87***	1.02
	(2.71)	(2.42)	(4.04)	(3.87)	(2.69)
VIX	0.75*	0.84	0.94	1.28*	1.09**
	(0.43)	(0.59)	(0.70)	(0.76)	(0.43)
Intercept	-105.34*	-219.79***	-112.85	-334.26***	-438.06***
	(57.98)	(77.13)	(102.92)	(98.37)	(60.85)
# Obs.	419	419	419	419	419
Adj R^2	0.76	0.68	0.52	0.61	0.75

Note: Newey-West standard errors (12 lags) in parentheses. $***p < 0.01, **p < 0.05, *p < 0.1$. Includes a linear time-trend. output gap is computed with various filters common in the literature. Column 1 uses the Hamilton filter on the (monthly) industrial production index. The index is seasonally adjusted. Column 2 uses the Band-Pass filter at business cycle frequencies (18 and 96 months) on the (monthly) industrial production index. Column 3 uses month-over-month change in the log of (monthly) industrial production index. Column 4 estimates a counterfactual potential (monthly) industrial production index using a (sixth-degree) polynomial regression on time. Column 5 uses the civilian unemployment rate. Baa-Aaa spread measures the percentage difference between Moody's Baa-rated long-maturity corporate bond yield and Moody's Aaa-rated long-maturity corporate bond yield. See text.

dustrial production. The first-row coefficient is interpreted as units of bps increase in the Baa-Aaa corporate bond spread associated with a one percent increase in output above trend. The first row in column (5) is interpreted as units of bps increase in the Baa-Aaa corporate bond spread associated with a one percentage point increase in the unemployment rate. Using an Okun's law coefficient of two, row (1) in column (5) implies a 13.98 bps reduction in the Baa-Aaa spread when output falls one percent below potential. These estimates of the coefficient β_y imply a reduction in the bond premium in the range of 4 to 18 bps when output is one percent above potential.

Table 3 reports the results for the full baseline specification using various measures for the bond premium. The output gap is measured using the year-on-year change in the (log) industrial production index. We find a relatively stable estimate for the coefficient β_y. It is important to note that we find significant results even after controlling for several endogenous variables that are likely to mitigate the independent effects of output. We interpret this as robust evidence in favor of a counter-cyclical bond premium. In the Online Appendix, we show robustness exercises that consider various combinations of output gap measures and financial spreads. Moreover, we show that our results are robust to an instrumental variable strategy where we use oil supply shocks and variations in utilization-adjusted TFP to identify the causal effect of variations in the output gap on bond-premia.

TABLE 3: Financial spreads on output gap (y-o-y $\Delta \log$ IP)

	Aaa-Tbill	BA-Tbill	AACP-Tbill	CD-Tbill	CPP2-Tbill
	(1)	(2)	(3)	(4)	(5)
	1948-2007	1948-2007	1948-2007	1976-2007	1998-2007
Output gap	-2.25***	-1.45***	-1.08**	-1.90**	-1.83
	(0.68)	(0.40)	(0.43)	(0.89)	(1.94)
Fed funds rate	4.15***	10.90***	6.96***	14.38***	34.21**
	(0.95)	(1.13)	(1.07)	(2.54)	(15.12)
log(T-Bill/GDP)	-59.08***	-25.68**	-13.57	-28.38	-41.32
	(18.40)	(11.60)	(11.72)	(37.71)	(71.13)
VIX	1.85***	1.75***	1.78***	3.42***	6.88***
	(0.47)	(0.48)	(0.44)	(1.00)	(2.64)
Slope	8.84**	-1.05	-4.30	10.30*	27.61
	(3.97)	(2.77)	(2.68)	(6.08)	(17.64)
Intercept	-136.26***	-67.73**	22.50	-300.65**	-1223.77**
	(35.32)	(26.64)	(29.01)	(133.19)	(520.80)
# Obs.	768	768	768	432	168
Adj R^2	0.48	0.63	0.49	0.37	0.47

Note: Newey-West standard errors (12 lags) in parentheses. $***p < 0.01, **p < 0.05, *p < 0.1$. Includes a linear time-trend. Output gap is computed with year-on-year change in the log of (monthly) industrial production index. Column 1 uses the percentage spread between Moody's Aaa-rated long-maturity corporate bond yield and the yield on long-maturity Treasury bonds. Column 2 uses the three-month banker's acceptance rate and T-bills. The data series for the banker's acceptance rate ends in the 1990s. To create a series until 2007, we use the GC repo/T-bill spread from 1991 onward constructed by Nagel (2016). Column 3 uses the percentage yield spread between 3-month high-grade commercial paper and Treasury bills. Column 4 uses the spread between three-month certificate of deposit (CD) rates and T-bills as an alternative measure of the illiquid rate. Column 5 uses the percentage yield spread between lower-grade commercial paper and Treasury bills. It is calculated as the sum of the CP-bills yield spread described above (i.e., high-grade commercial paper minus Treasury bills) and the yield spread between 30-day A2/P2 nonfinancial commercial paper and 30-day AA nonfinancial commercial paper, with data obtained from the Federal Reserve Bank of New York. See text.

2.4 The cyclicality of safe asset supply

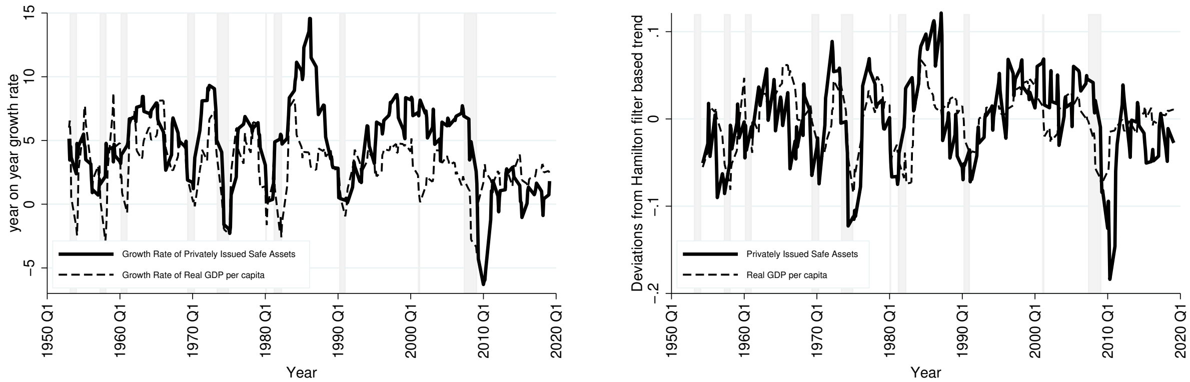
2.4.1 Data and measurement

Gorton et al. (2012) use the U.S. Financial Accounts to obtain a quarterly time-series of the total supply of safe assets. We follow their definitions to reproduce the time-series of safe assets, public and private. The public safe assets largely include treasury and municipal debt. The private safe assets include bank deposits, money market mutual fund shares, commercial paper, federal funds and repurchase agreements, short-term interbank loans, securitized debt, and high-grade financial sector corporate debt. Their rationale for defining these asset classes as safe is that they were information-insensitive assets before the Great Recession (see also Gorton and Metrick, 2012). Total private safe assets are disaggregated into five subcategories: deposits, money-like debt, mortgage- and asset-backed securities (MBS/ABS), corporate bonds and loans, and other safe assets (miscellaneous liabilities of the financial sector). Money-like debt refers to commercial paper, net repurchase agreements, money market mutual fund assets, federal funds, interbank transactions, broker-dealer payables, and broker-dealer security credits. We briefly describe the data construction and disaggregation

Figure 2: Cyclicality of privately supplied safe debt

(a) Growth rates

(b) Hamilton (2018) filter-based cycle



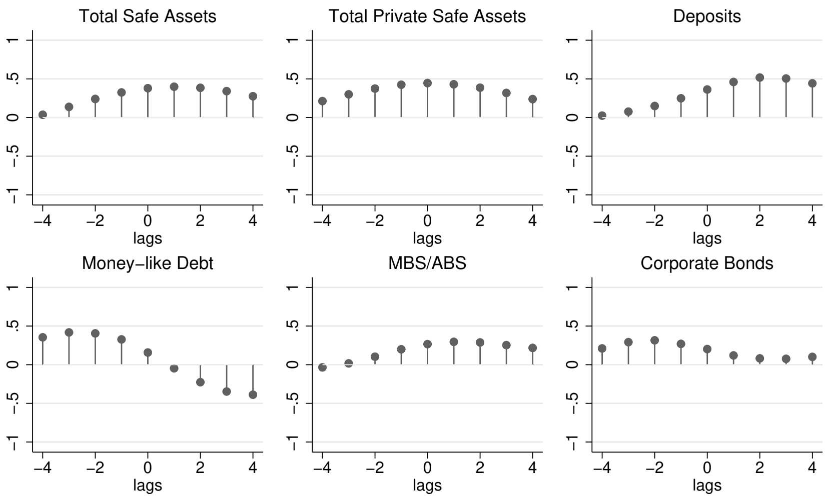
Source: Our calculations. We extended Gorton et al. (2012)'s definition to measure safe assets using US Financial Accounts data retrieved from FRED, St. Louis Fed. Shaded bars denote NBER Recession dates. See text.

of total private safe assets in Appendix B but refer the reader to Gorton et al. (2012) for a detailed methodology.

2.4.2 Results

Figure 2 plots the detrended time series of private safe assets along with real GDP. In Panel (a), we plot both series in terms of year-on-year growth rates, consistent with our empirical specifications in Section 2.3.2. In Panel (b), we plot both series using the Hamilton (2018) filter for robustness. A strong positive correlation between the two series suggests the pro-cyclicality of private safe asset supply in the U.S. Figure 3 presents the cross-correlations between various components of the safe asset supply at $t + h$ and real GDP at time t. The top-left Panel plots the cross-correlations for the total supply of safe assets, private and public. The remaining panels show the cross-correlations for the total private supply and four sub-categories. All series are transformations of the underlying asset series into year-on-year growth rates. These cross-correlations between various components of private safe assets and output align with the pro-cyclicality of safe asset supply suggested by Figure 2. To avoid issues with the GFC, we stop the sample in 2007, but all the results are robust to including the full sample. Moreover, the results are robust to using alternative measures of output gap. We present these additional robustness results in the Online Appendix. We also show that our results are robust to an instrumental variable strategy where we use exogenous variation in uncertainty and risk-aversion to identify shifters in the demand for safe assets that trace out the safe asset supply curve, and find that safe asset supply is pro-cyclical (that is, the sign of η_y is positive).

Figure 3: Correlations of x_{t+h} with time-t real GDP (y-o-y growth)



Source: Our calculations, using US Financial Accounts data retrieved from FRED, St. Louis Fed. The definitions follow Gorton et al. (2012). Real GDP and all of the safe asset component series are plotted in year-on-year growth rates. See text.

2.5 Summary of the Empirical Analysis

Let us briefly summarize the key takeaways from our empirical analysis and how it connects to the linear model. We find strong suggestive evidence that: *a*) the bond premium is counter-cyclical and *b*) total safe asset supply and safe assets supplied by the U.S. private sector are pro-cyclical. As the simple model shows, the bond premium's counter-cyclicality gives rise to the possibility of a self-fulfilling scarcity of safe assets that pushes the economy into a liquidity trap. Furthermore, we identified a force driving this result: the supply of private safe assets is pro-cyclical in the data. Collectively, these results provide empirical support for the possibility of SFLTs. In the next section, we use these findings to build a microfounded model that clarifies the transmission channels and provides a laboratory suitable for policy analysis.

3 A Microfounded Model

This section presents a model featuring an endogenous supply of assets and a preference for safe (or liquid) bonds that arises endogenously from a retirement motive. We study an infinite-horizon closed economy in discrete time, indexed by $t \in \{0, 1, 2, \dots\}$. The economy is inhabited by households, firms, and a government. We assume that the economy does not face any aggregate risk.[16]

3.1 The Environment

Households. The economy is populated by a measure one of households. Households are comprised of a measure $1-\chi$ of workers and a measure χ of retirees (with $\chi \in (0,1)$). In each period, workers are endowed with one unit of time, which they can sell in the labor market. In contrast, retirees cannot work, and they live for only one period. Every period a random measure χ of workers retire, and a measure χ of new agents is born. Thus, the composition of each household is constant over time.[17]

Workers supply their time inelastically to the labor market. In the presence of nominal wage rigidities, workers might be able to sell only a fraction $h_t \leq 1$ of their time. When $h_t < 1$, the economy is operating below potential, and there is involuntary unemployment. Households are the owners of the firms, which distribute nominal dividends D_t, they trade one-period nominal risk-free assets $\widetilde{B}_t$ at a nominal price $\frac{1}{1+i_t^s}$, where i_t^s is the nominal (safe) interest rate, and receive nominal lump-sum transfers T_t. Moreover, households operate a technology to produce physical capital according to

$$K_{t+1} = (1-\delta)K_t + I_t, \tag{1}$$

where K_{t+1} denotes the capital available in period $t+1$, δ is the depreciation rate, and I_t denotes investment. Households rent the capital in a competitive market at the nominal rate R_t^k.

Being a worker or a retiree determines the resources available for consumption. We assume that at the beginning of each period, and before agents know whether they are workers or retirees, the household distributes its portfolio of financial assets equally among its members. Once they leave the household, agents find out their type. Since retirees live for only one period, they cannot borrow in the financial markets, and their wealth is

[16] This is a common assumption in the literature, which usually studies the response of the economy to an unexpected shock, or a first-order approximation of the dynamics around a non-stochastic steady-state equilibrium, which effectively imposes a certainty-equivalence property on the model. We provide an extension of the model with aggregate risk in Appendix C.

[17] This modeling assumption is similar to the tractable stochastic OLG model of Gertler (1999), recently used by Rachel and Summers (2019) to study the decline in the natural interest rate. The "big family" assumption has a long tradition in macroeconomics. Lucas (1990) uses this framework to study the effect of open-market operations on the economy's interest rate. More recently, it has been used by Del Negro et al. (2017*b*), Bilbiie (2021) and Heathcote and Perri (2018). This formulation allows us to study a model with incomplete insurance at the individual level but without the need to keep track of the cross-sectional distribution of wealth as a relevant state variable.

limited to the assets in their portfolio.[18] In contrast, workers can also consume out of their other sources of income. At the end of the period, and after consumption takes place, the surviving members of each household pool their resources and make the savings decisions for the following period. Thus, workers have two motives to save. First, they face the traditional intertemporal substitution channel: savings allow them to substitute consumption tomorrow for consumption today. Second, workers have a retirement motive: a measure χ of them will retire and consume only out of their safe (or liquid) financial wealth. This second motive will generate the reduced form bond premium presented in Section 2.

Households maximize a utilitarian welfare function of their members' utility

$$\sum_{t=0}^{\infty} \beta^t \left[(1-\chi) u\left(C_t^w\right) + \chi v\left(C_t^r\right)\right] \tag{2}$$

where C_t^w and C_t^r denote the consumption of a worker and a retiree in period t, respectively, and β is the discount factor. For analytical tractability, we assume that $u(C_t^w) = \log(C_t^w)$ and $v(C_t^r) = \log(C_t^r)$. Within a period, each household member makes their consumption decisions based on their portfolio holdings and income. The intra-period budget constraints faced by individual agents are given by

$$P_t C_t^w \leq W_t h_t + \widetilde{B}_t + \frac{1}{1-\chi}(R_t^k K_t + D_t + T_t), \tag{3}$$

$$P_t C_t^r \leq \widetilde{B}_t, \tag{4}$$

where P_t is the price level, and W_t is the nominal wage. Equation (3) is the intra-period budget constraint for a worker who receives wage income, capital income, dividends and transfers in addition to their allocation of bonds from the household. Equation (4) is the intra-period budget constraint for a retiree who can consume only out of their holdings of one-period bonds, the *safe assets* of this economy. In particular, they do not receive any of the firm's dividends or the returns from capital. Moreover, we have abstracted from other financial assets, such as stocks, either because of their illiquidity or their relatively small prevalence in households' portfolios in the data.[19] This assumption is not necessary for our results, but it simplifies the exposition.[20] At the end of the period, the household as a whole

[18] We also assume that retirees cannot leave debts to their families.

[19] For example, households' direct ownership of liquid equities is very low. Heathcote and Perri (2018) document that in 2010, only 15.1% of households held stocks directly.

[20] All our results would go through as long as the retirees' income is not sufficiently high to allow for full insurance. The extension with aggregate risk in Appendix C provides a justification to our focus on bonds as safe assets, as it shows that the presence of risk can significantly reduce the safety properties of stocks.

faces the following budget constraint:

$$(1-\chi)P_tC_t^w + \chi P_tC_t^r + P_tI_t + \frac{\widetilde{B}_{t+1}}{1+i_t^s} \leq (1-\chi)W_th_t + R_t^kK_t + D_t + \widetilde{B}_t + T_t. \tag{5}$$

The problem of the household consists of choosing processes $\left\{C_t^w, C_t^r, I_t, K_{t+1}, \widetilde{B}_{t+1}\right\}_{t=0}^{\infty}$ in order to maximize (2) subject to the budget constraints (3), (4) and (5) for every $t \geq 0$, the capital accumulation technology (1), and a no-Ponzi condition, given K_0 and $\widetilde{B}_0$. Since all households solve the same problem, we can treat the economy as populated by a single representative household.

In what follows, we limit attention to equilibria in which the budget constraint of retirees (4) is binding, so that $C_t^r = \frac{\widetilde{B}_t}{P_t}$. Replacing this into the household's utility function, we get

$$\sum_{t=0}^{\infty} \beta^t \left[(1-\chi)u\left(C_t^w\right) + \chi v\left(\frac{\widetilde{B}_t}{P_t}\right)\right].$$

Thus, the household's problem looks *as-if* it was generated by an agent who values bonds directly (as in the *bonds in the utility function* tradition), even though it is the result of the workers' retirement concerns. Two differences that will become apparent below are that the retirement motive introduces a satiation point for bonds that is absent in non-microfounded environments, and that a fraction χ of the bonds enter in the resource constraint through the consumption of the retirees.

Firms. The final consumption good is produced by a measure one of perfectly competitive firms using labor and capital as the factors of production according to $Y_t = A_t(K_t^{\alpha}H_t^{1-\alpha})^{\nu}$, where A_t is the TFP level, K_t is the amount of capital rented, H_t is the amount of labor hired, $\alpha \in [0,1)$, and $\nu \in (0,1)$. The parameter ν denotes a span of control, which measures the degree of decreasing returns in the variable factors of production, and it will imply that the firms make profits in equilibrium. Firms rent capital at a rate R_t^k, they hire workers at a nominal wage W_t, and they must pay a per-period fixed cost F (in units of the final good), which is rebated lump-sum to the households.[21] Thus, a firm's per-period operating profit is given by

$$\Pi_t = P_t\left[A_t(K_t^{\alpha}H_t^{1-\alpha})^{\nu} - F\right] - W_tH_t - R_t^kK_t.$$

[21] This assumption about the fixed cost is reminiscent of the treatment of the Rotemberg price adjustment cost made in the literature (see, for example, Benhabib, Schmitt-Grohé and Uribe, 2001*a*, Ascari and Rossi, 2012 and Eggertsson and Singh, 2019). One can reinterpret the fixed cost as a technological requirement of a fixed amount of managerial services provided by households, which generates a disutility F in units of consumption.

We will assume that the fixed cost is never too high so as to preclude the existence of an equilibrium.

Since safe bonds carry a premium, the conditions for the Modigliani-Miller theorem do not hold in this economy. In particular, firms have an incentive to issue safe bonds to profit from the premium. For the bonds to be safe, we require that firms be able to pay their debt in full with internal funds alone. This way, firms' debt is not subject to roll-over risk. Formally, firms need to satisfy the following constraint:

$$\widetilde{B}^p_{t+1} \leq \Pi_{t+1}, \tag{6}$$

where $\widetilde{B}^p_{t+1}$ denotes the face value of the nominal bonds issued by the firm in period t and payable in $t+1$.

A firm's objective is to maximize the present discounted value of the stream of dividends paid out to their shareholders,

$$V_t \equiv \max_{\{D_t, K_t, H_t, \widetilde{B}^p_{t+1}\}_{t=0}^{\infty}} \sum_{s=0}^{\infty} \Lambda_{t+s} D_{t+s}$$

subject to

$$D_t = P_t \left[A_t (K_t^\alpha H_t^{1-\alpha})^\nu - F\right] - W_t H_t - R_t^k K_t - \widetilde{B}^p_t + \frac{\widetilde{B}^p_{t+1}}{1+i^s_t}$$
$$\widetilde{B}^p_{t+1} \leq P_{t+1} \left[A_{t+1} (K_{t+1}^\alpha H_{t+1}^{1-\alpha})^\nu - F\right] - W_{t+1} H_{t+1} - R_{t+1}^k K_{t+1},$$

given $\widetilde{B}^p_0$, where Λ_t is the household's stochastic discount factor. The firm is active in period t if and only if $P_t\left(Y_t - F\right) - W_t H_t - R_t^k K_t \geq 0$.

The presence of a fixed cost of production has two important implications in our analysis. First, the fixed cost precludes the existence of steady-state equilibria with output levels that are implausibly low relative to potential. Second, the fixed cost implies that firms can issue bonds only if their production in the next period is above a certain threshold. This property will allow self-fulfilling liquidity traps to arise as an equilibrium outcome.[22]

Nominal Rigidities. In this section, we assume that wages are perfectly rigid, that is,

$$W_t = W_{t-1} \quad \forall t.$$

[22] A self-fulfilling liquidity trap also obtains with an alternative formulation that features no fixed cost of production and a borrowing constraint that is convex in profits. However, such a model would admit equilibria in which output can be arbitrarily small. We choose the specification with the fixed cost because it generates a minimum level of output that is bounded away from zero.

This assumption implies that labor markets will not always clear, so that $h_t < 1$ can be part of an equilibrium. This choice has the benefit of isolating our channel for an SFLT from the one in Benhabib et al. (2001*b*), which works through deflation. In Section 4.4 we explore the implications of allowing a more flexible specification, in the spirit of Schmitt-Grohé and Uribe (2017), which we use in the quantitative exercise of Section 5.

Government. To close the model, we need to introduce a monetary policy rule and a budget constraint for the government. We assume that the central bank sets the nominal interest rate according to

$$1 + i_t^s = \max\left\{1, R_t^* + \phi_Y\left(\frac{Y_t}{Y_t^*} - 1\right)\right\} \tag{7}$$

where R_t^* is the gross real interest rate consistent with the full employment equilibrium, and Y_t^* denotes the full-employment output level.[23]

Finally, the government's budget constraint is given by

$$\widetilde{B}_t^g = \frac{\widetilde{B}_{t+1}^g}{1 + i_t^s} + T_t^g, \tag{8}$$

where T_t^g denotes lump-sum taxes levied on the household. The lump-sum transfers received by the household are, then, comprised of the government taxes and the rebates of the fixed costs borne out by firms: $T_t = F - T_t^g$.

3.2 Equilibrium

Let $w_t \equiv \frac{W_t}{P_t}$ denote the real wage. A competitive equilibrium of this economy is an allocation $\{C_t^w, C_t^r, I_t, K_{t+1}, \widetilde{B}_{t+1}, h_t, H_t, \widetilde{B}_{t+1}^p\}_{t=0}^\infty$ and prices $\{w_t, R_t^k, \pi_t, i_t^s\}_{t=0}^\infty$ such that, given fiscal policy $\{\widetilde{B}_t^g, T_t^g\}_{t=0}^\infty$, and initial capital and bonds, $\{K_0, \widetilde{B}_0, \widetilde{B}_0^p\}$,

1. $\{C_t^w, C_t^r, I_t, K_{t+1}, \widetilde{B}_{t+1}, h_t\}_{t=0}^\infty$ solves the household's problem given $\{w_t, R_t^k, \pi_t, i_t^s\}_{t=0}^\infty$, $\{T_t^g\}_{t=0}^\infty$ and $\{K_0, \widetilde{B}_0\}$
2. $\{K_t, H_t, \widetilde{B}_{t+1}^p\}_{t=0}^\infty$ solves the firms' problem given $\{w_t, R_t^k, \pi_t, i_t^s\}_{t=0}^\infty$ and $\widetilde{B}_0^p$
3. $\{i_t^s\}_{t=0}^\infty$ follows (7)
4. fiscal policy $\{\widetilde{B}_t^g, T_t^g\}_{t=0}^\infty$ satisfies (8)

[23]None of the results rely on the specification of the interest rate rule. In the quantitative exercise of Section 5 we assume a rule that depends on the inflation rate. Moreover, in Appendix E we show that all the results go through if monetary policy is set optimally but the central bank operates under discretion.

5. Markets clear

$$(1-\chi)C_t^w + \chi C_t^r + I_t = A_t(K_t^\alpha H_t^{1-\alpha})^\nu, \quad \widetilde{B}_t = \widetilde{B}_t^p + \widetilde{B}_t^g, \quad H_t = (1-\chi)h_t.$$

We focus our analysis on equilibria in which the budget constraint of retirees (4) is satisfied with equality. In this case, the optimality conditions associated with the household's problem are the budget constraint (5), and

$$\beta^t u'(C_t^w) = P_t \Lambda_t \tag{9}$$

$$\frac{\Lambda_t}{1+i_t^s} = (1-\chi)\Lambda_{t+1} + \chi \beta^{t+1} v'(B_{t+1}) \frac{1}{P_{t+1}} \tag{10}$$

$$\Lambda_t P_t = \Lambda_{t+1}[R_{t+1}^k + (1-\delta)P_{t+1}] \tag{11}$$

where $B_{t+1} \equiv \frac{\widetilde{B}_{t+1}}{P_{t+1}}$, and $\Lambda_t > 0$ is the Lagrange multiplier associated with the budget constraint (5). Combining equations (9) and (10), we get the following *Generalized Euler Equation* (GEE):

$$1 = \beta \frac{1+i_t^s}{1+\pi_{t+1}} \left[\underbrace{\frac{u'(C_{t+1}^w)}{u'(C_t^w)}}_{\substack{\text{intertemporal} \\ \text{substitution motive}}} + \chi \underbrace{\frac{v'(B_{t+1}) - u'(C_{t+1}^w)}{u'(C_t^w)}}_{\text{retirement motive}} \right]. \tag{12}$$

Equation (12) determines the demand for safe assets, which is one of the main building blocks of the economy's equilibrium. It reflects the two reasons households demand bonds. First, there is the intertemporal substitution motive. This is the typical motive in standard neoclassical models: households demand bonds to smooth the workers' consumption path. Second, there is a retirement motive. Since workers face retirement risk, households demand bonds above and beyond the intertemporal substitution motive to smooth the consumption of those who cannot work. In contrast, capital does not provide this precautionary savings services, so its optimality condition is given by the standard expression

$$1 = \beta \frac{u'\left(C_{t+1}^w\right)}{u'\left(C_t^w\right)} \left[r_{t+1}^k + (1-\delta)\right], \tag{13}$$

where $r_t^k \equiv \frac{R_t^k}{P_t}$.[24]

[24] The analog of i in Section 2, that is, the return of an asset that provides no convenience services, solves $1 = \beta \frac{1+i_t}{1+\pi_{t+1}} \frac{u'(C_{t+1}^w)}{u'(C_t^w)}$.

We define the *bond premium* as

$$bp_t \equiv \chi \frac{v'(B_{t+1}) - u'(C^w_{t+1})}{u'(C^w_t)}. \tag{14}$$

Ceteris paribus, a higher bond premium implies a lower real interest rate. Note that equation (12) implies a retirement motive that is *pro-cyclical*, in the sense that it is increasing in C^w_t and C^w_{t+1}: the higher the workers' consumption, the higher the demand for the retirees' consumption. This feature will represent a force towards a *pro-cyclical* bond premium. In the Online Appendix we explore a variant of this model that generates a *counter-cyclical* demand for bonds, arising from unemployment risk. Introducing a counter-cyclical bond demand makes the economy even more vulnerable to the existence of a self-fulfilling liquidity trap.

Next, consider the firms' problem. The choices of labor and capital are given by the following optimality conditions:

$$(1-\alpha)\,\nu A_t K_t^{\alpha\nu} H_t^{(1-\alpha)\nu-1} = w_t \quad \text{and} \quad \alpha\nu A_t K_t^{\alpha\nu-1} H_t^{(1-\alpha)\nu} = r^k_t$$

Thus, profits can be written as $\Pi_t = P_t\left[(1-\nu)\,Y_t - F\right]$.

Now, let μ_t be the Lagrange multiplier associated with the constraint (6). The first-order condition with respect to bonds is

$$\frac{\Lambda_t}{1+i^s_t} = \Lambda_{t+1} + \mu_t. \tag{15}$$

Comparing (12) and (15), it is immediate that $P_{t+1}\mu_t = \beta^{t+1}\chi\left[v'(B_{t+1}) - u'(C^w_{t+1})\right]$, that is, the Lagrange multiplier with respect to the borrowing constraint is proportional to the retirement motive. If the retirement motive is equal to zero (i.e., the retirement demand is satiated), the firm is unconstrained. If the retirement motive is strictly positive, then the firm is constrained. Focusing on equilibria in which the retiree's budget constraint (4) is binding, the supply of bonds is an affine function of aggregate output

$$B_t = (1-\nu)Y_t - F + B^g_t, \tag{16}$$

where $B^g_t \equiv \frac{\widetilde{B}^g_t}{P_t}$. Finally, equilibrium requires that $(1-\nu)Y_t - F \geq 0$ for all t.

3.3 Steady States

A steady state is an equilibrium in which all endogenous and exogenous variables are constant over time. Given our wage rigidity assumption, if w_t and W_t are constant over time, then the inflation rate is zero in any steady state. In what follows, variables without a time subscript denote their values in a non-stochastic steady state.

If firms are active, the bond supply is given by equation (16) evaluated at steady state: $B(Y) = (1-\nu)Y - F + B^g$. Noting that the firm's profits in steady state are given by $\Pi = (1-\nu)Y - F$, equilibrium requires that $Y \geq Y^{\min} \equiv \frac{F}{1-\nu}$, that is, there is a lower bound on admissible output levels.

Putting together the households' and firms' optimality conditions for capital evaluated at steady state, we get the following expression for capital: $K = \kappa Y$, where $\kappa \equiv \frac{\alpha\nu}{\frac{1}{\beta}-(1-\delta)}$. Thus, from the resource constraint, we get

$$C^w(Y) = \frac{(1-\delta\kappa)Y - \chi B(Y)}{1-\chi}.$$

From equation (14) evaluated at steady state, we can write the bond premium as a function of output

$$bp(Y) = \chi\left(\frac{v'(B(Y))}{u'(C^w(Y))} - 1\right). \tag{17}$$

Thus, the GEE in steady state can be written as

$$1 = \beta(1+i^s)\left[1 + bp(Y)\right], \tag{18}$$

which defines an implicit function between output Y and the (safe) nominal interest rate i^s. This is the analogue of the (SA) relation in Section 2.

In order to characterize the steady-state equilibria of the economy, it is useful first to study the cyclical properties of the bond premium. We say that the bond premium is *pro-cyclical* if $bp'(Y) > 0$, which happens if and only if

$$\underbrace{-\frac{u''(C^w(Y))}{u'(C^w(Y))}C^w(Y)}_{\text{elasticity of intertemporal substitution}} \quad \underbrace{\frac{C^{w\prime}(Y)}{C^w(Y)}Y}_{\text{elasticity of consumption to output}} > \underbrace{-\frac{v''(B(Y))}{v'(B(Y))}B(Y)}_{\text{elasticity of bond demand}} \quad \underbrace{\frac{B'(Y)}{B(Y)}Y}_{\text{elasticity of bond supply to output}}. \tag{19}$$

In contrast, if $bp'(Y) < 0$, we say that the bond premium is *counter-cyclical*.[25] Equation (19) characterizes two economic forces that determine the cyclical properties of the bond

[25] Strictly speaking, the cyclicality of the bond premium is defined locally at each level of Y. To simplify the analysis, we work under assumptions that define the cyclicality globally.

premium. Consider the effects of an increase in output. First, a higher output generates an increase in workers' consumption. Since households also value the retirees' consumption, this will trigger an increase in the demand for bonds. The increased demand for bonds then translates into a higher bond premium. However, there is a second and offsetting effect. The increase in output relaxes the firms' issuance constraint and hence increases the supply of private bonds. As the supply of private bonds increases, the demand for bonds gets (partially) satiated, reducing the bond premium. The cyclicality of the bond premium depends on which of these two forces dominates.[26] Note that by rearranging terms in equation (19), we get that the bond premium is pro-cyclical (counter-cyclical) if the elasticity with respect to output of the bond demand is larger (smaller) than the elasticity of the bond supply.[27] That is, the cyclicality of the bond premium is a property that depends on the relative elasticity of the demand and the supply of bonds. In this section's model, the bond premium is pro-cyclical if and only if $B^g > F$.

An equilibrium of the economy can be found from the intersection of equation (18) and the monetary rule evaluated at steady state

$$1 + i^s = \max\left\{1, R^* + \phi_Y\left(\frac{Y}{Y^*} - 1\right)\right\}, \tag{20}$$

where $Y^* = \left[A\kappa^{\alpha\nu}(1-\chi)^{(1-\alpha)\nu}\right]^{\frac{1}{1-\alpha\nu}}$ and $R^* = \frac{1}{\beta}\frac{1}{1+bp(Y^*)}$.

We make the following assumptions.

Assumption 1 *The parameters of the model and fiscal policy are such that:*

1. $Y^* > Y^{\min}$, *and for all* $Y \in [Y^{\min}, Y^*]$,

$$bp(Y) > 0; \tag{21}$$

[26]In the Online Appendix, we consider other economic forces that affect the exact characterization of the bond premium, such as counter-cyclical self-insurance motives. Still, the intuition behind the results follows a logic analogous to the one behind equation (19).

[27]Conditional on i^s, totally differentiating (18), we get

$$\beta(1+i^s)\chi\frac{v''(B)u'(C^w(Y))dB - v'(B)u''(C^w(Y))C^{w'}(Y)dY}{(u'(C^w(Y)))^2} = 0,$$

and rearranging

$$\frac{dB}{dY}\frac{Y}{B} = \frac{\frac{u''(C^w(Y))}{u'(C^w(Y))}C^w(Y)\frac{C^{w'}(Y)}{C^w(Y)}Y}{\frac{v''(B)}{v'(B)}B},$$

which is the elasticity of the bond demand to output. The cyclicality of the bond premium depends on the elasticity of the bond demand relative to the elasticity of the supply, given by $\frac{B'(Y)}{B(Y)}Y$.

2. there exists $Y \in (Y^{\min}, Y^*]$ such that

$$bp(Y) < \frac{1-\beta}{\beta}; \tag{22}$$

3. $\phi_Y > \overline{\phi}_Y$, where $\overline{\phi}_Y$ is the solution to

$$bp\left(Y^{LT}\left(\overline{\phi}_Y\right)\right) = \frac{1-\beta}{\beta}, \tag{23}$$

and $Y^{LT}\left(\overline{\phi}_Y\right) = \left(1 - \frac{R^*-1}{\overline{\phi}_Y}\right) Y^*$.

Condition (21) states that the bond premium is positive for all admissible levels of output, so that the retirees' budget constraint (4) is binding in any steady state of the economy. Using our assumed functional forms, condition (21) implies that $(1-\delta\kappa)Y > (1-\nu)Y - F + B^g$ for all $Y \in [Y^{\min}, Y^*]$. If B^g and $1-\nu$ are small, then the supply of safe assets in the economy is not sufficiently large to sustain perfect consumption smoothing, which leads to a positive bond premium. Condition (22) implies that there exists a level of output greater than $Y^{\min}$ such that the Euler equation (18) admits a solution with a weakly positive nominal rate. This condition guarantees the existence of a steady-state equilibrium. Finally, condition (23) guarantees that the full-employment steady state is the unique steady state with a positive interest rate.

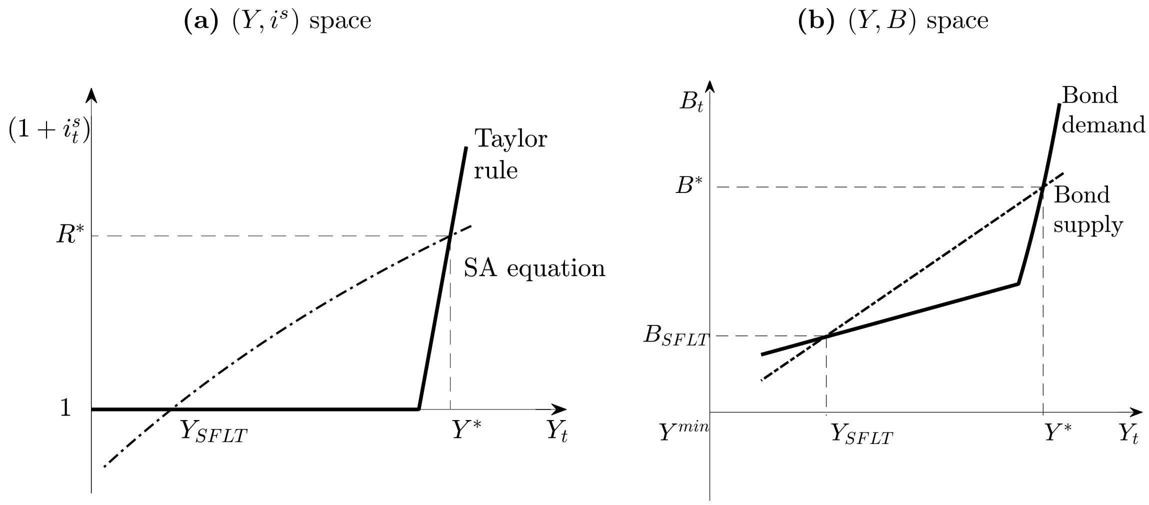
We are ready to characterize the steady-state equilibria of this economy. The next proposition establishes the existence of a full-employment steady-state equilibrium when the bond premium is counter-cyclical.

Proposition 1 (Existence of a Full-Employment Steady State) *Suppose Assumption 1 is satisfied and the bond premium is counter-cyclical. Then, there exists a unique full-employment steady-state equilibrium. Moreover, the full-employment steady state is the unique steady state with a positive nominal interest rate. If $\alpha = 0$, the full-employment steady state is locally determinate.*

A full-employment steady state exists as long as the bond premium is not so large as to push the natural rate of interest below zero. This is the equilibrium that the monetary authority seeks to implement.[28] However, Proposition 1 does not preclude the existence of other steady-state equilibria that feature involuntary unemployment. Next, we show that

[28] Proposition 1 establishes conditions for determinacy for the case in which there is no capital. In the quantitative exercise of Section 5, we check numerically the determinacy of the full-employment steady-state equilibrium in a model with capital.

Figure 4: Self-Fulfilling Liquidity Trap



when the bond premium is counter-cyclical, the full-employment steady-state equilibrium can co-exist with a liquidity trap steady state featuring positive unemployment. Because this steady state co-exists with the full-employment steady state and can be the equilibrium of the economy if agents' beliefs coordinate on it, we label it a *Self-Fulfilling Liquidity Trap (SFLT).*

Proposition 2 (Existence of a Self-Fulfilling Liquidity Trap (SFLT)) *Suppose Assumption 1 is satisfied. Moreover, assume that the bond premium is counter-cyclical and*

$$bp(Y^{\min}) \geq \frac{1-\beta}{\beta}.$$

Then, the economy features two steady-state equilibria: one with full employment and another with involuntary unemployment, a negative output gap, and a zero nominal interest rate. If $\alpha = 0$*, the SFLT is locally indeterminate.*

Figure 4 plots an economy featuring a counter-cyclical bond premium.[29] There are two steady states: one with full employment and one with involuntary unemployment. It is crucial for the existence of the two equilibria that the (SA) relation in Panel (a) be upward sloping, i.e., that the bond premium decrease with the level of output.[30] As Panel (b) shows, this can happen only if the supply of bonds is more elastic than the demand of bonds conditional on i^s. This result also shows the importance of modeling an endogenous (pro-cyclical) supply of safe assets to capture rich economic interactions.

[29] Calibration: $\beta = 0.965$, $\alpha = 0.4$, $\nu = 0.5$, $A = 2$, $F = 0.4A$, $B^g = 0.05A$, $\delta = 0.1$, $\phi_Y = 0.2$, $\chi = 0.011$.

[30] The reader might conclude that the positive relation between Y and i^s in the SA equation implies an "inverted aggregate demand" logic, as in Bilbiie (2008), with non-standard business cycle dynamics. However, in Appendix G, we show that the model generates standard impulse response functions to a monetary shock.

The existence of an SFLT has been widely studied in the literature, starting with Benhabib et al. (2001*b*), who show how the non-linearity of the Taylor rule can give rise to an unintended steady-state equilibrium. Relative to the literature, our contribution in this paper is two-fold. First, we study the consequences of the SFLT in relation to the bond premium and its cyclical properties. This analysis is particularly relevant for the design of policies that can deal with sudden increases in the bond premium. Second, our analysis shows that the economy is not always exposed to an SFLT, and it characterizes the conditions that facilitate its appearance. In particular, if the bond premium is relatively low for all levels of output, then the full-employment steady-state equilibrium is unique even if the bond premium is counter-cyclical.

Corollary 1 (Full-Employment as the Unique Steady State) *Suppose Assumption 1 is satisfied and* $bp(Y^{\min}) < \frac{1-\beta}{\beta}$*. Then, the unique steady-state equilibrium of the economy features full employment.*

4 Policy in a Liquidity Trap

This section shows that policies aimed at boosting the supply of safe assets may backfire and generate an endogenous scarcity of privately produced safe assets that end up reducing welfare. We find that the bond premium response to government policy is a key determinant of the policy's effect. Moreover, we show that small and large interventions can have opposite effects, and we highlight the importance of *fiscal capacity* for a successful policy implementation.

We demonstrate these results with three different policies. We first consider government bond issuances. This is the natural instrument in an economy that suffers from a scarcity of safe assets. We then analyze the effects of government spending. We show that while both types of policies can have similar effects, government spending policies require a bigger intervention (and, therefore, fiscal capacity) to generate the same results as government bonds. Finally, we consider an extension with short- and long-term bonds, and study the effects of "Quantitative Easing" (QE) type of policies.

4.1 Government Bonds

In this economy, a safety trap is an equilibrium in which the bond premium is too high relative to a level that can sustain full employment. Thus, a natural intervention is to increase the supply of (safe) government bonds and rebate the proceeds to the households. Since the bond premium is decreasing in the total supply of safe assets, an increase in the supply of government bonds should reduce the bond premium and increase aggregate demand. However, when the supply of private safe assets is endogenous, the overall effect on the *total* supply of safe assets depends on the general equilibrium response of the private sector. In this section, we show that while small interventions can *reduce* the total supply of safe assets and generate a decrease in the steady-state level of output, a *credible* commitment to a *sufficiently large* intervention is always expansionary.

Consider an economy that is in a liquidity trap equilibrium, and the government implements a "small" increase in the supply of government bonds. From equation (18) we know that absent any change in the interest rate, a steady-state equilibrium requires no change of the bond premium relative to the initial steady state (i.e., before the change in the supply of government bonds). Formally, let $bp(Y; B^g)$ denote the bond premium defined in (17), augmented to explicitly account for the dependence on the supply of government bonds. Totally differentiating $bp(\cdot;\cdot)$, we get

$$\frac{\partial bp(Y; B^g)}{\partial Y} dY + \frac{\partial bp(Y(B^g); B^g)}{\partial B^g} dB^g. \tag{24}$$

Since a steady-state equilibrium requires that there be no change in the bond premium after the policy (recall that locally to a liquidity trap equilibrium the interest rate is constant at zero), we can equalize expression (24) to zero and rearrange to get

$$\frac{dY}{dB^g} = -\frac{\frac{\partial bp(Y(B^g); B^g)}{\partial B^g}}{\frac{\partial bp(Y; B^g)}{\partial Y}}, \tag{25}$$

where $\frac{\partial bp(Y;B^g)}{\partial Y} \neq 0$. Noting that $\frac{\partial bp(Y;B^g)}{\partial B^g} < 0$, equation (25) implies that the effect of government bonds on output depends entirely on the cyclicality of the bond premium, that is, on the sign of $\frac{\partial bp(Y;B^g)}{\partial Y}$. Proposition 3 shows that, for a small change in the supply of government bonds, and as long as households' expectations remain pessimistic (i.e., agents coordinate on the liquidity trap steady state), increased provision of public safe assets is contractionary when the bond premium is counter-cyclical.

Proposition 3 (Government Bonds in an SFLT) *Suppose that Assumption 1 is satisfied, the bond premium is counter-cyclical, and the economy is in a liquidity trap steady-state equilibrium. Then, in the neighborhood of the SFLT,* $\frac{dY}{dB^g} < 0$.

As noted above, an increase in the supply of government bonds reduces the bond premium, *ceteris paribus*. However, a steady state requires that the bond premium is equal to $\frac{1-\beta}{\beta}$ in a liquidity trap. Thus, when the bond premium is counter-cyclical, it is a reduction in output that pushes the bond premium up, which offsets the direct effect of government policy. This reduction in output reduces private bond issuances. Thus, small increases in the supply of government bonds are contractionary and *crowd out* private safe asset production.

Importantly, we focus here on equilibria in which agents' expectations are anchored around the initial equilibrium. Recall that the SFLT co-exists with the full-employment steady state. Thus, the policy intervention may change agents' expectations in such a way that the economy transitions to the full-employment steady state. While theoretically interesting, our model does not provide a sufficiently rich theory of expectations formation to allow us to study transitions to the other steady state. Moreover, we believe that small policy interventions are unlikely to coordinate agents' expectations on the good equilibrium. Such drastic changes usually require specific policies that generate a credible regime shift (see, e.g., Sargent, 1983).

However, a *suffiently large* intervention can successfully eliminate SFLTs by imposing a sufficiently low upper bound on the bond premium such that self-fulfilling pessimism becomes inconsistent with equilibrium. In particular, let B^{g*} be such that

$$bp(Y^{\min}; B^{g,*}) = \frac{1-\beta}{\beta}. \tag{26}$$

The next proposition shows that if the government commits to issue bonds above B^{g*} when it faces a liquidity trap, SFLTs cannot arise in equilibrium.

Proposition 4 (Large Interventions) *Suppose that Assumption 1 is satisfied, and the bond premium is counter-cyclical. Suppose that the government follows a bond rule $B^g(Y)$, with $B^g(Y^{\min}) > B^{g*}$, where B^{g*} is defined by* (26). *Then, the unique steady-state equilibrium of the economy features full employment and a positive nominal interest rate.*

Note that this policy requires a *discrete* intervention, i.e., an intervention sufficiently large that it precludes the existence of an SFLT. This logic is reminiscent of Krugman (2014)'s *timidity trap*. In our model, a small increase in government debt can be contractionary, justifying the government's timidity in carrying out such actions. However, if the government could commit to a sufficiently large intervention, the policy would be welfare-enhancing. Moreover, note that the intervention can be a purely off-equilibrium promise. For example, the bond rule could be such that $B^g(Y^*) < B^{g*}$, so that, if successful, the government never actually needs to increase the supply of bonds relative to its target under full employment.[31]

[31] Government bonds facilitate the redistribution of purchasing power from workers to retirees. While our assumption that only workers are taxed contributes to this channel, it is not necessary to obtain the results. Our results would go through if all agents were taxed equally, and, under some parametric assumptions, even if only retirees are taxed. The reason is that most of government debt is not paid but rolled over, and only workers buy new debt. This amounts to an effective transfer from workers to retirees.

However, for the intervention to achieve its objective, the policy announcement needs to be credible, which requires sufficient *fiscal capacity.* Suppose that the government is subject to the same constraints as the private sector, in that only debt that can be backed by (potentially off-equilibrium) taxation power is deemed safe. That is, the government faces the following constraint:

$$B^g \leq \tau^{\max} Y,$$

where $\tau^{\max}$ is the maximum tax rate the government can implement. We say that a policy rule $B^g(Y)$, with $B^{g'}(\cdot) \leq 0$, is *credible* if and only if

$$B^g(Y^{\min}) \leq \tau^{\max} Y^{\min},$$

that is, the government's taxation power is sufficient to back the outstanding government bonds even in the worst-case scenario, i.e., $Y = Y^{\min}$.[32] This is a relatively standard constraint on the government's ability to provide safe assets that are not subject to roll-over risk (see, e.g., Calvo (1988) and Cole and Kehoe (2000)). More recently, He et al. (2019) argue that the safety of a country's debt is decreasing in roll-over risk. Suppose, to the contrary, that the minimum supply necessary to preclude the SFLT is such that $B^{g*} > \tau^{\max} Y^{\min}$, so that the government cannot credibly commit to issuing enough bonds to avoid an SFLT. In this case, an increase in the supply of government bonds to $\tau^{\max} Y^{\min}$ would be contractionary. Thus, sufficient fiscal capacity is crucial for the implementation of a successful policy intervention. The next proposition summarizes these results.

Proposition 5 (Fiscal capacity and safe asset provision) *Suppose that Assumption 1 is satisfied, and the bond premium is counter-cyclical. Suppose that the government's fiscal capacity is given by* $\overline{B}^g = \tau^{\max} Y^{\min}$. *Let* B^{g*} *be defined by* (26)*. If* $\overline{B}^g > B^{g*}$*, a bond rule* $B^g(Y)$ *with* $B^{g'}(\cdot) < 0$ *precludes the possibility of an SFLT equilibrium if and only if* $B^g(Y^{\min}) > B^{g*}$.

The fiscal capacity consideration illustrates a reason why large interventions may not be feasible in practice. An additional challenge, highlighted in the literature proposing *gradualism* in policy decisions, is that the government might not know with certainty where the fiscal limit is. For example, an intervention that is "too large" could make the government bonds lose their "safety" properties because it may "spook" the markets (Brainard, 1967; Bernanke, 2004).

[32] One could relax the constraint to allow for some unbacked debt as long as the amount of safe debt that the government can issue is still related to its taxation capacity.

4.2 Government Spending

An alternative policy instrument available to the government is spending. Government spending can affect the bond premium indirectly through increases in aggregate demand. As we found with government bonds, the effect of government spending depends on the cyclicality of the bond premium. Despite the two policy instruments' similar effects on output, we show that government bond issuances dominate government spending in terms of their fiscal requirements.

We extend the model of Section 3 to incorporate government spending as a policy tool. The budget constraint of the government is now given by $B_t^g = \frac{B_{t+1}^g}{1+r_t} + T_t^g - G_t$, where $1+r_t \equiv \frac{1+i_t^s}{1+\pi_{t+1}}$, and G_t denotes real government spending. Moreover, the resource constraint of the economy is now given by

$$(1-\chi)C_t^w + \chi B_t + I_t + G_t = Y_t.$$

Consider an economy that is in a liquidity trap steady state, and the government increases its spending. For a given level of C^w, B, and I, an increase in G increases aggregate demand. The increase in aggregate demand relaxes the firms' bond issuance constraint, increasing, *ceteris paribus*, the supply of bonds, which is a force towards a lower bond premium. To restore equilibrium, output needs to adjust. When the bond premium is counter-cyclical, the effect that government spending has on the equilibrium of the economy depends on the magnitude of the intervention.

Proposition 6 (Government Spending in a Liquidity Trap) *Suppose that Assumption 1 is satisfied, the bond premium is counter-cyclical, $B^g \in (0, B^{g^*})$ and $G = 0$. Consider an economy that is in a liquidity trap, and the government increases government spending by dG. Then, in the neighborhood of an SFLT, $\frac{dY}{dG} < 0$. Moreover, there exists G^* such that, if $G > G^*$, the unique steady state of the economy features full employment.*

Proposition 6 states that government spending generates results in terms of output similar to the results obtained with government bonds. However, the fiscal costs are higher when government spending is used as a policy tool. Thus, government bond issuances are a superior policy.

Proposition 7 (Superiority of B^g over G) *Suppose that Assumption 1 is satisfied, and the bond premium is counter-cyclical. Suppose the economy is in a liquidity trap, and $B^g \in (0, B^{g*})$. The fiscal capacity necessary to rule out an SFLT is smaller under a government bond policy than under a government spending policy.*

4.3 Long-Term Bonds, Fiscal Capacity and QE

Next, we consider an extension of the model that incorporates long-term government bonds.[33] This extension allows us to perform two important exercises. First, we consider how the presence of long-term bonds affects the government's fiscal capacity requirement. Since long-term bonds are less exposed to roll-over risk, they should use less fiscal space. However, if long-term bonds' convenience yield is lower than that of short-term bonds (because they are less safe or liquid), the fiscal cost of long-term bonds will be higher. We find that the first effect always dominates for fiscal capacity. Second, we study the effects of an asset purchase program or *quantitative easing*. This policy was extensively used after the Great Recession and became part of the standard monetary toolkit since then.

We now assume that the government can issue one-period nominal bonds as well as perpetuities that pay exponentially decaying (nominal) coupons, with decaying factor $\rho \in (0, 1)$ (see Woodford, 2001). To capture the differences in non-pecuniary services between these two bonds, we assume that there is a cost $\zeta > 0$ associated with *selling* long-term bonds (this cost is rebated back to households). That is, if an agent sells a unit of long-term bond at price Q_t, the agent receives only $(1 - \zeta)Q_t$ from the sale. Then, if $\zeta > 0$, the yield on long-term bonds is higher than that of short-term bonds. Let $B_t^{s,g}$ denote the (real) supply of short-term government bonds and $B_t^{l,g}$ denote the (real) supply of long-term government bonds. We show in Appendix D that the characterization of equilibrium is analogous to the one in Section 3, except that the total supply of safe assets B_t is now given by $B_t^{\zeta} \equiv B_t^p + B_t^{s,g} + [1 + (1 - \zeta)\rho Q_t]B^{l,g}$, where B_t^p is the supply of (short-term) private safe assets.

Fiscal Capacity. We define fiscal capacity as the maximum amount of debt obligations that the government can pay out of taxation in the case that it cannot roll-over its debt and the economy is at its minimum level of output. That is, fiscal capacity amounts to the following constraint:

$$B^{s,g} + B^{l,g} \leq \tau^{\max} Y^{\min},$$

where $\tau^{\max}$ is the maximum tax rate the government can impose on the economy. For short-term bonds, the full face value $B^{s,g}$ enters into the constraint. In contrast, for long-term bonds, only the coupon $B^{l,g}$ uses fiscal capacity rather than the full market value of the bonds. That is, long-term bonds partially insulate the government from roll-over risk.

Consider two economies. In one economy, the government only issues short-term debt. In

[33]To simplify the analysis, we assume that labor is the only factor of production. We study the model with long-term bonds and capital in Section 5.

the other economy, the government only issues long-term debt. To make the two economies comparable, assume that $B^{s,g}$ and $B^{l,g}$ are such that the full-employment steady-state equilibrium coincides in both economies. That is, output, workers' and retirees' consumption, the bond premium, and the natural rate are the same in both economies. This is achieved if and only if

$$B^{s,g} = [1 + (1 - \zeta)\rho Q(Y^*)]B^{l,g},$$

where $Y^* = A(1 - \chi)^\nu$. It is straightforward to see that if $\zeta < 1$ and $\rho > 0$, then $B^{l,g} < B^{s,g}$, which implies that long-term debt requires less fiscal capacity for the same supply of government safe assets. Moreover, a lower value of ζ increases the net value of long-term government bonds, so a lower stock of debt $B^{l,g}$ is needed to get the same total value of safe assets. Thus, a lower ζ relaxes the government's fiscal capacity constraint.

Proposition 8 (Fiscal Capacity and the Duration of Government Debt) *Suppose that Assumption 1 is satisfied and the bond premium is counter-cyclical. The fiscal capacity required to preclude the possibility of an SFLT is decreasing in the duration of government debt and increasing in the transaction cost ζ.*

Notably, while long-term debt relaxes the fiscal capacity constraint, the lower convenience on long-term bonds implies a higher fiscal cost.

Quantitative Easing. Suppose the economy is in an SFLT. Consider a one time intervention that exchanges a fraction $\psi \in (0, 1)$ of the long-term bonds for short-term bonds. That is, we have

$$B^{s,g'} - B^{s,g} = \psi Q^{SFLT} B^{l,g}.$$

where $B^{s,g'}$ denotes the supply of short-term government bonds after the intervention and Q^{SFLT} denotes the price of the long-term government bond in the SFLT. We assume that after the swap, the government keeps the debt at the post-intervention levels forever (lump-sum transfers adjust as necessary). The next proposition shows that, in the neighborhood of the SFLT, output decreases with the intervention.

Proposition 9 (Quantitative Easing in an SFLT) *Suppose that Assumption 1 is satisfied, the bond-premium is counter-cyclical, and the economy is in a liquidity trap steady-state. Suppose that the government intervenes by exchanging a fraction $\psi > 0$ of long-term bonds for short-term bonds. The change in the value of the supply of government safe assets is $\Delta B^g = \zeta\psi\beta\rho Q^{SFLT} B^{l,g} > 0$, where Q^{SFLT} denotes the price of the long-term government bond in the SFLT. Then, in the neighborhood of the SFLT, $\frac{dY}{d\psi} < 0$.*

The result in Proposition 9 is analogous to the one in Proposition 3. QE swaps low-convenience assets for high-convenience assets. This swap effectively increases the supply of public safe assets in the economy. A small intervention that keeps the economy in the SFLT crowds out the private production of safe assets and reduces the level of output. It is immediate, then, that a sufficiently large intervention rules out SFLT equilibria, triggering a transition to the full-employment equilibrium.

4.4 Other Extensions

Here, we discuss two additional extensions to the previous analysis. First, we show that if the bond premium is pro-cyclical, the economy can feature a liquidity trap in which increases in the supply of government bonds are always expansionary. Then, we extend the model to allow for inflation in steady state and show that SFLTs become even more likely.

Fundamental Liquidity Traps. Consider the model from Section 3, but assume that the bond premium is pro-cyclical, that is, $bp'(Y) > 0$. Then, under Assumption 1, there exists a unique steady-state equilibrium in this economy. If $bp(Y^*) \leq \frac{1-\beta}{\beta}$, the unique steady state features full employment. In contrast, if $bp(Y^*) > \frac{1-\beta}{\beta}$, the unique steady-state equilibrium features positive unemployment, negative output gap and a zero nominal interest rate. Because this steady state does not co-exist with the full-employment steady state, we call it a *Fundamental Liquidity Trap (FLT)*. The next proposition characterizes these results.

Proposition 10 (Existence of a Fundamental Liquidity Trap (FLT)) *Suppose Assumption 1 is satisfied and the bond premium is pro-cyclical. Then, if $bp(Y^*) \leq \frac{1-\beta}{\beta}$, the unique steady-state equilibrium features full employment. In contrast, if $bp(Y^*) > \frac{1-\beta}{\beta}$, the unique steady-state equilibrium features involuntary unemployment, a negative output gap and a zero nominal rate.*

An FLT is the type of liquidity trap most commonly studied in the literature. It is the result of structural characteristics of the economy that generate a negative natural rate of interest. This has important consequences for the effects of policy interventions.

Proposition 11 *Suppose Assumption 1 is satisfied, the bond premium is pro-cyclical, and the economy is in a liquidity trap. Then, $\frac{dY}{dB^g} > 0$ and $\frac{dY}{dG} > 0$.*

Proposition 11 shows that even small interventions are expansionary in FLTs. Note that even though these results are the opposite to the ones in Proposition 3, Proposition 4 implies that large interventions can be expansionary independently of the nature of the trap affecting the economy. This result can be of particular relevance for policymakers who might find it challenging to identify in real time the exact shock that brought the economy to a liquidity trap just from observing aggregate dynamics. In this sense, large interventions can be *robust* policies governments can implement in a liquidity trap characterized by a scarcity of safe assets when the exact type of trap (SFLT or FLT) cannot be determined.

Inflation. Our analysis in Section 3 assumed perfect nominal wage rigidity to isolate the role of the endogenous scarcity of safe assets in generating SFLTs. This assumption implies steady-state equilibria in which the inflation rate of the economy is always equal to zero. We showed that even in that case, pessimism about the private sector's capacity to issue safe or liquid financial assets, captured by the bond premium, can drive the economy to a liquidity trap. We close this section by extending our framework with an upward sloping inflation-output Phillips curve through a downward nominal wage rigidity assumption (Schmitt-Grohé and Uribe, 2017):

$$\frac{W_t}{W_{t-1}} \geq \gamma(u_t) \tag{27}$$

where $u_t = 1 - h_t$ is the unemployment rate of the economy, and $\gamma(\cdot)$ satisfies $\gamma(0) = 1$, $\gamma'(\cdot) < 0$, $\gamma''(\cdot) \leq 0$, and $\gamma(\cdot) > \beta - 1$. The assumption in Section 3 was a special case in which $\gamma(u_t) = 1$ for all u_t.[34]

A key takeaway emerges: the possibility of deflation in steady state expands the set of parameters consistent with an SFLT. On the one hand, it is now pessimism about either inflation, output, or the private sector's ability to produce safe assets that can lead to an SFLT. In all these cases, the SFLT is characterized by deflation and a high bond premium. Second, a lower level of the bond premium is sufficient for the existence of an SFLT. To see this, note that in this case, the GEE becomes

$$1 = \beta \frac{1}{1 + \pi(Y)} \left[1 + bp(Y)\right] \tag{28}$$

where $\pi(Y) = \gamma\left(u(Y)\right)$ and $u(Y) = 1 - \frac{1}{1-\chi}\left(\frac{Y^{1-\alpha\nu}}{A\kappa^{\alpha\nu}}\right)^{\frac{1}{(1-\alpha)\nu}}$. Thus, if Assumption 1 is satisfied and the bond premium is countercyclical, an SFLT in this economy exists if and only if

$$bp(Y^{\min}) \geq \frac{1 - \beta + \pi(Y^{\min})}{\beta}.$$

Given that $\pi(Y^{\min}) < 0$, then $\frac{1-\beta+\pi(Y^{\min})}{\beta} < \frac{1-\beta}{\beta}$, which was the condition for existence of

[34]Strictly speaking, in Section 3 we also assumed *upward* wage rigidity. None of the results depend on this assumption.

an SFLT in Section 3. Moreover, this result affects the policy implications. Let B^{g**} denote the level of government bonds such that

$$bp(Y^{\min}, B^{g**}) = \frac{1 - \beta + \pi(Y^{\min})}{\beta}$$

Then, $B^{g**} > B^{g*}$, where B^{g*} is the solution to (26). That is, the presence of deflation requires a larger intervention.[35] The next proposition summarizes these results.

Proposition 12 (Inflation and Bond Premia in SFLT) *Consider the economy in Section 3 augmented with the downward wage rigidity condition* (28)*. Suppose Assumption 1 is satisfied and the bond premium is counter-cyclical. A full-employment steady-state equilibrium exists. Moreover, an SFLT exists if and only if*

$$bp(Y^{\min}) \geq \frac{1 - \beta + \pi(Y^{\min})}{\beta},$$

with $\pi(Y^{\min}) < 0$*. The SFLT features lower inflation and higher bond premium than the full-employment steady state. Finally, the increase in the supply of government bonds necessary to preclude the existence of an SFLT is larger than when the nominal wage is fully rigid.*

5 Quantitative Model

This section presents a quantitative exercise to illustrate the dynamics associated with a self-fulfilling liquidity trap. The economy transitions from the full-employment steady state to the liquidity trap equilibrium after a small shock to agents' expectations. We use the calibrated model to calculate the size of the intervention necessary to preclude the existence of SFLTs.

Relative to the baseline model in Section 3, we make five main modifications (we present the full system of equations characterizing the equilibrium in Appendix F). First, we adopt the downward nominal wage rigidity assumption (27), to allow for changes in the rate of inflation in the different steady-state equilibria. We assume the following simple functional form: $\gamma(u_t) = [\bar{\gamma} + (1 - \bar{\gamma})(1 - u_t)]\pi^*$, where $\bar{\gamma} \in (0, 1)$ and π^* is the exogenous inflation rate in the full-employment steady state. Second, we assume that $u(C_t^W)$ and $v(B_t)$ are CRRA functions with common parameter σ. Third, we assume a monetary rule targeting deviations of the inflation rate and the unemployment rate from their targets, subject to a

[35] More generally, the bigger the drop in inflation at $Y^{\min}$ relative to the full-employment steady state, the larger the intervention needs to be to preclude the existence of an SFLT.

TABLE 4: PARAMETERS

Fixed parameters	Value	Source/Target
Discount Factor	$\beta = 0.965$	Standard
Price inflation at full emp.	$\pi^* = 1.02$	Standard
Depreciation rate	$\delta = 0.10$	Standard
Elas. intertemporal substitution	$\sigma^{-1} = 0.1$	(Best, Cloyne, Ilzetzki and Kleven, 2019)
Taylor rule coefficients	$(\phi_\pi, \phi_Y) = (1.5, 0.5)$	Standard
Jointly calibrated parameters	SFLT	
Span of Control	$\nu = 0.64$	
Production function parameter	$\alpha = 0.52$	
Retirement risk	$\chi = 0.017$	
Fixed Cost	$F = 10.70$	
Borrowing constraint	$(\eta, \phi) = (3.45, 0.29)$	
Government Bonds	$B^g = 3.07$	
TFP	$A = 8.56$	
Wage rigidity parameter	$\gamma = 0.88$	
Average duration of Govt Bonds	$\rho = 0.83$	
Targeted moments	data	model
Full emp Real interest rate	1.50%	1.50%
Full emp Profits to GDP	7.20%	6.99%
Investment to GDP	25%	24%
Capital Share	33%	32.84%
Full emp Govt Bonds to GDP	40%	40%
Duration of Govt Bonds in US	5 years	5 years
Unemployment in LT	7.00%	7.34%
Profits to GDP in LT	6.50%	6.57%
Inflation (net) in LT	1.00%	1.10%
$\uparrow$ in bond premia in LT	250 bps	268 bps

Notes: The table shows the parameter values of the model, as well as model and data moments used in the calibration.

ZLB constraint, as in a standard Taylor rule.[36] Fourth, we introduce long-term government bonds based on our modeling in Section 4.3.[37] Finally, we generalize the firm's borrowing constraint to $\frac{\widetilde{B}^p_{t+1}}{P_{t+1}} \leq \phi \left[\frac{\Pi_{t+1}}{P_{t+1}} + \left(\frac{\Pi_{t+1}}{P_{t+1}}\right)^\eta\right]$, where $\phi > 0$ and $\eta > 0$ are parameters governing the degree of financial frictions and the elasticity of the borrowing constraint with respect to profits. Recall that the elasticity of the supply of bonds is a key determinant of the cyclicality of the bond premium. The parameters ϕ and η provide a higher degree of flexibility to match key moments in the data.

We solve the model numerically. The top two panels in Table 4 summarize the parameters'

[36] We use the following policy rule $1 + i^s_t = \max\left\{1, R^* + \phi_\pi\left(\frac{\pi_t}{\pi^*} - 1\right) + \phi_Y\left(\frac{Y_t}{Y^*} - 1\right)\right\}$, where Y^* is output at the full employment steady state, and R^* is the gross real interest rate at the full-employment steady state.

[37] We assume zero transaction costs on long-term bonds, that is, $\zeta = 0$. In Appendix D, we discuss a quantitative case with $\zeta > 0$.

values in the quantitative model at an annual frequency. We set the discount factor to 0.965, the inflation target to 2%, and the capital depreciation rate to $\delta = 0.10$, the inflation and the output gap reaction coefficients in the policy rule at 1.50 and 0.50 respectively. These are standard values in the business cycle literature. The inverse of the elasticity of inter-temporal substitution is set to $\sigma = 10$ consistent with estimates by Best et al. (2019).[38]

The remaining ten parameters are calibrated using a minimum distance approach that minimizes the squared deviation of the model-implied values from the targets in the data. The model-implied moments and corresponding data values are shown in Table 4. Here, we briefly describe the selection of the data targets. In the post-war US data (1953Q1 – 2008Q2), the average real interest rate is 150 bps, average profits (net of interest expenses) to GDP ratio is 7.2%, average investment to GDP ratio (including durable consumption) is 25%, and the capital share is 30%. Following Del Negro et al. (2017*b*) and Krishnamurthy and Vissing-Jorgensen (2012), we interpret B^g as Treasury securities, and target Treasury securities over GDP ratio of 40%. For the liquidity trap steady state, we target an unemployment rate of 7% and an average inflation decline of 100 bps relative to the target. In the four quarters following 2008Q3, the average profits to GDP ratio declined to 6.5%. During the same period, the convenience yield increased by 250 basis points on average, as estimated by Del Negro et al. (2017*b*). Table 4 shows that the model fits these targets closely.[39][40] Moreover, in Appendix G we show that the model produces impulse responses to a monetary policy shock that are consistent with the data.

The solid blue line in Figure 5 plots the transitional dynamics from a full-employment steady state to the self-fulfilling liquidity trap steady state. The transition to the liquidity trap is triggered by a decline of households' confidence in the economy, which is modeled as a 1% shock to employment expectations.[41] The interaction between this pessimism and the counter-cyclical bond premium puts upward pressure on the bond premium. The central bank reacts to this recessionary pressure by lowering the policy rate until it hits the ZLB. In the liquidity trap steady state, bond premium and unemployment remain elevated, while

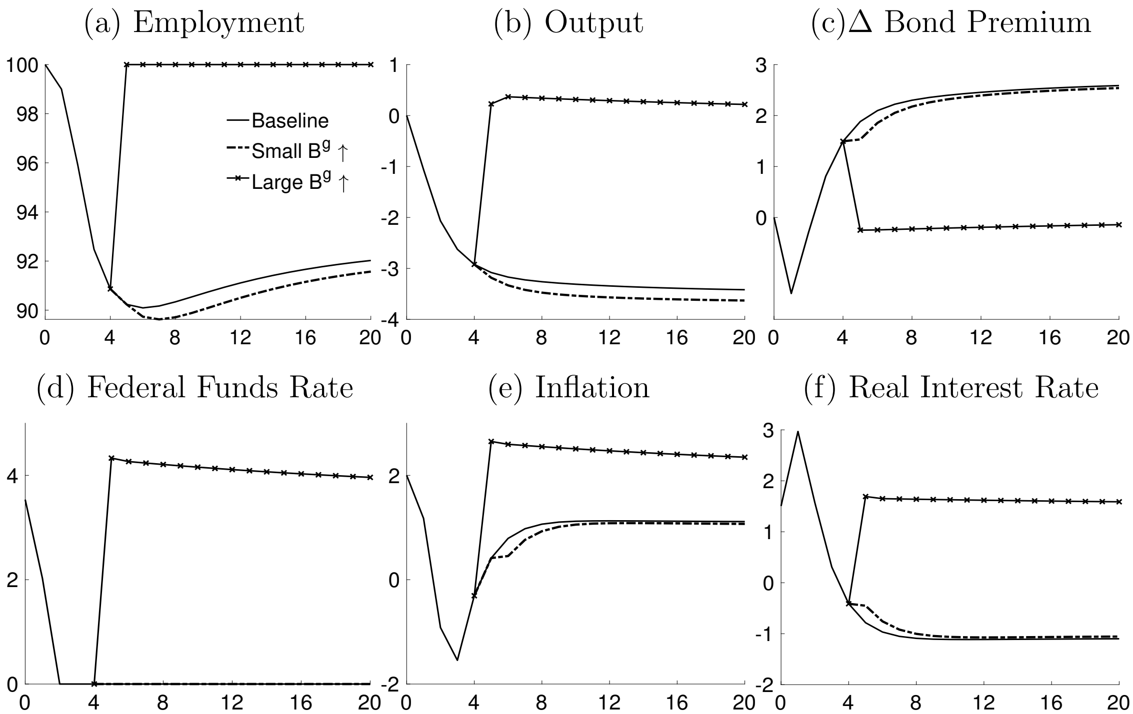
[38]Note that σ governs the EIS as well as the risk aversion from the retirement risk.

[39]The fixed value of $\beta = 0.965$ and targeted real interest of 150 bps implies that the convenience yield at full employment is 209 bps, equal to the average spread of Moody's BAA Corporate Bond yield over 10 year treasury constant maturity rate until 2008Q2. We obtain the data from FRED series `BAA10Y`, `CP_GDP`, `GDPDEF`, `GDPI`, `PCGDP`, `TB3MS`, and `UNRATE`. The real interest rate is constructed using the 3-month Treasury Bill yield and the GDP deflator.

[40]Estimates of span of control are usually in the range of 0.85 and 0.92 in the literature. See also Basu and Fernald (1997), and Atkeson and Kehoe (2005). Given that the fixed cost is rebated back to the households in our model, the labor share in the model is close to the standard estimates of 70 percent, assuming that the fixed cost is part of the returns to labor.

[41]While the SFLT equilibrium is locally indeterminate, the path from the full-employment steady state can be pinned down by setting an initial value for one of the endogenous variables of the model.

Figure 5: Transitional dynamics



Note: The solid line (Baseline) plots the transition of employment, output, change in bond premium, nominal interest rate, inflation rate, and real interest rate from full-employment steady state to the liquidity trap steady state. The dashed line (Small $B^g \uparrow$) plots the transition from year 4 of the baseline path to a new liquidity trap steady state with 1 p.p. higher government debt. The solid line marked with x (Big $B^g \uparrow$) plots the transition from year 4 of the baseline path to the full-employment steady state under the robust policy. Time is in years. Employment is in percentage points. Output is measured as the percentage deviation of output from the full-employment steady-state output. Δ bond premium represents the annual percentage point change in the bond premium relative to the full-employment steady-state bond premium. The nominal interest rate is the annualized level of the nominal interest rate in percentage points. Inflation is measured in annualized percentage points. The inflation target of the central bank is 2%. The real interest rate is the annualized level of the real interest rate in percentage points.

(annual) inflation runs persistently below the central bank's target.

We further use this section's model to quantify the amount of public debt that can eliminate the SFLT. This is the quantitative counterpart of the results in Proposition 3 and 4. We model the government's intervention as a state-dependent rule: $B_t^g = (1 + \psi u_t)B_0^g$, where B_0^g is the level of debt in full-employment, and $\psi > 0$ regulates the government's commitment to increase the supply of debt with unemployment rate u_t. When the government's commitment is not sufficiently large ($\psi = 0.13$ in our exercise), the intervention can exacerbate the recession, as seen in the dashed red line in Figure 5. In contrast, a sufficiently large commitment to supplying debt can eliminate the liquidity trap equilibrium. This can be achieved with a $\psi \geq 1$ in the quantitative model. This value of ψ translates into an increase of government debt of 20% as a fraction of the output at the SFLT, Y^{SFLT}, or alternately an increase of government debt of 16% as a fraction of the full employment output. From Proposition 5, if the maximum tax rate that the government can implement is at least 14.70% (i.e., $\tau^{max} > 0.147$), the government can credibly issue new safe assets by up to 20% of Y^{SFLT}.[42] In this case, the intervention eliminates the SFLT, and the economy transitions back to the full-employment steady state. Note that the fiscal capacity of 14.70% is just for covering the cost of servicing the debt rather than financing the entire government budget. In this sense, the fiscal capacity required is economically significant. The solid line with cross marks plots the transition of the economy to the full-employment steady state when the government announces a sufficiently large intervention. Notably, if the announcement is credible, the actual intervention can be orders of magnitude smaller than the off-equilibrium promise. The belief that the government is willing to do whatever it takes to sustain full-employment precludes the adverse outcomes.

6 Conclusion

In this paper, we developed a theory of endogenous supply of safe assets and derived its implications for the macroeconomy. When the bond premium is counter-cyclical, the economy admits two types of steady-state equilibria: a full-employment steady state and an SFLT. Pessimism about the state of the economy can trigger a transition from full employment to the liquidity trap. In an SFLT, small issuances of government debt crowd out private debt and exacerbate the pessimism-driven recession. In the data, we found evidence of a counter-cyclical bond premium and a pro-cyclical supply of safe assets, consistent with the model's assumptions. We proposed robust policies that prevent the existence of self-fulfilling traps. We further underscored the importance of fiscal capacity in a government's ability to manage liquidity traps. Finally, we built a quantitative model calibrated to match the evolution of employment and asset prices during the Great Recession and showed that the model is able to generate aggregate dynamics consistent with the Great Recession. Moreover, we used the model to calculate the size of the fiscal response necessary to preclude the existence of SFLTs and found that a promise to increase the government debt-to-GDP ratio by 20 percentage points would be sufficient.

Data Availability Statement

The data underlying this article are available on Zenodo at https://dx.doi.org/.

[42]Recall that fiscal capacity is calculated at $Y^{\min}$. We target 40% of debt-to-GDP ratio at the full-employment steady state, which becomes 50% at the SFLT. Adding a 20% increase from the robust policy leads to a debt-to-GDP ratio in the SFLT of 70%.

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A Proofs

Proof of Proposition 1. If the bond premium is counter-cyclical, Assumption 1 guarantees that $bp(Y^*) < \frac{1-\beta}{\beta}$. Thus, there exists $R^* > 1$ such that $1 = \beta R^*[1 + bp(Y^*)]$, and the monetary rule implies $1 + i^s = R^*$. Thus, a full-employment steady state exists. Since R^* is unique, there is a unique full-employment steady state. In the full-employment steady state, $H = 1 - \chi$, hence $Y^* = \left[A\kappa^{\alpha\nu}(1-\chi)^{(1-\alpha)\nu}\right]^{\frac{1}{1-\alpha\nu}}$.

To see that there is no other steady state with a positive nominal interest rate, rewrite (18) as

$$1 + i^s = \frac{1}{\beta}\frac{1}{1 + bp(Y)}$$

Then

$$\frac{\partial\ 1 + i^s}{\partial Y} = -\frac{1}{\beta}\frac{bp'(Y)}{[1 + bp(Y)]^2} > 0$$

and

$$\frac{\partial^2\ 1 + i^s}{\partial Y} = -\frac{1}{\beta}\frac{bp''(Y)\left[1 + bp(Y)\right] - 2\left(bp'(Y)\right)^2}{\left[1 + bp(Y)\right]^3}$$

where, given our expressions,

$$\begin{aligned}
bp(Y) &= \frac{\chi}{1-\chi}\frac{(\nu - \delta\kappa)Y + F - B^g}{(1-\nu)Y - F + B^g} \\
bp'(Y) &= -\frac{\chi}{1-\chi}\frac{(1-\delta\kappa)(F - B^g)}{\left[(1-\nu)Y - F + B^g\right]^2} < 0 \\
bp''(Y) &= 2\frac{\chi}{1-\chi}\frac{(1-\nu)(1-\delta\kappa)(F - B^g)}{\left[(1-\nu)Y - F + B^g\right]^3} > 0
\end{aligned}$$

hence

$$\begin{aligned}
bp''(Y)\left[1 + bp(Y)\right] - 2\left(bp'(Y)\right)^2 = 2&\left(\frac{\chi}{1-\chi}\frac{(1-\delta\kappa)(F - B^g)}{\left[(1-\nu)Y - F + B^g\right]^2}\right)^2 \\
&\frac{(1-\chi)(1-\nu) + \chi(\nu - \delta\kappa)}{\chi(1-\delta\kappa)}\frac{(1-\nu)Y - (F - B^g)}{F - B^g} > 0
\end{aligned}$$

and $\frac{\partial^2\ 1+i^s}{\partial Y^2} < 0$. Let Y^{LT} be defined as

$$Y^{LT} \equiv \left(1 - \frac{R^* - 1}{\phi_Y}\right)Y^*$$

Then, a necessary and sufficient condition for the uniqueness of steady-state with positive nominal rate is that,

$$\frac{1}{\beta}\frac{1}{1 + bp(Y^{LT})} > 1 \tag{29}$$

Since Y^{LT} is increasing and continuous in ϕ_Y, $bp(\cdot)$ is decreasing and continuous in Y, and $bp(Y^*) =$

R^*, there exists $\overline{\phi}_Y$ such that (29) holds if and only if $\phi_Y > \overline{\phi}_Y$.

For the local determinacy of the full-employment steady state, we need to study the dynamic properties of the model in a neighborhood of the steady state. Assuming $\alpha = 0$, the system of equations characterizing the equilibrium is given by

$$\beta \frac{1+i_t^s}{1+\pi_{t+1}} \left[(1-\chi) \frac{C_t^w}{C_{t+1}^w} + \chi \frac{C_t^w}{B_{t+1}} \right] = 1$$

$$C_t^w = \frac{Y_t - \chi B_t}{1-\chi}$$

$$B_t = (1-\nu) Y_t - F + B^g$$

$$1+\pi_{t+1} = \left(\frac{Y_{t+1}}{Y_t} \right)^{\frac{1-\nu}{\nu}}$$

$$Y_t = A H_t^{\nu}$$

$$1 + i_t^s = R^* + \phi_Y \left(\frac{Y_t}{Y^*} - 1 \right)$$

Log-linearizing the system around the full-employment steady state, we get

$$i_t^s - \pi_{t+1} - r^* + \beta R^* (1-\chi) \left(c_t^w - c_{t+1}^w \right) + \beta R^* \chi \frac{C^{w*}}{B^*} \left(c_t^w - b_{t+1} \right) = 0$$

$$c_t^w = \frac{(1-\chi(1-\nu)) Y^*}{(1-\chi) C^{w*}} y_t$$

$$b_t = \frac{(1-\nu) Y^*}{B^*} y_t$$

$$\pi_{t+1} = \frac{1-\nu}{\nu} (y_{t+1} - y_t)$$

$$i_t^s - r^* = \frac{\phi_Y}{R^*} y_t,$$

where $r^* \equiv \log(R^*)$. We can combine these equations to obtain a difference equation in y_t

$$\left[\frac{1-\nu}{\nu} + \beta R^* \frac{(1-\chi(1-\nu)) Y^*}{C^{w*}} + \beta R^* \chi \frac{C^{w*}}{B^*} \frac{(1-\chi(1-\nu)) Y^*}{(1-\chi) C^{w*}} + \frac{\phi_Y}{R^*} \right] y_t = \left[\frac{1-\nu}{\nu} + \beta R^* \frac{(1-\chi(1-\nu)) Y^*}{C^{w*}} + \beta R^* \chi \frac{C^{w*}}{B^*} \frac{(1-\nu) Y^*}{B^*} \right] y_{t+1}$$

The system is locally determinate if and only if

$$\beta R^* \chi \frac{(1-\chi(1-\nu)) Y^*}{B^*} + \frac{\phi_Y}{R^*} > \beta R^* \chi \frac{C^{w*}}{B^*} \frac{(1-\nu) Y^*}{B^*}$$

or, after some algebra,

$$\frac{\phi_Y}{R^*} > \beta R^* \frac{\chi}{1-\chi} \frac{Y^*}{B^*} \frac{F - B^g}{B^*}.$$

Note that

$$bp(Y) = \chi\left(\frac{C^w(Y)}{B(Y)} - 1\right) = \frac{\chi}{1-\chi}\left(\frac{Y - \chi((1-\nu)Y - F + B^g)}{(1-\nu)Y - F + B^g} - (1-\chi)\right)$$

hence

$$bp'(Y) = -\frac{\chi}{1-\chi}\frac{F - B^g}{B(Y)^2}$$

and

$$\beta^* R^* = \frac{1}{1 + bp(Y)},$$

hence, the system is locally determinate if and only if

$$\phi_Y > -\frac{bp'(Y^*)Y^*}{1 + bp(Y^*)}R^* \tag{30}$$

But if $\phi_Y > \overline{\phi}_Y$, (30) is immediately satisfied. To see this, note that if $\phi_Y > \overline{\phi}_Y$, the slope of $1 + i^s = \frac{1}{\beta}\frac{1}{1+bp(Y)}$ is smaller than ϕ_Y at $Y = Y^*$. That is

$$-\frac{1}{\beta}\frac{bp'(Y^*)}{[1 + bp(Y^*)]^2} < \frac{\phi_Y}{Y^*}$$

Using that $R^* = \frac{1}{\beta}\frac{1}{1+bp(Y^*)}$, we can rewrite this expression as

$$-\frac{bp'(Y^*)}{1 + bp(Y^*)}R^* < \frac{\phi_Y}{Y^*}$$

which coincides with the condition for determinacy. ■

Proof of Proposition 2. From Proposition 1, we know that the full-employment steady state exists and there is no other steady state with a positive nominal interest rate. Moreover, since $bp(Y^{\min}) \geq \frac{1-\beta}{\beta}$, by continuity of $bp(Y)$, there exists $\tilde{Y} \in [Y^{\min}, Y^*)$ such that $1 = \beta[1 + bp(\tilde{Y})]$, which establishes the existence of a liquidity trap steady state. Finally, since $bp'(Y) < 0$, $\phi_Y = 0$ is not sufficient for determinacy when $\alpha = 0$, so the SFLT is locally indeterminate. ■

Proof of Corollary 1. Again, from Proposition 1, we know that the full-employment steady state exists and there is no other steady state with a positive nominal interest rate. If the bond premium is counter-cyclical, $bp(Y^{\min}) < \frac{1-\beta}{\beta}$ implies that $i^s = 0$ is never part of a solution to equation (18), and hence the economy does not admit an SFLT. ■

Proof of Proposition 3. Immediate from equation (25). ■

Proof of Proposition 4. Under the rule $B^g(Y)$, the economy is under the conditions of Corollary 1. ■

Proof of Proposition 5. Immediate from Propositions 3 and 4. ■

Proof of Proposition 6. In a liquidity trap, the bond premium must remain unchanged after the change in G. The bond premium is given by

$$bp(Y;G) = \chi\left(\frac{C^w(Y;G)}{B(Y)} - 1\right)$$

where

$$C^w(Y;G) = \frac{(1-\delta\kappa)\,Y - \chi B\,(Y) - G}{1-\chi} = \frac{(1-\delta\kappa-(1-\nu)\,\chi)\,Y + \chi\,(F - B^g) - G}{1-\chi},$$
$$B(Y) = (1-\nu)Y - F + B^g.$$

Fully differentiating $bp(Y;G)$ and equalizing to zero, we get

$$\frac{\partial bp(Y;G)}{\partial Y}dY + \frac{\partial bp(Y;G)}{\partial G}dG = 0,$$

or

$$\frac{dY}{dG} = -\frac{\frac{\partial bp(Y;G)}{\partial G}}{\frac{\partial bp(Y;G)}{\partial Y}},$$

where we have used that $\frac{\partial bp(Y;G)}{\partial Y} \neq 0$. Note that

$$\frac{\partial bp(Y;G)}{\partial G} = \chi\frac{\partial C^w(Y;G)}{\partial G}\frac{1}{B(Y)}.$$

Given Y, $C^w(Y;\cdot)$ is decreasing in G, so $\frac{\partial bp(Y;G)}{\partial G} < 0$. Thus, $\frac{dY}{dG} > 0$ if the bond premium is pro-cyclical and $\frac{dY}{dG} < 0$ if the bond premium is counter-cyclical.

Suppose the bond premium is counter-cyclical, $bp(Y^{\min}) > \frac{1-\beta}{\beta}$, and $B^g \in (0, B^{g*})$. Let G^* be given by the solution to

$$bp(Y^{\min}; G^*) = \frac{1-\beta}{\beta}$$

or

$$G^* = (1-\delta\kappa)\,Y^{\min} - \left[1 + \frac{1-\chi}{\chi}\frac{1-\beta}{\beta}\right]B^g$$

Then, if $G > G^*$, $bp(Y^{\min}; G^*) < \frac{1-\beta}{\beta}$ and by Corollary 1 the unique steady state of the economy features full employment. ■

Proof of Proposition 7. Take an economy with a counter-cyclical bond premium, $bp(Y^{\min}) > \frac{1-\beta}{\beta}$, $B^g \in (0, B^{g*})$ and $G = 0$. Let ΔB^g be the solution to

$$\frac{\chi}{1-\chi}\left(\frac{(1-\delta\kappa)Y^{\min} - \chi\left((1-\nu)\,Y^{\min} - F + B^g + \Delta B^g\right)}{(1-\nu)\,Y^{\min} - F + B^g + \Delta B^g} - (1-\chi)\right) = \frac{1-\beta}{\beta}$$

hence

$$\Delta B^g = \frac{(1-\delta\kappa)Y^{\min} - \left[1+\frac{1-\chi}{\chi}\frac{1-\beta}{\beta}\right]B^g}{1+\frac{1-\chi}{\chi}\frac{1-\beta}{\beta}}$$

The total fiscal cost of this policy is

$$B^g + \Delta B^g = \frac{1-\delta\kappa}{1+\frac{1-\chi}{\chi}\frac{1-\beta}{\beta}}Y^{\min}$$

Now, let ΔG be the solution to

$$\frac{\chi}{1-\chi}\left(\frac{(1-\delta\kappa)Y^{\min} - \chi\left(\left(1-\nu\right)Y^{\min} - F + B^g\right) - \Delta G}{\left(1-\nu\right)Y^{\min} - F + B^g} - (1-\chi)\right) = \frac{1-\beta}{\beta}$$

hence

$$\Delta G = (1-\delta\kappa)Y^{\min} - \left[1+\frac{1-\chi}{\chi}\frac{1-\beta}{\beta}\right]B^g$$

The total fiscal cost of this policy is

$$B^g + \Delta G = (1-\delta\kappa)Y^{\min} - \frac{1-\chi}{\chi}\frac{1-\beta}{\beta}B^g$$

Therefore, the fiscal cost of government bonds is smaller than the fiscal cost of government spending if and only if

$$\frac{1-\delta\kappa}{1+\frac{1-\chi}{\chi}\frac{1-\beta}{\beta}}Y^{\min} \leq (1-\delta\kappa)Y^{\min} - \frac{1-\chi}{\chi}\frac{1-\beta}{\beta}B^g$$

or

$$B^g \leq \frac{1-\delta\kappa}{1+\frac{1-\chi}{\chi}\frac{1-\beta}{\beta}}Y^{\min} = B^{g*}$$

which always holds if the economy is in an SFLT. ■

Proof of Proposition 8. Fiscal capacity amounts to the following constraint $B^{s,g} + B^{l,g} \leq \tau^{\max}Y^{\min}$. Consider two economies. In one economy, the government only issues short-term debt. In the other economy, the government only issues long-term debt. Assume that $B^{s,g}$ and $B^{l,g}$ are such that the full-employment steady-state equilibrium coincides in both economies. This is achieved if and only if

$$B^{s,g} = [1+(1-\zeta)\rho Q(Y^*)]B^{l,g},$$

where $Y^* = A(1-\chi)^\nu$. Then, if $\zeta < 1$ and $\rho > 0$, we have $B^{l,g} < B^{s,g}$, which implies that long-term debt requires less fiscal capacity. Moreover, a lower value of ζ increases the net value of long-term government bonds, so we need a lower stock of debt $B^{l,g}$ to get the same value of safe assets.

■

Proof of Proposition 9. Consider a one-time policy intervention that exchanges a fraction

ψ of the long-term bonds for short-term bonds. That is, we have $B^{s,g'} - B^{s,g} = \psi Q B^{l,g}$, where $B^{s,g'}$ denotes the stock of short-term government bonds after the intervention, and $\psi \in (0,1)$. We assume that after this swap, the government keeps the debt at the post-intervention levels (lump-sum transfers adjust as necessary). Moreover, we assume that the policy does not lift the economy from the liquidity trap, that is, $i^s = 0$ after the intervention.

To find the new equilibrium of the economy we need to compute the change in Y that keeps the bond premium unchanged after the asset swap. Since the bond premium is counter-cyclical, it is sufficient to determine whether the policy increases or reduces the supply of government safe assets. We have that the change in government safe assets is given by

$$\Delta B^g \equiv B^{s,g'} + (1 + \rho(1 - \zeta Q)(1 - \psi)B^{l,g} - \left[B^{s,g} + (1 + \rho(1 - \zeta)Q)B^{l,g}\right],$$

where we used that Q does not change if the bond premium does not change. Then,

$$\begin{aligned}\Delta B^g &= \underbrace{B^{s,g'} - B^{s,g}}_{=\psi Q B^{l,g}} - \psi(1 + \rho(1 - \zeta)Q)B^{l,g} \\ &= \zeta\psi\beta\rho Q B^{l,g} > 0,\end{aligned}$$

where we used that $1 + bp = \frac{1}{\beta}$ and the expression for Q. Since this asset swap increases the supply of safe assets, output has to decrease to keep the bond premium unchanged. ■

Proof of Proposition 10.

Suppose the bond premium is pro-cyclical and $bp(Y^*) \leq \frac{1-\beta}{\beta}$. Then, there exists $R^* \geq 1$ such that $1 = \beta R^*[1 + bp(Y^*)]$. Hence, a full-employment steady-state exists. Since the bond premium is increasing in Y, there exists no other steady-state in the economy. If the bond premium is increasing in Y and $bp(Y^*) > \frac{1-\beta}{\beta}$, Assumption 1 guarantees the existence of $Y^{FLT} < Y^*$ such that $1 = \beta[1 + bp(Y^{FLT})]$. Then, Y^{FLT} characterizes the unique steady state of the economy. ■

Proof of Proposition 11.

Immediate from the proofs of Propositions 3 and 6. ■

Proof of Proposition 12. First, note that, from the production technology,

$$H = \left(\frac{Y^{1-\alpha\nu}}{A\kappa^{\alpha\nu}}\right)^{\frac{1}{(1-\alpha)\nu}}$$

Hence

$$u = 1 - \frac{H}{1-\chi} = 1 - \frac{1}{1-\chi}\left(\frac{Y^{1-\alpha\nu}}{A\kappa^{\alpha\nu}}\right)^{\frac{1}{(1-\alpha)\nu}}$$

If Assumption 1 is satisfied, then there exists a full-employment steady state with $\pi = 0$, as the system of equations characterizing equilibrium is the same as with rigid prices.

If the bond premium is counter-cyclical, an SFLT exists if and only if

$$bp\left(Y^{\min}\right) \geq \frac{1-\beta+\pi\left(Y^{\min}\right)}{\beta}$$

where

$$\pi\left(Y^{\min}\right) = \gamma\left(1 - \frac{1}{1-\chi}\left(\frac{Y^{\min}}{A\kappa^{\frac{\alpha\nu}{1-\alpha\nu}}}\right)^{\frac{1-\alpha\nu}{(1-\alpha)\nu}}\right)$$

Since $\pi\left(Y^{\min}\right) < 0$, the condition for existence of an SFLT is less demanding than when nominal wages are rigid.

Finally, since the expression for $bp\left(Y\right)$ is the same with price rigidity but the minimum bond premium is smaller, the amount of government bonds such that

$$bp\left(Y^{\min}, B^{g**}\right) = \frac{1-\beta+\pi\left(Y^{\min}\right)}{\beta} > 0$$

since $\pi(Y^{\min}) > \beta - 1$. Since $bp(\cdot, B^g)$ is decreasing in B^g (conditional on Y) and $\pi(Y^{\min}) < 0$, $B^{g**} > B^{g*}$. ■

B Data Sources

B.1 Section 2.3

Time period: Monthly. For most series in this section, our sample extended from 01/1948 until 12/2011. We note some exceptions below as we describe the data construction and sources.

1. Industrial Production Index: FRED series INDPRO.

2. Unemployment rate: FRED Series UNRATE.

3. Baa: Moody's Seasoned Baa Corporate Bond Yield Index from FRED (series BAA). The Moody's Baa index is constructed from a sample of long-maturity ($\geq$ 20 years) industrial and utility bonds (industrial only from 2002 onward).

4. Aaa: Moody's Seasoned Aaa Corporate Bond Yield Index from FRED (series AAA). The Moody's Aaa index is constructed from a sample of long-maturity ($\geq$ 20 years) industrial and utility bonds (industrial only from 2002 onward).

5. long-term Treasury yields: we follow the data construction of Krishnamurthy and Vissing-Jorgensen (2012). Their data series is annual. We went back to their sources and constructed a monthly data series. This series is a combination of LTGOVTBD and GS20 in the FRED database. GS20 is available from 2000 onwards.

6. three-month high-grade commercial paper (AACP) yields: obtained from the FRED database. For 1971–96 it is the series CP3M (the average of offering rates on 3-month commercial paper placed by several leading dealers for firms whose bond rating is AA or the equivalent), and for 1997–2011 the series CPN3M (the 3-month AA non-financial commercial paper rate).

7. lower-grade commercial paper yields (CPP2): calculated as the sum of the CP-bills yield spread described above (i.e., high-grade commercial paper minus Treasury bills) and the yield spread between 30-day A2/P2 non-financial commercial paper and 30-day AA non-financial commercial paper, with data obtained from the Federal Reserve Bank of New York. Sample used: 01/1998–12/2011.

8. short-term Treasury yield: we follow the data construction of Krishnamurthy and Vissing-Jorgensen (2012). Their data series is annual. We went back to their sources and constructed a monthly data series. The Treasury bill yield is for 3-month Treasury bills for 1971–2008 (from FRED, series TB3MS), 6-month Treasury bills for 1959–70 (from FRED, series TB6MS), and 3–6 month Treasury bills for 1948–58 from the NBER Macro History database (series m13029b for 1931–58).

9. slope of the Treasury yield curve: follows Krishnamurthy and Vissing-Jorgensen (2012). Their data series is annual. We went back to their sources and constructed a monthly data series. This series is measured as the spread between the 10-year Treasury yield and the 3-month Treasury yield. The interest rate on Treasuries with 10-year maturity is from FRED for 1953–2011 (series GS10). Prior to 1953 we use series m13033b (1948–52) from the NBER Macro History Database. It is referred to as the yield on long-term Treasuries. The interest rate on Treasuries with 3-month maturity is from FRED for 1948–2011 (series TB3MS).

10. three-month certificate of deposit (CD) rates and Treasury spread: Obtained from published supplementary material in Nagel (2016).

11. three-month banker's acceptance rates and Treasury spread (BA-Tbill): Obtained from published supplementary material in Nagel (2016).

12. VIX index: Obtained from published supplementary material in Nagel (2016).

13. outstanding stock of T-bills: Obtained as Tbill/GDP ratio from published supplementary material in Nagel (2016). Quarterly GDP is interpolated by Nagel (2016) to a monthly series for computing this ratio.

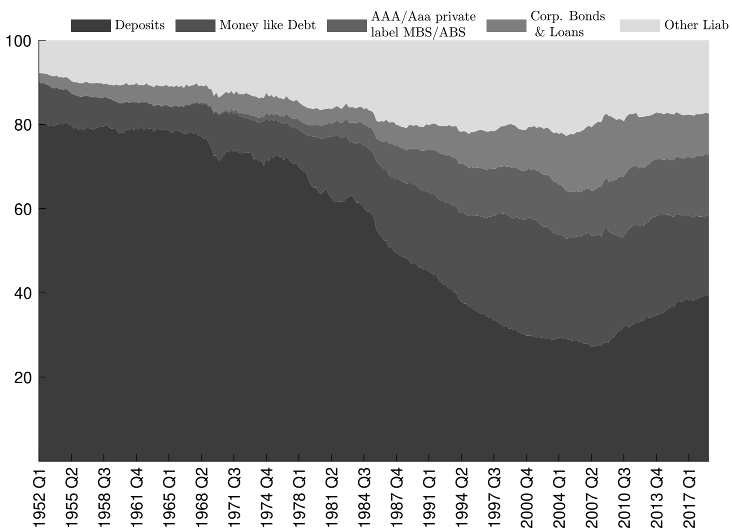
14. federal funds rate: FRED database.

15. Chicago Fed National Activity Index (CFNAI-MA3): The Chicago Fed National Activity Index (CFNAI) is a weighted average of 85 existing monthly indicators of national economic activity. The Chicago Fed normalizes the index to have an average value of zero and a

standard deviation of one. A positive value of the index corresponds to above trend growth (and vice-versa). We obtain the 3 month moving average series from the Chicago Fed Website. Data is available only March 1967 onwards. Sample used: 03/1967–12/2011.

B.2 Section 2.4

Figure 6: Components of privately produced safe debt



Source: Our calculations. Extended Gorton et al. (2012) using US Financial Accounts data retrieved from FRED, St. Louis Fed.

The online appendix of Gorton et al. (2012) prints a table with the identifiers in the US Financial Accounts for safe assets. Following their methodology, we constructed our data series for private safe assets. We used a series that they refer to as the “High” estimate of private safe assets. The key difference between the high and the low categories is in three asset class categories: “Financial business; other loans and advances; liability’,’ “Real estate investment trusts; total mortgages; liability,” and “Financial business; total miscellaneous liabilities” are not considered safe in the low category and some of these are considered safe in the high estimate. Results with their “Low” estimate are similar and are available upon request.

Figure 6 plots the share of each of these components in total private safe assets over time. Time period: Quarterly. Sample: 1952Q1 – 2019Q2. Total private safe assets is the sum of the following categories.

1. Deposits: “Financial business; checkable deposits and currency; liability,” “Financial business; total time and savings deposits; liability”

2. Money-like Debt: This refers to commercial paper, net repurchase agreements, federal funds, money market mutual fund assets, interbank transactions, broker-dealer payables, and broker-dealer security credits.

3. MBS/ABS: MBS/ABS Debt includes all GSE and private-label MBS debt, as well as all ABS debt.

4. Corporate Bonds: "Corporate Bonds and Loans" includes "Financial business; corporate and foreign bonds; liability," "Private nonbank financial institutions; bank loans not elsewhere classified; liability," and "Financial business; other loans and advances; liability."

5. Other Safe Assets: "Financial business; total miscellaneous liabilities"

C Aggregate Risk and the Demand for Safe Assets

In this section, we present an extension of the model from Section 3 that incorporates aggregate risk. This model allows us to connect the bond premium to the safety properties of the assets.

To avoid repetition, we only describe the differences relative to the model presented in Section 3. The economy is now subject to TFP shocks. In particular, we assume that the firms TFP level A_t is an i.i.d. random variable with associated cumulative distribution function $G(\cdot)$ in the support $[\underline{A}, \overline{A}]$, with $\underline{A} > 0$ and $\overline{A} > \underline{A}$. Households' attitude towards risk depends on their members' employment status. We assume that households are expected utility maximizers with respect to workers but are infinitely risk-averse (i.e. "Knightian" uncertainty) concerning the retirees. Let $\mathcal{C}_t^w(A)$ and $\mathcal{C}_t^r(A)$ denote the consumption of workers and retirees as functions of the TFP level. The households' utility is given by

$$\sum_{t=0}^{\infty} \beta^t \left\{ (1-\chi) \left[\int_{\underline{A}}^{\overline{A}} u\left(\mathcal{C}_t^w\left(A\right)\right) dG\left(A\right) \right] + \chi \min_{A \in \left[\underline{A}, \overline{A}\right]} v\left(\mathcal{C}_t^r\left(A\right)\right) \right\}.$$

Moreover, we assume that labor is the only factor of production (i.e. $\alpha = 0$). Because profits are risky, we assume that the firms' borrowing constraint is such that they are able to repay their debts in full from the period's profits even in the worst-case scenario. That is, we assume that the borrowing constraint is given by

$$\widetilde{B}_{t+1}^p \leq \Pi_{t+1}(\underline{A}).$$

Recall that the dividends are $D_t(A) = \Pi_t(A) - \widetilde{B}_t^p + \frac{\widetilde{B}_{t+1}^p}{1+i_t}$.

Let $\epsilon(\delta) \equiv 1 - G(\overline{A} - \delta)$, that is, the probability that the TFP level is above $\overline{A} - \delta$. We make the following assumptions.

Assumption 2 *The parameters of the model are such that:*

1. *In the full-employment equilibrium,* $D_t(\underline{A}) = 0$ *or, equivalently,* $\underline{A} = \frac{R^*}{(1-\nu)(1-\chi)^\nu} F$;

2. $\lim_{\delta \to 0} \varepsilon(\delta) = 1$.

Under this assumption, the households' problem is to maximize

$$\sum_{t=0}^{\infty} \beta^t \left[(1-\chi) u(C_t^w) + \chi v(C_t^r) \right],$$

subject to the intra-period constraints

$$P_t C_t^w \leq W_t h_t + \widetilde{B}_t + D_t(\overline{A}) + \frac{T_t}{1-\chi},$$
$$P_t C_t^r \leq \widetilde{B}_t + \underbrace{D_t(\underline{A})}_{\substack{=0 \text{ in the full-employment} \\ \text{steady state}}},$$

where $C_t^w \equiv \mathcal{C}_t^w(\overline{A})$ and $C_t^r \equiv \mathcal{C}_t^r(\underline{A})$, and the households budget constraint

$$(1-\chi) P_t C_t^w + \chi P_t (C_t^r + D_t(\overline{A})) + \frac{\widetilde{B}_{t+1}}{1+i_t^s} \leq (1-\chi) W_t h_t + R_t^k K_t + D_t + \widetilde{B}_t + T_t.$$

Note that, unlike in the model of Section 3, we now assume that retirees receive a proportional fraction of the firms' dividends (for simplicity we still assume that only workers receive lump-sum transfers). However, retirees' consumption is *as-if* they do not receive any dividend. The reason is that, under Assumption 2, in the worst-case scenario the dividends are zero, which is the case used to compute the retirees' utility. Thus, firms' stock will not feature a premium.[43] Finally, note that even if only C_t^r derives utility to the retirees, their actual consumption when $A = \overline{A}$ is $C_t^r + D_t(\overline{A})$.

C.1 Equilibrium

We define an equilibrium under Assumption 2. Let $w_t \equiv \frac{W_t}{P_t}$ denote the real wage. A competitive equilibrium of this economy is an allocation $\{C_t^w, C_t^r, \widetilde{B}_{t+1}, h_t, H_t, \widetilde{B}_{t+1}^p\}_{t=0}^{\infty}$ and prices $\{w_t, \pi_t, i_t^s\}_{t=0}^{\infty}$ such that, given fiscal policy $\{\widetilde{B}_t^g, T_t^g\}_{t=0}^{\infty}$, and initial bonds, $\{\widetilde{B}_0, \widetilde{B}_0^p\}$,

1. $\{C_t^w, C_t^r, \widetilde{B}_{t+1}, h_t\}_{t=0}^{\infty}$ solves the household's problem given $\{w_t, \pi_t, i_t^s\}_{t=0}^{\infty}$, $\{T_t^g\}_{t=0}^{\infty}$ and $\widetilde{B}_0$

2. $\{H_t, \widetilde{B}_{t+1}^p\}_{t=0}^{\infty}$ solves the firms' problem given $\{w_t, \pi_t, i_t^s\}_{t=0}^{\infty}$ and $\widetilde{B}_0^p$

3. $\{i_t^s\}_{t=0}^{\infty}$ follows (7)

4. fiscal policy $\{\widetilde{B}_t^g, T_t^g\}_{t=0}^{\infty}$ satisfies (8)

5. Markets clear

$$(1-\chi) C_t^w + \chi (C_t^r + D_t(\overline{A})) = \overline{A} H_t^{\nu}, \quad \widetilde{B}_t = \widetilde{B}_t^p + \widetilde{B}_t^g, \quad H_t = (1-\chi) h_t.$$

[43]Note that if the firm's dividends are zero in the worst-case scenario, the stock value would also be zero, so stocks provide no convenience to retirees.

Note that since C_t^r denotes the retirees' consumption when $A = \underline{A}$, we need to adjust for the dividends when $A = \overline{A}$ in the market clearing condition for goods. Moreover, note that the definition of equilibrium does not consider states in which $A \neq \overline{A}$. This is because under Assumption 2, these other states do not affect the equilibrium when $A = \overline{A}$, except for C_t^r, which is fully accounted for.

Consider a steady-state equilibrium with zero inflation. Output is given by

$$Y = \overline{A} H^{\nu}.$$

The supply of private safe assets is given by

$$B^p(Y) = (1-\nu)\, \underline{A} H^{\nu} - F = \frac{\underline{A}}{\overline{A}} (1-\nu)\, Y - F,$$

and, therefore, total safe asset supply is

$$B(Y) = B^p(Y) + B^g.$$

The firms' dividends are given by

$$D(Y) = (1-\nu)\, Y - F - B^p(Y) + \frac{B^p(Y)}{1+i^s} = \left[1 - \frac{i^s}{1+i^s} \frac{\underline{A}}{\overline{A}}\right] (1-\nu)\, Y - \frac{1}{1+i^s} F.$$

Workers' consumption is

$$C^w = \frac{Y - \chi\left[B(Y) + D(Y)\right]}{1-\chi}.$$

Then, the bond premium is given by

$$bp(Y) = \chi \left(\frac{\frac{Y - \chi[B(Y) + D(Y)]}{1-\chi}}{B(Y)} - 1 \right).$$

Recall that the bond premium is counter-cyclical conditional on i^s if and only if $bp'(Y) < 0$. After some algebra, we get

$$bp'(Y) = \frac{\chi}{1-\chi} \frac{ - \overbrace{\left(1 - \chi(1-\nu) + \chi(1-\nu) \frac{\underline{A}}{\overline{A}}\right)}^{>0} (F - B^g) - \chi(1-\nu) \frac{1}{1+i^s} \frac{\underline{A}}{\overline{A}} B^g }{B(Y)^2},$$

which implies that $F > B^g$ is a sufficient condition for counter-cyclicality of the bond premium. Then, the economy can be analyzed analogously to the model in Section 3.

C.2 Aggregate Risk and the SFLT

Suppose the economy starts with $\underline{A} = \overline{A}$, that is, there is no aggregate risk. Moreover, suppose that

$$bp(Y^{\min}) < \frac{1-\beta}{\beta} \quad \text{or, equivalently,} \quad \frac{\chi}{1-\chi}\frac{\frac{F}{1-\nu} - B^g}{B^g} < \frac{1-\beta}{\beta}.$$

This implies that the economy *does not admit a SFLT.*

Suppose that, unexpectedly, agents believe that there is a (very small) probability that the TFP takes the value $\underline{A} < \overline{A}$. In particular, suppose that Assumption 2 holds. The next proposition establishes that if $\underline{A}$ is sufficiently small, the economy becomes exposed to a SFLT.

Proposition 13 *Suppose Assumptions 1 and 2 are satisfied, and the bond premium is countercyclical. There exists $\underline{A}^* < \overline{A}$ such that if $\underline{A} \leq \underline{A}^*$, the economy admits two steady-state equilibria: one with full employment and another with involuntary unemployment, a negative output gap, and a zero nominal interest rate.*

Proof. The existence of a full employment steady-state equilibrium is immediate from Assumption 1 and Proposition 1. To see that there exists a SFLT steady-state equilibrium, note that

$$\lim_{\underline{A}\to 0} B^p(Y^{\min}) = 0 \implies \lim_{\underline{A}\to 0} bp(Y^{\min}) = \infty > \frac{1-\beta}{\beta}.$$

Since $bp(Y^{\min}) < \frac{1-\beta}{\beta}$ when $\underline{A} = \overline{A}$ by assumption, and $bp(Y^{\min})$ is continuous and monotonic in $\underline{A}$, $\underline{A}^*$ exists.

■

The intuition for Proposition 13 is simple. The presence of aggregate risk reduces the private sector's ability to produce safe assets (keeping the supply of government bonds fixed), which increases the lower bound of the bond premium. If the lower bound for the TFP level, $\underline{A}$, is sufficiently low, then the conditions for a liquidity trap equilibrium will be satisfied. Thus, the exposure to aggregate risk can lead to a self-fulfilling liquidity trap.

D A Model with Long-Term Debt

We study a version of the model in Section 3 in which the government issues short-term and long-term bonds. Long-term bonds are modeled as perpetuities that pay exponentially decaying coupons, where $\rho \in (0, 1)$ (see Woodford, 2001). Note that ρ governs the duration of the bond, with $\rho = 0$ corresponding to a one-period bond, and $\rho = 1$ corresponding to a consol.

We use this model to answer two questions. First, do long-term bonds expand the fiscal capacity? Second, does quantitative easing stimulate the economy in a SFLT?

D.1 The Model

Households now hold short-term and long-term bonds. Retirees receive a proportional share of short- and long-term bonds. Short-term bonds mature immediately, so the full face value can be used in consumption (like in Section 3). Long-term bonds pay a coupon, which can be used for consumption, and the remainder can be sold in the secondary market. To capture potential differences in the convenience provided by the two assets, we assume that selling the long-term bonds entails a proportional cost ζ (we assume that there is no cost associated with buying). We interpret ζ as an intermediation cost and assume that is rebated back to the households. Moreover, we simplify the analysis from Section 3 and assume that labor is the only factor of production. Thus, the intra-period budget constraints faced by individual agents are now given by

$$P_t C_t^w \leq W_t h_t + \widetilde{B}_t^s + (1 + (1-\zeta)\rho Q_t)\widetilde{B}_t^l + \frac{1}{1-\chi}(D_t + T_t), \tag{31}$$

$$P_t C_t^r \leq \widetilde{B}_t^s + (1 + (1-\zeta)\rho Q_t)\widetilde{B}_t^l, \tag{32}$$

where Q_t denotes the price of the long-term nominal bond. At the end of the period, the household as a whole faces the following budget constraint:

$$(1-\chi)P_t C_t^w + \chi P_t C_t^r + \frac{\widetilde{B}_{t+1}^s}{1+i_t^s} + Q_t(\widetilde{B}_{t+1}^l - \rho \widetilde{B}_t^l) \leq (1-\chi)W_t h_t + D_t + \widetilde{B}_t^s + \widetilde{B}_t^l + T_t. \tag{33}$$

In what follows, we limit attention to equilibria in which the budget constraint of retirees (32) is binding, so that $C_t^r = \frac{\widetilde{B}_t^\zeta}{P_t}$, where $\widetilde{B}_t^\zeta \equiv \widetilde{B}_t^s + (1 + (1-\zeta)\rho Q_t)\widetilde{B}_t^l$. In this equilibrium, the workers never sell their long-term bonds. Replacing into the household's utility function, we get

$$\sum_{t=0}^{\infty} \beta^t \left[(1-\chi) u\left(C_t^w\right) + \chi v \left(\frac{\widetilde{B}_t^\zeta}{P_t} \right) \right]. \tag{34}$$

The problem of the household consists of choosing processes $\left\{ C_t^w, \widetilde{B}_t^\zeta, \widetilde{B}_t^s, \widetilde{B}_t^l \right\}_{t=0}^{\infty}$ in order to maximize (34) subject to the budget constraints (31), (32), and (33) for every $t \geq 0$, the definition of $\widetilde{B}_t^\zeta$, and a No Ponzi condition, given $\left\{ K_0, \widetilde{B}_0^s, \widetilde{B}_0^l \right\}$.

The firms' problem is the same as in Section 3. In particular, we assume that the firms' issue only short-term bonds. Moreover, we keep the assumption that wages are perfectly rigid.

Finally, we need to modify the government's budget constraint to include the long-term bonds. We have

$$\widetilde{B}_t^{g,s} + (1 + \rho Q_t)\, \widetilde{B}_t^{g,l} = \frac{\widetilde{B}_{t+1}^{g,s}}{1+i_t^s} + Q_t \widetilde{B}_{t+1}^{g,l} + T_t^g. \tag{35}$$

We assume that the government does not face an intermediation cost when issuing long-term bonds (that is, there is no intermediation cost in the primary market).

Market clearing implies:

$$(1-\chi)C_t^w + \chi C_t^r = A_t H_t^\nu, \quad \widetilde{B}_t^s = \widetilde{B}_t^p + \widetilde{B}_t^{g,s}, \quad \widetilde{B}_t^l = \widetilde{B}_t^{g,l}, \quad H_t = (1-\chi)h_t.$$

Below we will study a policy that swaps long-term bonds for short-term bonds, capturing large-scale asset purchases programs that the fed implemented after the Great Recession and the COVID-19 crisis.

D.2 Equilibrium

Let $w_t \equiv \frac{W_t}{P_t}$ denote the real wage. A competitive equilibrium of this economy is an allocation $\{C_t^w, \widetilde{B}_{t+1}^\zeta, \widetilde{B}_{t+1}^s, \widetilde{B}_{t+1}^l, h_t, H_t, \widetilde{B}_{t+1}^p\}_{t=0}^\infty$ and prices $\{w_t, \pi_t, i_t^s, Q_t\}_{t=0}^\infty$ such that, given fiscal policy $\{\widetilde{B}_t^{s,g}, \widetilde{B}_t^{l,g}, T_t^g\}_{t=0}^\infty$, and initial bonds, $\{\widetilde{B}_0^s, \widetilde{B}_0^l, \widetilde{B}_0^p\}$,

1. $\{C_t^w, \widetilde{B}_{t+1}^\zeta, \widetilde{B}_{t+1}^s, \widetilde{B}_{t+1}^l, h_t\}_{t=0}^\infty$ solves the household's problem given $\{w_t, \pi_t, i_t^s, Q_t\}_{t=0}^\infty$, $\{T_t^g\}_{t=0}^\infty$ and $\{\widetilde{B}_0^s, \widetilde{B}_0^l\}$
2. $\{H_t, \widetilde{B}_{t+1}^p\}_{t=0}^\infty$ solves the firms' problem given $\{w_t, \pi_t, i_t^s\}_{t=0}^\infty$ and $\widetilde{B}_0^p$
3. $\{i_t^s\}_{t=0}^\infty$ follows (7)
4. fiscal policy $\{\widetilde{B}_t^{s,g}, \widetilde{B}_t^{l,g}, T_t^g\}_{t=0}^\infty$ satisfies (35)
5. Markets clear
$$(1-\chi)C_t^w + \chi C_t^r = A_t H_t^\nu, \quad \widetilde{B}_t^s = \widetilde{B}_t^p + \widetilde{B}_t^{s,g}, \quad \widetilde{B}_t^l = \widetilde{B}_t^{l,g}, \quad H_t = (1-\chi)h_t.$$

We focus our analysis on equilibria in which the budget constraint of retirees (32) is satisfied with equality. In this case, the optimality conditions associated with the household's problem are the budget constraint (33), and

$$\beta^t u'(C_t^w) = P_t \Lambda_t \tag{36}$$

$$\frac{\Lambda_t}{1+i_t^s} = (1-\chi)\Lambda_{t+1} + \chi\beta^{t+1} v'\left(B_{t+1}^\zeta\right)\frac{1}{P_{t+1}} \tag{37}$$

$$Q_t \Lambda_t = \left(1-\chi + \left(1-\chi\left(1-\zeta\right)\right)\rho Q_{t+1}\right)\Lambda_{t+1} + \beta^{t+1}\left(1+\left(1-\zeta\right)\rho Q_{t+1}\right)\chi v'\left(B_{t+1}^\zeta\right)\frac{1}{P_{t+1}} \tag{38}$$

where $B_{t+1}^\zeta \equiv \frac{\widetilde{B}_{t+1}^\zeta}{P_{t+1}}$, and $\Lambda_t > 0$ is the Lagrange multiplier associated with the budget constraint (33). Combining equations (36) and (37), we get the same GEE as in Section 3:

$$1 = \beta\frac{1+i_t^s}{1+\pi_{t+1}}\left[\frac{u'(C_{t+1}^w)}{u'(C_t^w)} + \chi\frac{v'(B_{t+1}^\zeta) - u'(C_{t+1}^w)}{u'(C_t^w)}\right]. \tag{39}$$

Recall that the *bond premium* is defined as

$$bp_t \equiv \chi \frac{v'(B^{\zeta}_{t+1}) - u'(C^w_{t+1})}{u'(C^w_t)}. \tag{40}$$

Then, combining (36) and (38), we get the following pricing equation for long-term bonds:

$$Q_t = \beta \frac{1}{1+\pi_{t+1}} \left[(1+\rho Q_{t+1}) \frac{u'\left(C^w_{t+1}\right)}{u'\left(C^w_t\right)} + (1 + (1-\zeta)\, \rho Q_{t+1})\, bp_t \right]. \tag{41}$$

Next, consider the firms' problem. Focusing on equilibria in which the retiree's budget constraint (32) is binding, the supply of short-term bonds is an affine function of aggregate output

$$B^s_t = (1-\nu) Y_t - F + B^{s,g}_t, \tag{42}$$

where $B^{s,g}_t \equiv \frac{\widetilde{B}^{s,g}_t}{P_t}$. Finally, equilibrium requires that $(1-\nu)Y_t - F \geq 0$ for all t.

D.3 Steady States

A steady state is an equilibrium in which all endogenous and exogenous variables are constant over time. Given our wage rigidity assumption, if w_t and W_t are constant over time, then the inflation rate is zero in any steady state.

If firms are active, the short-term bond supply is given by equation (42) evaluated at steady state:

$$B^s(Y) = (1-\nu)Y - F + B^{s,g}.$$

Evaluating equation (41) in steady-state, we get

$$Q(Y) = \frac{\beta\left[1 + bp(Y)\right]}{1 - \beta\left[1 + (1-\zeta)\, bp(Y)\right] \rho},$$

where $bp(Y)$ is the bond premium in steady-state. Then, the total value of safe assets available to the retirees is given by

$$B^{\zeta}(Y) = B^s(Y) + [1 + (1-\zeta)\rho Q(Y)] B^{l,g}.$$

From the resource constraint, we get

$$C^w(Y) = \frac{Y - \chi B^{\zeta}(Y)}{1-\chi}.$$

Thus, the bond premium can be written as

$$bp(Y) = \chi \left(\frac{v'(B^{\zeta}(Y))}{u'(C^w(Y))} - 1 \right). \tag{43}$$

An equilibrium of the economy can be found from the intersection of

$$1 = \beta(1 + i^s)[1 + bp(Y)]$$

and

$$1 + i^s = \max\left\{1, R^* + \phi_Y\left(\frac{Y}{Y^*} - 1\right)\right\},$$

where $Y^* = A(1-\chi)^\nu$ and $R^* = \frac{1}{\beta}\frac{1}{1+bp(Y^*)}$.

Note that the characterization of the steady-state equilibria of this economy is analogous to the one in Section 3, where now the total value of safe assets available to retirees involves both short- and long-term bonds. Thus, under Assumption 1, and assuming that the bond-premium is counter-cyclical, a full-employment steady-state equilibrium exists (analogous to Proposition 1).[44] Moreover, under Assumption 1, and assuming that the bond-premium is counter-cyclical and satisfies $bp(Y^{\min}) \geq \frac{1-\beta}{\beta}$, a SFLT exists (analogous to Proposition 2).

Finally, let $1 + i^l$ denote the yield on the long-term bond. We have

$$1 + i^l = \frac{1 + \rho Q(Y)}{Q(Y)} = \frac{1 - \beta\left[1 + (1-\zeta)\, bp\,(Y)\right]\rho}{\beta\left[1 + bp\,(Y)\right]} + \rho = \underbrace{\left[1 + \beta\zeta bp\,(Y)\,\rho\right]}_{>1}(1 + i^s) > 1 + i^s,$$

where we used that $Q\,(Y) = \frac{\beta[1+bp(Y)]}{1-\beta[1+(1-\zeta)bp(Y)]\rho}$ and $1 + i^s = \frac{1}{\beta}\frac{1}{1+bp(Y)}$. Thus, the model generates an upward sloping yield curve driven by the difference in convenience provided by the two assets.

D.4 Fiscal Capacity

We define fiscal capacity as the maximum amount of debt obligations that the government can pay out of taxation in the case that it cannot roll-over its debt and the economy is at its minimum level of output. That is, fiscal capacity amounts to the following constraint:

$$B^{s,g} + B^{l,g} \leq \tau^{\max} Y^{\min},$$

where $\tau^{\max}$ is the maximum tax rate the government can impose on the economy. For short-term bonds, the full face value $B^{s,g}$ enters into the constraint. In contrast, for long-term bonds, only the coupon $B^{l,g}$ uses fiscal capacity. The reason is that households are forced to hold the continuation value of the long-term bond, so there is no roll-over risk over this component. Note that when $\rho = 0$, long-term bonds are effectively short-term since the continuation value is equal to zero.

Now, we want to show that long-term bonds increase the fiscal capacity of the government, and that the increase in fiscal capacity is decreasing in the intermediation cost ζ. Consider two economies. In one economy, the government only issues short-term debt. In the other economy, the government only issues long-term debt. To make the two economies comparable, assume that $B^{s,g}$

[44] The condition for a counter-cyclical bond premium is $F > B^\phi$.

and $B^{l,g}$ are such that the full-employment steady-state equilibrium coincides in both economies. That is, output, workers' and retirees' consumption, the bond premium, and the natural rate are the same in both economies. This is achieved if and only if

$$B^{s,g} = [1 + (1-\zeta)\rho Q(Y^*)]B^{l,g},$$

where $Y^* = A(1-\chi)^\nu$. It is straightforward to see that if $\zeta < 1$ and $\rho > 0$, then $B^{l,g} < B^{s,g}$, which implies that long-term debt requires less fiscal capacity for the same supply of government safe assets. Moreover, a lower value of ζ increases the net value of long-term government bonds, so we need a lower stock of debt $B^{l,g}$ to get the same value of safe assets. Thus, lower ζ relaxes government's fiscal capacity constraint.

D.5 Quantitative Easing

Suppose that the economy is in a SFLT. Then, $i^s = 0$ so we know that

$$1 = \beta[1 + bp] \quad \text{or} \quad 1 + bp = \frac{1}{\beta}.$$

Replacing in the price for the long-term bond, we get

$$Q = \frac{1}{1 - \beta\left[1 + (1-\zeta)\, bp\right]\rho}.$$

Note that, in a liquidity trap, the bond premium is independent of the level of output. Thus, the price of long-term bonds is also independent of the level of output.

Consider a one-time policy intervention that exchanges a fraction ψ of the long-term bonds for short-term bonds. That is, we have

$$B^{s,g'} - B^{s,g} = \psi Q B^{l,g},$$

where $B^{s,g'}$ denotes the stock of short-term government bonds after the intervention, and $\psi \in (0,1)$. We assume that after this swap, the government keeps the debt at the post-intervention levels (lump-sum transfers adjust as necessary). Moreover, we assume that the policy does not lift the economy from the liquidity trap, that is, $i^s = 0$ after the intervention.

To find the new equilibrium of the economy we need to compute the change in Y that keeps the bond premium unchanged after the asset swap. Since the bond premium is counter-cyclical, it is sufficient to determine whether the policy increases or reduces the supply of government safe assets. We have that the change in government safe assets is given by

$$\Delta B^g \equiv B^{s,g'} + (1 + \rho(1-\zeta Q)(1-\psi)B^{l,g} - \left[B^{s,g} + (1 + \rho(1-\zeta)Q)B^{l,g}\right],$$

where we used that Q does not change if the bond premium does not change. Then,

$$\begin{aligned}\Delta B^{g} &= \underbrace{B^{s,g'} - B^{s,g}}_{=\psi Q B^{l,g}} - \psi(1+\rho(1-\zeta)Q)B^{l,g} \\ &= \zeta\psi\beta\rho Q B^{l,g} > 0,\end{aligned}$$

where we used that $1 + bp = \frac{1}{\beta}$ and the expression for Q. Since this asset swap increases the supply of safe assets, output has to decrease to keep the bond premium unchanged. That is, a small asset purchase program that keeps the economy in a liquidity trap *reduces* the equilibrium level of output. In terms of the fiscal consequences of this policy, we have that as long as $\zeta > 0$, the debt servicing costs decrease (as the bond premium on short-term bonds is higher than that of long-term bonds), but the change in the maturity structure tightens the government's fiscal capacity constraint.

D.6 QE in the Quantitative model

We analyze a Quantitative Easing program in the quantitative model presented in Section 5. The main difference with respect to that model is that we introduce a term structure of interest rates by calibrating ζ (the illiquidity of long-term bonds) to a non-zero value. As noted in the literature, for example Nagel (2016), "long-term Treasuries are not as liquid as T-bills, but they may command a convenience yield for reasons other than moneyness." As measured by Kim and Wright (2005), the average term-premium on 5 year US treasuries relative to federal funds rate from 2001-Q1 to 2007Q3 was 50 basis points.[45] Using the bond-pricing equation that we derived above, $\frac{1+i^l}{1+i^s} = 1+\beta\zeta bp\,(Y)\,\rho$, we can calibrate ζ. Given that bond premium target is 209 basis points in the quantitative model,[46] and the ρ implied by the average duration of US government bonds of five years is 0.83, and a calibrated annual value of β of 0.965, we get $\zeta = 0.30$. We re-calibrate all the parameters in Section 5 to hit the targeted moments, using a minimum distance approach.[47]

The solid blue line in Figure 7 plots the transitional dynamics from a full-employment steady state to the self-fulfilling liquidity trap steady state as also discussed in Section 5. The key difference is that now we model a term-spread between short-term bonds and long-term bonds. The transition to the liquidity trap is triggered by a decline of households' confidence in the economy, which is modeled as a 1% shock to employment expectations.

We further model a QE intervention where the government swaps out ψ fracion of long-term bonds with short-term bonds. The short-term bonds do not have any transaction costs

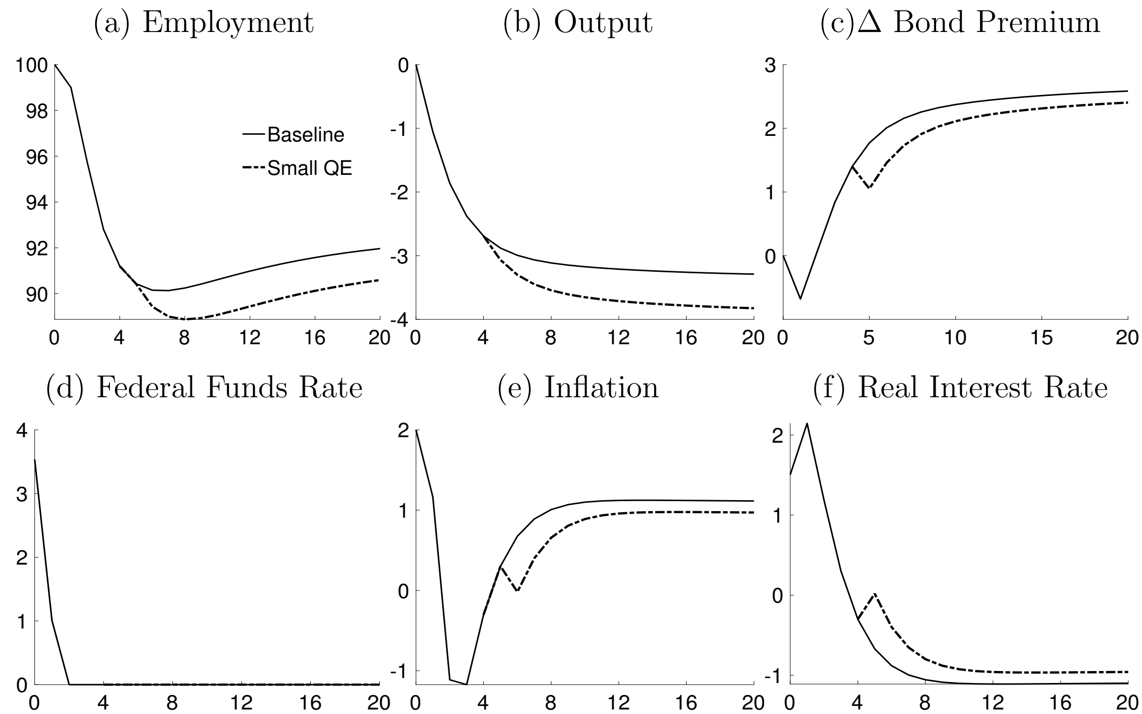
$$B^{s,g'} - B^{s,g} = \psi Q B^{l,g},$$

[45] This data series is available as `THREEFYTP5` from the St Louis Fed's FRED database.

[46] The average spread of Moody's BAA Corporate Bond yield over 10 year treasury constant maturity rate until 2008Q2 is 209 basis points.

[47] Fixed parameters are the same as in Table 4. The calibrated parameters are as follows: $\nu = 0.63$, $\alpha = 0.52$, $\chi = 0.001$, $F = 6.69$, $\eta = 3.16$, $\phi = 0.54$, $B^g = 2.45$, $A = 7.21$, $\gamma = 0.88$, $\rho = 0.83$, and $\zeta = 0.30$.

Figure 7: Transitional dynamics to a small Quantitative Easing program



Note: The solid line (Baseline) plots the transition of employment, output, change in bond premium, nominal interest rate, inflation rate, and real interest rate from full-employment steady state to the liquidity trap steady state. The dashed line (Small $B^g \uparrow$) plots the transition from year 4 of the baseline path to a new liquidity trap steady state with QE swap with parameter $\psi = 0.1$. Time is in years. Employment is in percentage points. Output is measured as the percentage deviation of output from the full-employment steady-state output. Δ bond premium represents the annual percentage point change in the bond premium relative to the full-employment steady-state bond premium. The nominal interest rate is the annualized level of the nominal interest rate in percentage points. Inflation is measured in annualized percentage points. The inflation target of the central bank is 2%. The real interest rate is the annualized level of the real interest rate in percentage points.

When the government's commitment is small ($\psi = 0.1$ in our exercise), the intervention can exacerbate the recession, as seen in the dashed red line in Figure 7.

E Optimal discretionary monetary policy

Consider the problem of a central bank that operates under discretion. The central bank chooses the level of the interest rate taking all future variables as given. The central bank's problem in period t is given by

$$\max \sum_{s=t}^{\infty} \beta^{s-t} \left[(1-\chi) u\left(C_s^w\right) + \chi v\left(B_s\right)\right] = (1-\chi) u\left(C_t^w\right) + \chi v\left(B_t\right) + V_t^H$$

subject to

$$1 = \beta \left(1 + i_t^s\right) Y_t^{\frac{1-\nu}{\nu}} \frac{V_t^E}{u'\left(C_t^w\right)} \tag{44}$$

$$C_t^w = \frac{Y_t - \chi B_t}{1 - \chi} \tag{45}$$

$$B_t = (1 - \nu) Y_t - F + B^g \tag{46}$$

$$Y_t = A_t H_t^{\nu} \tag{47}$$

$$H_t \leq 1 - \chi \tag{48}$$

$$i_t^s \geq 0 \tag{49}$$

where

$$V_t^H \equiv \sum_{s=t+1}^{\infty} \beta^{s-t} \left[(1 - \chi) u\left(C_s^w\right) + \chi v\left(B_s\right)\right]$$

and

$$V_t^E \equiv \frac{(1 - \chi) u'\left(C_{t+1}^w\right) + \chi v'\left(B_{t+1}\right)}{Y_{t+1}^{1-\nu}}$$

are taken as given by the central bank in period t. Equation (44) is the Euler equation, where V_t^E collects the terms dated in $t+1$ and we used that the inflation rate satisfies $1 + \pi_{t+1} = \left(\frac{Y_{t+1}}{Y_t}\right)^{\frac{1-\nu}{\nu}}$. Equation (45) states workers' consumption from the goods market clearing. Equation (46) is the firms' borrowing constraint satisfied with equality. It is important to note that while B_t is chosen in period $t-1$, a rational expectations equilibrium requires that this choice is consistent with the central bank's policy in period t. Equation (47) is the production function, equation (48) is the market clearing condition for labor satisfied with inequality, and equation (49) denotes the ZLB.

From equation (46) we can write

$$B_t = B\left(Y_t\right) \equiv (1 - \nu) Y_t - F + B^g,$$

and then from equation (45) we get

$$C_t^w = C^w\left(Y_t\right) \equiv \frac{\left[1 - \chi (1 - \nu)\right] Y_t + \chi\left(F - B^g\right)}{1 - \chi}.$$

Naturally, B_t and C_t^w are increasing in Y_t. Plugging this expression into the Euler equation, we get

$$1 = \beta \left(1 + i_t^s\right) Y_t^{\frac{1-\nu}{\nu}} \frac{V_t^E}{u'\left(C^w\left(Y_t\right)\right)},$$

which defines an implicit function $Y\left(i_t^s\right)$ with $Y'\left(i_t^s\right) < 0$. Then, we can simplify the central bank's problem as

$$\max_{i_t^s} (1 - \chi) u\left(C^w\left(Y\left(i_t^s\right)\right)\right) + \chi v\left(B\left(Y\left(i_t^s\right)\right)\right) + V_t^H$$

subject to

$$H_t \leq 1 - \chi$$
$$i_t^s \geq 0$$

It is immediate to see that the solution is given by

$$i_t^s \left[H_t - (1 - \chi)\right] = 0$$
$$H_t = \left(\frac{Y_t\left(i_t^s\right)}{A_t}\right)^{\frac{1}{\nu}}.$$

The solution of the optimal policy under discretion can thus feature a liquidity trap regime in which there is involuntary unemployment.

F Quantitative Model

The economy of Section 5 is an extension of the model described in Section 3. The equilibrium of the economy can be characterized by a system of 13 equations in the 13 unknowns $\{Y_t, i_t^s, \pi_t, C_t^w, bp_t, B_{t+1}, Q_t, r_{t+1}^k, I_t, K_{t+1}, w_t, h_t, u_t\}$

$$B_t = (1-\nu)Y_t - F + [1 + (1-\zeta)\rho Q_t]B_t^g, \tag{50}$$

$$bp_t = \chi \frac{v'(B_{t+1}) - u'(C_{t+1}^w)}{u'(C_t^w)} \tag{51}$$

$$1 = \beta \frac{1 + i_t^s}{1 + \pi_{t+1}} \left[\frac{u'(C_{t+1}^w)}{u'(C_t^w)} + bp_t\right] \tag{52}$$

$$1 + i_t^s = \max\left\{1, R^* + \phi_\pi \left(\frac{1+\pi_t}{1+\pi^*} - 1\right) + \phi_Y \left(\frac{Y_t}{Y^*} - 1\right)\right\} \tag{53}$$

$$Q_t = \beta \frac{1}{1 + \pi_{t+1}} \left[(1 + \rho Q_{t+1}) \frac{u'\left(C_{t+1}^w\right)}{u'\left(C_t^w\right)} + (1 + (1-\zeta)\rho Q_{t+1}) bp_t\right] \tag{54}$$

$$1 = \beta \frac{u'\left(C_{t+1}^w\right)}{u'\left(C_t^w\right)} \left[r_{t+1}^k + 1 - \delta\right], \tag{55}$$

$$C_t^w = \frac{Y_t - \chi B_t - I_t}{1 - \chi} \tag{56}$$

$$K_{t+1} = (1-\delta)K_t + I_t, \tag{57}$$

$$Y_t = A_t (K_t^\alpha ((1-\chi)h_t)^{1-\alpha})^\nu \tag{58}$$

$$w_t = (1-\alpha)\nu \frac{Y_t}{(1-\chi)h_t}, \tag{59}$$

$$r_t^k = \alpha\nu \frac{Y_t}{K_t}, \tag{60}$$

$$u_t = 1 - h_t, \tag{61}$$

$$w_t \geq \frac{\gamma + (1-\gamma)h_t}{1+\pi_t} w_{t-1}; \quad h_t \leq 1; \quad \left(w_t - \frac{\gamma + (1-\gamma)h_t}{1+\pi_t} w_{t-1} \right)(1-h_t) = 0 \tag{62}$$

given initial $\{K_0, B_0\}$, supply of government bonds B^g, and subject to the requirement that dividends be non-negative

$$D_t = (1-\nu)Y_t - F + (B_{t+1} - [1 + (1-\zeta)\rho Q_t]B^g)\frac{1+\pi_{t+1}}{1+i_t^s} - (B_t - [1 + (1-\zeta)\rho Q_t]B^g),$$

G Responses to monetary policy shocks

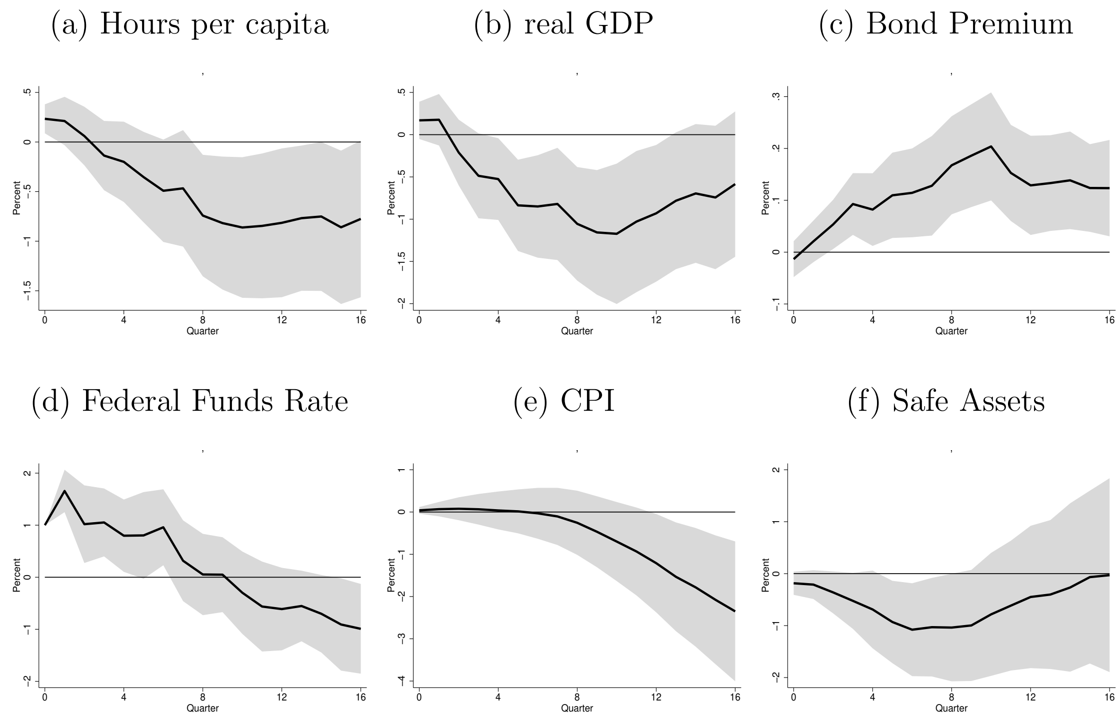
G.1 Empirical impulse responses

To convince the readers that our model exhibits "standard" impulse responses to monetary shocks, we estimate impulse responses of corresponding variables in the data using local projections instrumental variables method (Jordà, 2005; Jordà, Schularick and Taylor, 2020*a*). We use narrative monetary policy surprises from Wieland and Yang (2016) based on Romer and Romer (2004)'s methodology. The data is at quarterly frequency, and the sample extends from 1969Q2 to 2007Q4.[48] The contractionary monetary policy shock is scaled to generate a 100 basis points increase in federal funds rate on impact.

A contractionary monetary policy shock causes a contraction in hours per capita, and real GDP. Private supply of safe assets (measured following Gorton et al. (2012) methodology) contracts, while the bond premium is elevated. To our knowledge, this response of private supply of safe assets has not been documented in the literature.

[48] Additional results at monthly frequency using Romer and Romer (2004) surprises or the high-frequency surprises from Gorodnichenko and Weber (2016) are available upon request. Results are robust to considering these alternate surprises, as well as extending the sample, past the Great Recession, to up to 2016Q4. We truncate the sample at 2007Q4 to stop before Great Recession, following Ramey (2016).

Figure 8: Empirical impulse responses to a 100 basis points shock to federal funds rate



Note: Figure plots the impulse response of employment, output, bond premium, inflation rate, private safe assets and federal funds rate to a 100 basis point contractionary monetary policy shock. The shock is identified using Romer & Romer (2004) narrative series as instrumental variable. Two lags of control variables (all the plotted variables) are used in the local projections estimation. 90% confidence bands are constructed with robust standard errors. Sample: 1969Q2: 2007Q4. Bond premium is measured using Moody's Seasoned Aaa Corporate Bond Yield Index (FRED: AAA) and long-term Treasury yields (FRED: LTGOVTBD, GS20). Safe Assets is measured as total private safe assets following Gorton et al. (2012). For remaining variables, we use the quarterly data from replication material in Ramey (2016).

G.2 Model impulse responses

We show that the economy exhibits conventional responses to monetary policy shocks in the neighborhood of the full-employment steady state. We plot the impulse response to a one-time shock to the Taylor rule:

$$1 + i_t^s = R^* + \phi_\pi \left(\frac{1+\pi_t}{1+\pi^*} - 1 \right) + \phi_Y \left(\frac{Y_t}{Y^*} - 1 \right) + \epsilon_t,$$

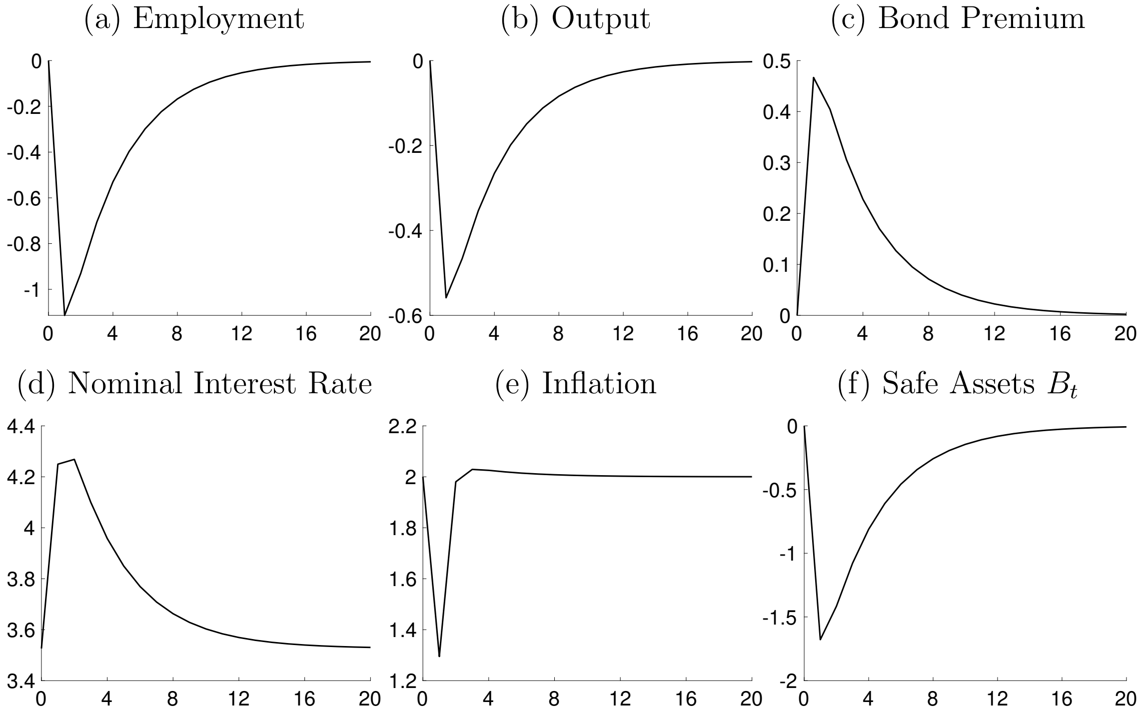
where $\epsilon_t = \rho \epsilon_{t-1} + \varepsilon_t$. We set $\rho = 0.75$ and simulate a response to a 100 basis points increase in ε_t.

G.2.1 Baseline model, no capital

First, we produce the impulse responses in the baseline model, with a downward nominal wage rigidity and only labor in the production function.[49] Figure 9 plots the impulse responses of output, employment, bond premium, nominal interest rate, inflation rate, and safe assets. The model

[49]We set $\sigma = 10$, $\beta = 0.965$, $\nu = 0.5$, $A = 1$, $F = 0.3$, and $B^g = 0.17$. Downward nominal wage rigidity is introduced in the same way as in the quantitative model: $w_t \geq \frac{\gamma + (1-\gamma) h_t}{\pi_t} w_{t-1}$; $h_t \leq 1$; $\left(w_t - \frac{\gamma + (1-\gamma) h_t}{\pi_t} w_{t-1} \right) (1 - h_t) = 0$, where we set $\gamma = 0.88$ as in the quantitative model.

Figure 9: Baseline model (no capital) responses to Taylor rule shock



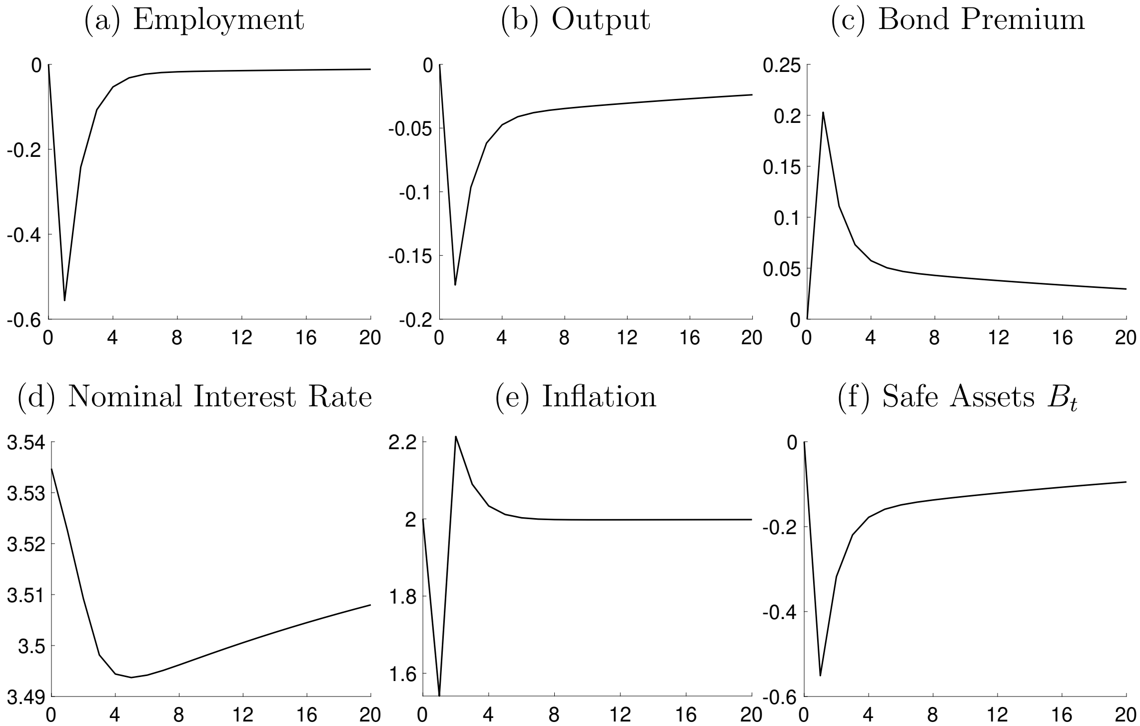
Note: Figure plots the model impulse response of employment, output, bond premium, inflation rate, private safe assets and nominal interest rate starting at full-employment steady state. The model is the baseline model presented in Section 3, but with only labor in the production function. Output, and safe assets are measured as percentage deviation from full-employment steady state values. Employment rate and Δ bond premium represents the annual percentage point change relative to the full-employment steady state. Nominal interest rate and the inflation rate are measured in annualized percentage points. The inflation target of the central bank is 2%.

exhibits impulse responses that resemble the empirical impulse responses. Following a contractionary monetary policy shock, short-term nominal interest rate rises and inflation falls. Aggregate demand contraction leads to a reduction in output, and employment. Since safe asset production is pro-cyclical, private safe asset production also declines. Counter-cyclicality of bond-premia implies that the bond premium rises in the model.

G.3 Monetary policy shocks in the quantitative model

Finally, we show the impulse responses in the quantitative model, presented in Section F. Figure 10 plots the impulse responses of output, employment, bond premium, nominal interest rate, inflation rate, and safe assets. The quantitative model is able to generate similar impulse responses as in the data. Following a contractionary monetary policy shock, short-term nominal interest rate rises and inflation falls. Aggregate demand contraction leads to a reduction in output, and employment. Since safe asset production is pro-cyclical, private safe asset production also declines. Counter-cyclicality of bond-premia implies that bond premium rises in the model.

Figure 10: Quantitative model responses to Taylor rule shock



Note: Figure plots the model impulse response of employment, output, bond premium, inflation rate, private safe assets and nominal interest rate starting at full-employment steady state. The model equations are provided in Section F. Calibration is discussed in the main text. Output, and safe assets are measured as percentage deviation from full-employment steady state values. Employment rate and Δ bond premium represents the annual percentage point change relative to the full-employment steady state. Nominal interest rate and the inflation rate are measured in annualized percentage points.The inflation target of the central bank is 2%.

Note, however, that the quantitative model generates a counter-factual response of real interest rate (seen from the nominal interest rate and the inflation responses). We verified that this response of real interest rate is not due to particular features of our extension of the new Keynesian model, but due to the presence of capital. Recently, Rupert and Šustek (2019) document that, in a new Keynesian model with capital, real interest rate may fall in response to a contractionary monetary policy shock while output, consumption and investment also contract. This anomaly is also present in our setup, as one can see from this figure.

Online Appendix

A Unemployment Risk and Counter-Cyclical Demand for Safe Assets

In this section, we present an extension of the model from Section 3 that incorporates unemployment risk following Heathcote and Perri (2018). The model allows us to obtain a richer version of the GEE equation (12), and show how different economic forces generate different cyclicalities in the demand for assets and the bond premium.

The economy is populated by a measure one of households. Households are comprised of a measure $1-\chi$ of workers and a measure χ of retirees. Workers are endowed with one unit of time every period, which they supply inelastically in the labor market. Retirees cannot work, and they live for only one period. Every period a fraction χ of workers retires, and a measure χ of workers is born. Thus, the composition of each household is constant over time.

At the beginning of every period, each household workers look for jobs in the labor market. In the presence of nominal wage rigidities, not all workers might find a job, and a fraction u_t will remain unemployed. When $u_t < 1$, the economy is operating below potential, and there is involuntary unemployment. Households are the owners of the firms, which distribute nominal dividends D_t.[50] Finally, households can trade nominal assets $\widetilde{B}_t$ at a nominal price $\frac{1}{1+i_t^s}$, where i_t^s is the nominal interest rate.

Being employed, unemployed, or retired determines what resources are available for consumption. In particular, we assume that intra-period transfer of funds is not possible. To finance their consumption, agents have access to their savings, and only employed workers can use their wage income. At the end of the period, and after consumption takes place, the members of each household pool their resources and make the saving decisions for the following period.

Households maximize a utilitarian welfare function of their members' utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[(1-\chi) U_t(C_t^w, C_t^u) + \chi \log(C_t^r)\right] \tag{63}$$

where C_t^w, C_t^u and C_t^r are the consumption of an employed worker, an unemployed worker and a retiree, respectively, and β is the discount factor. The function $U_t(\cdot,\cdot)$ is an aggregator of the workers' consumption, which we assume takes the following functional form:

$$U_t(C_t^w, C_t^u) = \frac{\left[(1-u_t)\left(C_t^w\right)^{\frac{\rho-1}{\rho}} + u_t \left(C_t^u\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{(1-\sigma)\rho}{\rho-1}}}{1-\sigma} \tag{64}$$

[50] For simplicity, we assume there is no capital, that is, $\alpha = 0$.

If we assume that $\rho \to \infty$ and $\sigma = 1$, we get

$$U_t(C_t^w, C_t^u) = \log((1 - u_t)C_t^w + u_t C_t^u).$$

This specification corresponds to the case where employed and unemployed worker's consumption are perfect substitutes, so unemployment risk is irrelevant.[51] This can be mapped onto the case we studied in Section 3. Here, we consider the case with $\rho = 1$ and $\sigma = 1$, so that equation (64) simplifies to

$$U_t(C_t^w, C_t^u) = (1 - u_t)\log(C_t^w) + u_t \log(C_t^u).$$

This specification corresponds to the case where unemployment risk matters for intra-household allocations, as in Heathcote and Perri (2018).

Within a period, each household member makes their consumption decision based on their portfolio holdings and income. The intra-period budget constraints faced by agents are given by

$$P_t C_t^w \leq \widetilde{B}_t + W_t + D_t + T_t \tag{65}$$

$$P_t C_t^j \leq \widetilde{B}_t \quad \text{for } j \in \{u, r\} \tag{66}$$

where P_t denotes the price level, $\widetilde{B}_t$ denotes the holdings of nominal one period safe bonds, W_t is the nominal wage, and T_t are lump-sum transfers. At the end of the period, the household as a whole faces the following budget constraint:

$$(1 - \chi)\left[(1 - u_t)P_t C_t^w + u_t P_t C_t^u\right] + \chi P_t C_t^r + \frac{\widetilde{B}_{t+1}}{1 + i_t^s} \leq (1 - u_t)W_t + D_t + \widetilde{B}_t + T_t \tag{67}$$

The problem of the household consists of choosing $\{C_t^w, C_t^u, C_t^r, \widetilde{B}_{t+1}\}_{t=0}^{\infty}$ in order to maximize (2) subject to the budget constraints (65), (66) and (67) for every $t \geq 0$, and a no-Ponzi condition. Since all households solve the same problem, we can treat the economy as populated by one representative household.

A.1 Generalized Euler Equation (GEE)

We will look for equilibria in which the intra-period budget constraint of the unemployed workers and retirees is binding. The FOCs associated with the household's problem are

$$(C_t^w): \quad \beta^t \left(C_t^w\right)^{-1} = P_t \Lambda_t \tag{68}$$

$$(\widetilde{B}_{t+1}): \quad \beta^{t+1}\left((1-\chi)u_{t+1}B_{t+1}^{-1} + \chi B_{t+1}^{-1}\right)\frac{1}{P_{t+1}} + (1 - (1-\chi)u_{t+1} - \chi)\Lambda_{t+1} = \frac{\Lambda_t}{1 + i_t^s} \tag{69}$$

[51] Unemployment still matters because it reduces the household's income, but not for its effects on the distribution of consumption.

where Λ_t is the Lagrange multiplier associated with the household's budget constraint, and $B_t \equiv \frac{\widetilde{B}_t}{P_t}$. Plugging (68) into (69), we get the following *Generalized Euler Equation* (GEE):

$$1 = \frac{1+i_t^s}{1+\pi_{t+1}}\beta\left[\underbrace{\left(\frac{C_{t+1}^w}{C_t^w}\right)^{-1}}_{\text{intertemporal substitution motive}} + \underbrace{(1-\chi)u_{t+1}\frac{(B_{t+1})^{-1}-\left(C_{t+1}^w\right)^{-1}}{\left(C_t^w\right)^{-1}}}_{\text{unemployment risk motive}} + \underbrace{\chi\frac{(B_{t+1})^{-1}-\left(C_{t+1}^w\right)^{-1}}{\left(C_t^w\right)^{-1}}}_{\text{retirement motive}}\right] \tag{70}$$

We define the bond premium as

$$bp \equiv (1-\chi)u_{t+1}\frac{(B_{t+1})^{-1}-\left(C_{t+1}^w\right)^{-1}}{\left(C_t^w\right)^{-1}} + \chi\frac{(B_{t+1})^{-1}-\left(C_{t+1}^w\right)^{-1}}{\left(C_t^w\right)^{-1}}.$$

The bond premium now has two terms: the retirement motive, as in Section 3, and the unemployment risk motive, as in Heathcote and Perri (2018). Note that while the retirement motive is always pro-cyclical (increasing in C_t^W and C_{t+1}^w), the self-insurance motive can be counter-cyclical since u_{t+1} is decreasing in Y_{t+1}.

The rest of the economy can be characterized similarly to Section 3. Focusing on steady-state equilibria, the characteristics of the economy depend on the cyclicality of the bond demand. If the bond demand is pro-cyclical (because the self-insurance motive is not sufficiently counter-cyclical), the economy is isomorphic to the economy in Section 3. In contrast, if the bond demand is counter-cyclical, the economy admits only two types of steady-state equilibria: a full-employment steady state and a self-fulfilling liquidity trap. In particular, if the demand for safe assets is counter-cyclical, the economy does not admit (permanent) fundamental liquidity traps.

B Robustness to Table 1

TABLE 5: Baa-Aaa spread on output gap (CFNAI)

	(1)	(2)	(3)	(4)	(5)	(6)
Output gap	-2.78***		-2.30***	-2.15***	-2.21***	-2.23***
	(0.86)		(0.66)	(0.67)	(0.51)	(0.51)
Fed funds rate		4.63**	3.63**	4.09**	9.03***	9.37***
		(1.99)	(1.79)	(2.08)	(2.20)	(2.24)
log(T-Bill/GDP)				38.50	-42.08	-29.28
				(41.83)	(28.39)	(27.24)
Distance to Default				-2.48	1.22	2.68
				(2.08)	(2.02)	(2.18)
Slope					20.35***	20.47***
					(3.70)	(3.68)
VIX						0.84
						(0.66)
Intercept	243.99***	136.88**	162.04***	281.26***	-98.69	-107.79
	(44.52)	(60.97)	(55.60)	(98.13)	(99.78)	(97.46)
# Obs.	419	419	419	419	419	419
Adj R^2	0.29	0.27	0.34	0.36	0.53	0.54

Note: Newey-West standard errors (12 lags) in parentheses. $***p < 0.01, **p < 0.05, *p < 0.1$. Includes a linear time-trend. Baa-Aaa spread measures the percentage difference between Moody's Baa-rated long-maturity corporate bond yield and Moody's Aaa-rated long-maturity corporate bond yield. Economic slack is computed with the monthly Chicago Fed National Activity Index (MA3). We normalized β_y to correspond to 0.2 units of increase in the CFNAI, which is associated with the onset of an expansion. A zero value for the CFNAI has been associated with the national economy expanding at its historical trend (average) rate of growth; negative values with below-average growth; and positive values with above-average growth. Sample: 1973–2007 (monthly).

TABLE 6: Baa-Aaa spread on output gap (band pass filter)

	(1)	(2)	(3)	(4)	(5)	(6)
Output gap	-14.75***		-19.67***	-19.02***	-17.50***	-17.54***
	(4.55)		(3.41)	(3.59)	(2.20)	(2.03)
Fed funds rate		4.63**	7.05***	6.45***	10.67***	11.01***
		(1.99)	(1.75)	(2.01)	(1.71)	(1.65)
log(T-Bill/GDP)				-6.64	-76.08***	-63.48**
				(32.56)	(26.11)	(24.95)
Distance to Default				-2.74	0.34	1.78
				(2.10)	(1.93)	(1.83)
Slope					18.03***	18.14***
					(2.43)	(2.42)
VIX						0.84
						(0.59)
Intercept	242.30***	136.88**	85.09**	115.54	-210.30**	-219.79***
	(38.22)	(60.97)	(40.84)	(80.65)	(81.97)	(77.13)
# Obs.	419	419	419	419	419	419
Adj R^2	0.34	0.27	0.53	0.53	0.67	0.68

Note: Newey-West standard errors (12 lags) in parentheses. $***p < 0.01, **p < 0.05, *p < 0.1$. Includes a linear time-trend. Baa-Aaa spread measures the percentage difference between Moody's Baa-rated long-maturity corporate bond yield and Moody's Aaa-rated long-maturity corporate bond yield. Economic slack is computed with band-pass filtering of log of (monthly) industrial production index at business cycle frequencies (18 and 96 months). Sample: 1973–2007 (monthly).

TABLE 7: Baa-Aaa spread on output gap (Hamilton filter)

	(1)	(2)	(3)	(4)	(5)	(6)
Output gap	-4.54***		-4.63***	-4.57***	-4.02***	-4.02***
	(0.70)		(0.53)	(0.55)	(0.44)	(0.43)
Fed funds rate		4.63**	5.04***	4.92***	7.86***	8.16***
		(1.99)	(0.87)	(1.06)	(1.33)	(1.34)
log(T-Bill/GDP)				-0.29	-46.78***	-35.55**
				(23.03)	(16.77)	(15.00)
Distance to Default				-0.83	0.93	2.20
				(1.35)	(1.35)	(1.53)
Slope					12.36***	12.48***
					(2.69)	(2.71)
VIX						0.75*
						(0.43)
Intercept	218.07***	136.88**	104.66***	117.40*	-97.04	-105.34*
	(24.09)	(60.97)	(18.65)	(60.06)	(59.77)	(57.98)
# Obs.	419	419	419	419	419	419
Adj R^2	0.59	0.27	0.70	0.70	0.75	0.76

Note: Newey-West standard errors (12 lags) in parentheses. $***p < 0.01, **p < 0.05, *p < 0.1$. Includes a linear time-trend. Baa-Aaa spread measures the percentage difference between Moody's Baa-rated long-maturity corporate bond yield and Moody's Aaa-rated long-maturity corporate bond yield. Economic slack is computed with filtering of log of (monthly) industrial production index using the Hamilton filter. Sample: 1973–2007 (monthly).

TABLE 8: Baa-Aaa spread on output gap (Polynomial filter)

	(1)	(2)	(3)	(4)	(5)	(6)
Output gap	-3.07***		-3.34***	-5.56***	-4.26***	-4.59***
	(1.09)		(0.91)	(1.18)	(1.22)	(1.23)
Fed funds rate		4.63**	5.29***	2.55	5.75***	5.99***
		(1.99)	(1.76)	(1.76)	(2.19)	(2.18)
log(T-Bill/GDP)				-144.44***	-150.56***	-138.46***
				(38.21)	(34.23)	(30.52)
Distance to Default				-2.76	-1.17	1.01
				(1.94)	(1.94)	(2.12)
Slope					11.38***	10.87***
					(3.89)	(3.87)
VIX						1.28*
						(0.76)
Intercept	214.23***	136.88**	93.40**	-200.32*	-305.68***	-334.26***
	(35.72)	(60.97)	(46.15)	(109.23)	(97.36)	(98.37)
# Obs.	419	419	419	419	419	419
Adj R^2	0.34	0.27	0.45	0.54	0.58	0.61

Note: Newey-West standard errors (12 lags) in parentheses. $***p < 0.01, **p < 0.05, *p < 0.1$. Includes a linear time-trend. Baa-Aaa spread measures the percentage difference between Moody's Baa-rated long-maturity corporate bond yield and Moody's Aaa-rated long-maturity corporate bond yield. Economic slack is computed as deviation from trend estimated using a (sixth-degree) polynomial regression on time. Sample: 1973–2007 (monthly).

TABLE 9: Baa-Aaa spread on output gap (unemployment rate)

	(1)	(2)	(3)	(4)	(5)	(6)
Output gap	21.45***		22.19***	28.13***	27.49***	27.96***
	(3.79)		(2.97)	(2.46)	(3.09)	(3.17)
Fed funds rate		4.63**	5.23***	2.69**	3.02**	3.35**
		(1.99)	(1.20)	(1.10)	(1.50)	(1.46)
log(T-Bill/GDP)				-116.48***	-118.09***	-102.68***
				(22.00)	(22.27)	(20.03)
Distance to Default				-3.13**	-2.95**	-1.11
				(1.29)	(1.41)	(1.53)
Slope					1.20	1.02
					(2.74)	(2.69)
VIX						1.09**
						(0.43)
Intercept	-14.27	136.88**	-140.13***	-413.72***	-420.85***	-438.06***
	(45.67)	(60.97)	(43.58)	(64.60)	(67.23)	(60.85)
# Obs.	419	419	419	419	419	419
Adj R^2	0.54	0.27	0.65	0.74	0.74	0.75

Note: Newey-West standard errors (12 lags) in parentheses. $***p < 0.01, **p < 0.05, *p < 0.1$. Includes a linear time-trend. Baa-Aaa spread measures the percentage difference between Moody's Baa-rated long-maturity corporate bond yield and Moody's Aaa-rated long-maturity corporate bond yield. Economic slack variable is civilian unemployment rate. Sample: 1973–2007 (monthly).

C Robustness to Table 3

TABLE 10: Financial spreads on output gap (CFNAI)

	Aaa-Tbill	BA-Tbill	AACP-Tbill	CD-Tbill	CPP2-Tbill
	(1)	(2)	(3)	(4)	(5)
	1967-2007	1967-2007	1967-2007	1976-2007	1998-2007
Output gap	-1.36**	-1.32**	-1.39***	-1.65*	1.34**
	(0.67)	(0.60)	(0.46)	(0.95)	(0.55)
Fed funds rate	-0.97	10.40***	6.42***	14.34***	20.79*
	(1.83)	(1.05)	(1.16)	(2.43)	(11.46)
log(T-Bill/GDP)	-87.97***	-62.85**	-27.28	-67.05*	-22.93
	(30.23)	(27.30)	(26.79)	(34.25)	(45.14)
Slope	5.01	-0.46	-4.56	12.43*	15.46
	(3.40)	(3.42)	(3.28)	(6.70)	(13.25)
VIX	1.65***	2.06***	1.83***	1.61***	2.56**
	(0.62)	(0.64)	(0.60)	(0.57)	(1.17)
Intercept	-29.36	-133.39*	14.43	-341.21***	-492.15
	(91.33)	(76.38)	(74.54)	(115.19)	(326.54)
# Obs.	489	489	489	383	119
Adj R^2	0.40	0.63	0.54	0.38	0.48

Note: Newey-West standard errors (12 lags) in parentheses. $***p < 0.01, **p < 0.05, *p < 0.1$. Includes a linear time-trend. Output gap is proxied with Chicago Fed National Activity Index (MA3). We normalized β_y to correspond to 0.2 units of increase in the CFNAI, which is associated with the onset of an expansion. Column 1 uses the percentage spread between Moody's Aaa-rated long-maturity corporate bond yield and the yield on long-maturity Treasury bonds. Column 2 use the three-month banker's acceptance rate and T-bills. The data series for the banker's acceptance rate ends in the 1990s. To create a series until 2007, we use the GC repo/T-bill spread from 1991 onward constructed by Nagel (2016). Column 3 uses the percentage yield spread between 3-month high-grade commercial paper and Treasury bills. Column 4 uses the spread between three-month certificate of deposit (CD) rates and T-bills as an alternative measure of the illiquid rate. Column 5 uses the percentage yield spread between lower-grade commercial paper and Treasury bills. It is calculated as the sum of the CP-bills yield spread described above (i.e., high-grade commercial paper minus Treasury bills) and the yield spread between 30-day A2/P2 nonfinancial commercial paper and 30-day AA nonfinancial commercial paper, with data obtained from the Federal Reserve Bank of New York.

TABLE 11: Financial spreads on output gap (band pass filter)

	Aaa-Tbill	BA-Tbill	AACP-Tbill	CD-Tbill	CPP2-Tbill
	(1)	(2)	(3)	(4)	(5)
	1948-2007	1948-2007	1948-2007	1976-2007	1998-2007
Output gap	-7.23***	-3.30**	-0.90	2.48	8.07
	(2.17)	(1.54)	(1.60)	(6.76)	(7.14)
Fed funds rate	5.59***	11.93***	7.76***	15.37***	18.04*
	(1.04)	(1.25)	(1.16)	(2.76)	(9.84)
log(T-Bill/GDP)	-79.04***	-32.15***	-11.14	-71.39*	-13.11
	(24.74)	(11.00)	(11.15)	(41.20)	(40.83)
Slope	8.65**	-1.27	-4.12	13.89*	13.86
	(3.85)	(2.74)	(2.76)	(7.40)	(12.46)
VIX	1.52**	1.90***	1.69***	1.72***	2.12**
	(0.65)	(0.60)	(0.55)	(0.49)	(1.00)
Intercept	-184.56***	-87.62***	25.99	-389.34***	-353.84
	(48.78)	(24.07)	(25.70)	(139.90)	(257.21)
# Obs.	719	719	719	383	119
Adj R^2	0.47	0.62	0.47	0.37	0.49

Note: Newey-West standard errors (12 lags) in parentheses. $***p < 0.01, **p < 0.05, *p < 0.1$. Includes a linear time-trend. Output gap is computed with band-pass filtering of log of (monthly) industrial production index at business cycle frequencies (18 and 96 months). Column 1 uses the percentage spread between Moody's Aaa-rated long-maturity corporate bond yield and the yield on long-maturity Treasury bonds. Column 2 uses the three-month banker's acceptance rate and T-bills. The data series for the banker's acceptance rate ends in the 1990s. To create a series until 2011, we use the GC repo/T-bill spread from 1991 onward constructed by Nagel (2016). Column 3 uses the percentage yield spread between 3-month high-grade commercial paper and Treasury bills. Column 4 uses the spread between three-month certificate of deposit (CD) rates and T-bills as an alternative measure of the illiquid rate. Column 5 uses the percentage yield spread between lower-grade commercial paper and Treasury bills. It is calculated as the sum of the CP-bills yield spread described above (i.e., high-grade commercial paper minus Treasury bills) and the yield spread between 30-day A2/P2 nonfinancial commercial paper and 30-day AA nonfinancial commercial paper, with data obtained from the Federal Reserve Bank of New York.

TABLE 12: Financial spreads on output gap (Hamilton filter)

	Aaa-Tbill	BA-Tbill	AACP-Tbill	CD-Tbill	CPP2-Tbill
	(1)	(2)	(3)	(4)	(5)
	1973-2007	1973-2007	1973-2007	1976-2007	1998-2007
Output gap	-2.52***	-0.09	0.26	0.13	3.34**
	(0.62)	(0.40)	(0.43)	(0.81)	(1.59)
Fed funds rate	4.49***	11.92***	7.89***	15.55***	16.70*
	(1.05)	(1.32)	(1.22)	(2.87)	(9.66)
log(T-Bill/GDP)	-81.38***	-22.91*	-5.85	-75.54**	-58.18
	(24.17)	(12.04)	(11.37)	(36.00)	(45.07)
Slope	5.49	-0.69	-3.42	14.06*	19.39
	(3.51)	(3.03)	(3.02)	(8.07)	(12.66)
VIX	1.13**	1.86***	1.72***	1.70***	2.96**
	(0.56)	(0.62)	(0.56)	(0.50)	(1.16)
Intercept	-177.26***	-66.74**	36.39	-403.10***	-765.89**
	(47.28)	(26.62)	(26.24)	(133.06)	(361.56)
# Obs.	719	719	719	383	119
Adj R^2	0.52	0.61	0.47	0.37	0.52

Note: Newey-West standard errors (12 lags) in parentheses. $***p < 0.01, **p < 0.05, *p < 0.1$. Includes a linear time-trend. Output gap is computed with filtering of log of (monthly) industrial production index using the Hamilton filter. Column 1 uses the percentage spread between Moody's Aaa-rated long-maturity corporate bond yield and the yield on long-maturity Treasury bonds. Column 2 use the three-month banker's acceptance rate and T-bills. The data series for the banker's acceptance rate ends in the 1990s. To create a series until 2007, we use the GC repo/T-bill spread from 1991 onward constructed by Nagel (2016). Column 3 uses the percentage yield spread between 3-month high-grade commercial paper and Treasury bills. Column 4 uses the spread between three-month certificate of deposit (CD) rates and T-bills as an alternative measure of the illiquid rate. Column 5 uses the percentage yield spread between lower-grade commercial paper and Treasury bills. It is calculated as the sum of the CP-bills yield spread described above (i.e., high-grade commercial paper minus Treasury bills) and the yield spread between 30-day A2/P2 nonfinancial commercial paper and 30-day AA nonfinancial commercial paper, with data obtained from the Federal Reserve Bank of New York.

TABLE 13: Financial spreads on output gap (Polynomial filter)

	Aaa-Tbill	BA-Tbill	AACP-Tbill	CD-Tbill	CPP2-Tbill
	(1)	(2)	(3)	(4)	(5)
	1973-2007	1973-2007	1973-2007	1976-2007	1998-2007
Output gap	0.70	-0.45	0.06	-0.39	2.66
	(0.73)	(0.68)	(0.60)	(1.05)	(2.03)
Fed funds rate	6.01***	11.74***	7.80***	15.14***	18.02
	(1.26)	(1.40)	(1.28)	(3.01)	(11.58)
log(T-Bill/GDP)	-45.50	-29.23*	-7.42	-83.76*	5.13
	(27.98)	(15.03)	(12.21)	(46.37)	(47.88)
Slope	11.85**	-1.50	-3.78	13.12*	14.14
	(4.70)	(3.04)	(3.00)	(7.39)	(13.46)
VIX	1.41**	1.92***	1.68***	1.72***	2.30**
	(0.70)	(0.59)	(0.53)	(0.53)	(1.13)
Intercept	-107.16*	-83.29**	34.48	-418.68***	-488.84
	(59.54)	(32.28)	(29.04)	(139.24)	(333.32)
# Obs.	719	719	719	383	119
Adj R^2	0.41	0.61	0.47	0.37	0.47

Note: Newey-West standard errors (12 lags) in parentheses. $***p < 0.01, **p < 0.05, *p < 0.1$. Includes a linear time-trend. Output gap is computed as deviation from trend estimated using a (sixth-degree) polynomial regression on time. Column 1 uses the percentage spread between Moody's Aaa-rated long-maturity corporate bond yield and the yield on long-maturity Treasury bonds. Column 2 use the three-month banker's acceptance rate and T-bills. The data series for the banker's acceptance rate ends in the 1990s. To create a series until 2007, we use the GC repo/T-bill spread from 1991 onward constructed by Nagel (2016). Column 3 uses the percentage yield spread between 3-month high-grade commercial paper and Treasury bills. Column 4 uses the spread between three-month certificate of deposit (CD) rates and T-bills as an alternative measure of the illiquid rate. Column 5 uses the percentage yield spread between lower-grade commercial paper and Treasury bills. It is calculated as the sum of the CP-bills yield spread described above (i.e., high-grade commercial paper minus Treasury bills) and the yield spread between 30-day A2/P2 nonfinancial commercial paper and 30-day AA nonfinancial commercial paper, with data obtained from the Federal Reserve Bank of New York.

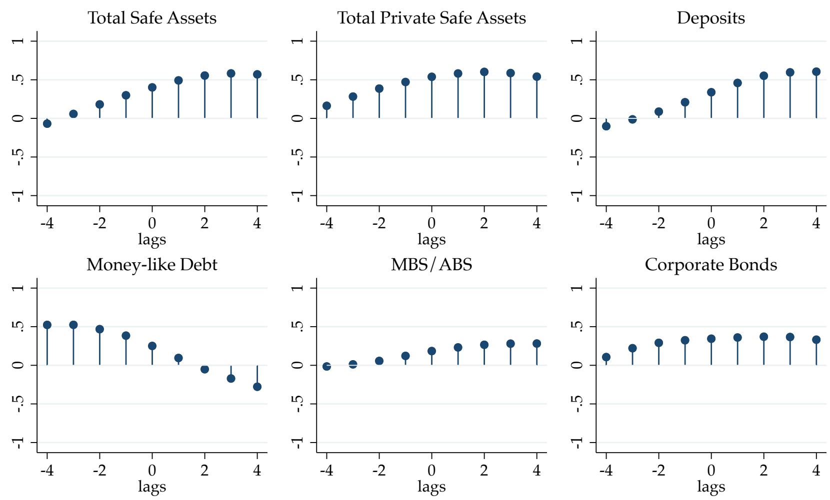
TABLE 14: FINANCIAL SPREADS ON OUTPUT GAP (UNEMPLOYMENT RATE)

	Aaa-Tbill	BA-Tbill	AACP-Tbill	CD-Tbill	CPP2-Tbill
	(1) 1973-2007	(2) 1973-2007	(3) 1973-2007	(4) 1976-2007	(5) 1998-2007
Output gap	10.21**	-1.53	-7.58***	-1.37	-11.93
	(3.96)	(3.09)	(2.69)	(5.46)	(9.11)
Fed funds rate	2.56	12.43***	10.08***	15.73***	18.93
	(1.66)	(1.57)	(1.51)	(2.68)	(11.79)
log(T-Bill/GDP)	-74.69***	-19.33	4.89	-72.63**	19.36
	(28.74)	(12.27)	(9.49)	(35.00)	(53.55)
Slope	1.28	0.84	2.79	14.59*	16.39
	(4.58)	(3.92)	(3.77)	(7.92)	(13.77)
VIX	1.59**	1.86***	1.59***	1.67***	2.43**
	(0.63)	(0.60)	(0.54)	(0.52)	(1.17)
Intercept	-211.49***	-53.81*	88.53***	-383.30***	-344.98
	(65.10)	(30.53)	(25.67)	(139.94)	(309.56)
# Obs.	719	719	719	383	119
Adj R^2	0.45	0.61	0.49	0.37	0.47

Note: Newey-West standard errors (12 lags) in parentheses. $***p < 0.01, **p < 0.05, *p < 0.1$. Includes a linear time-trend. Output gap variable is civilian unemployment rate. Column 1 uses the percentage spread between Moody's Aaa-rated long-maturity corporate bond yield and the yield on long-maturity Treasury bonds. Column 2 use the three-month banker's acceptance rate and T-bills. The data series for the banker's acceptance rate ends in the 1990s. To create a series until 2007, we use the GC repo/T-bill spread from 1991 onward constructed by Nagel (2016). Column 3 uses the percentage yield spread between 3-month high-grade commercial paper and Treasury bills. Column 4 uses the spread between three-month certificate of deposit (CD) rates and T-bills as an alternative measure of the illiquid rate. Column 5 uses the percentage yield spread between lower-grade commercial paper and Treasury bills. It is calculated as the sum of the CP-bills yield spread described above (i.e., high-grade commercial paper minus Treasury bills) and the yield spread between 30-day A2/P2 nonfinancial commercial paper and 30-day AA nonfinancial commercial paper, with data obtained from the Federal Reserve Bank of New York.

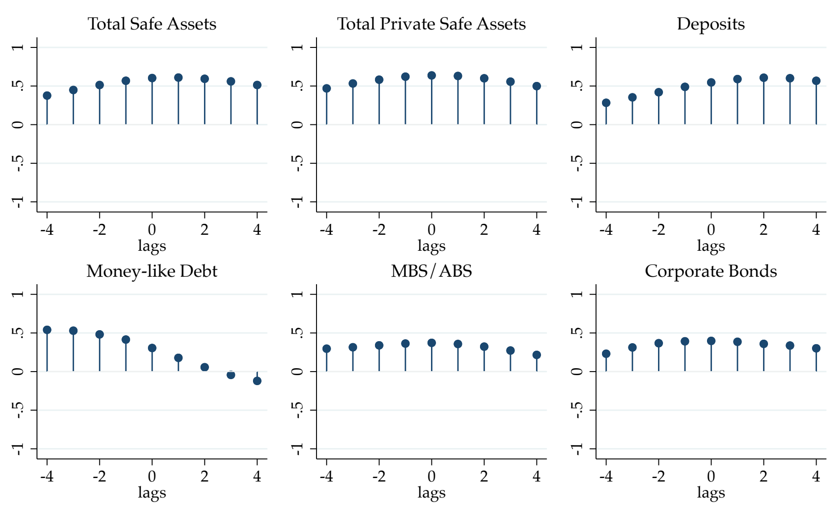
D Robustness to Figure 3

Figure 11: Correlations of x_{t+h} with time-t real GDP (filtered with Hamilton filter)



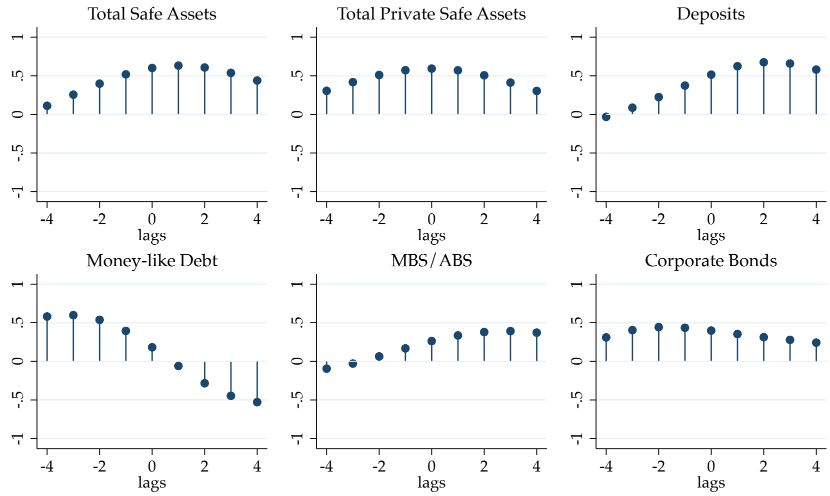
Source: Our calculations using US Financial Accounts data retrieved from FRED, St. Louis Fed. The definitions follow Gorton et al. (2012). Real GDP and all the safe asset component series are detrended with the Hamilton filter. See text.

Figure 12: Correlations of x_{t+h} with time-t real GDP (polynomial filter)



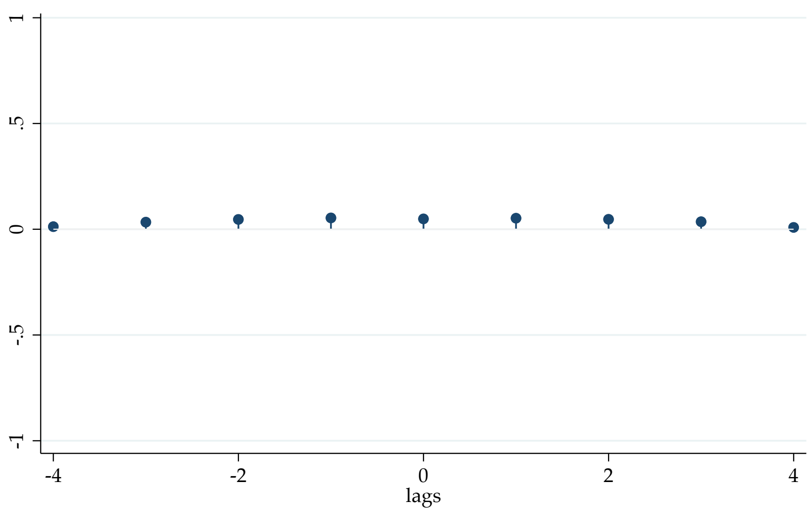
Source: Our calculations using US Financial Accounts data retrieved from FRED, St. Louis Fed. The definitions follow Gorton et al. (2012). Real GDP and all of the safe asset component series are detrended using a (sixth-degree) polynomial regression on time. See text.

Figure 13: Correlations of x_{t+h} with time-t real GDP (band pass filter)



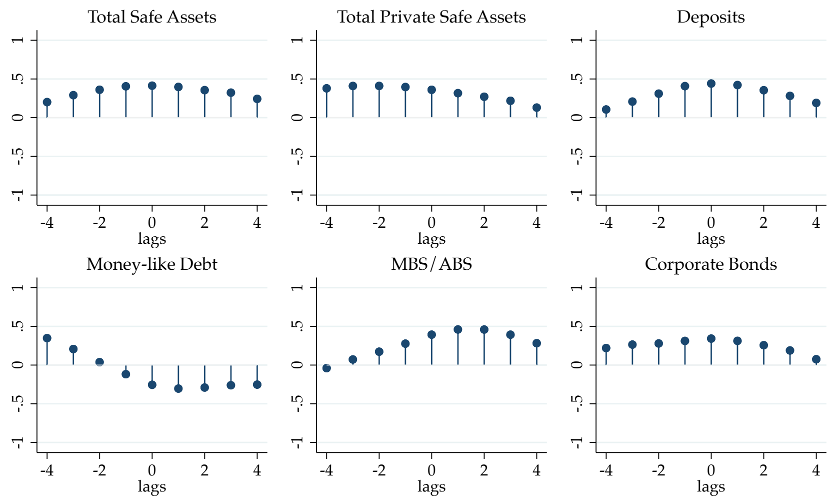
Source: Our calculations using US Financial Accounts data retrieved from FRED, St. Louis Fed. The definitions follow Gorton et al. (2012). Real GDP and all of the safe asset component series are detrended using a band-pass filter at business cycle frequencies (18 and 96 months). See text.

Figure 14: Correlations of x_{t+h} with time-t real GDP (linearly detrended)



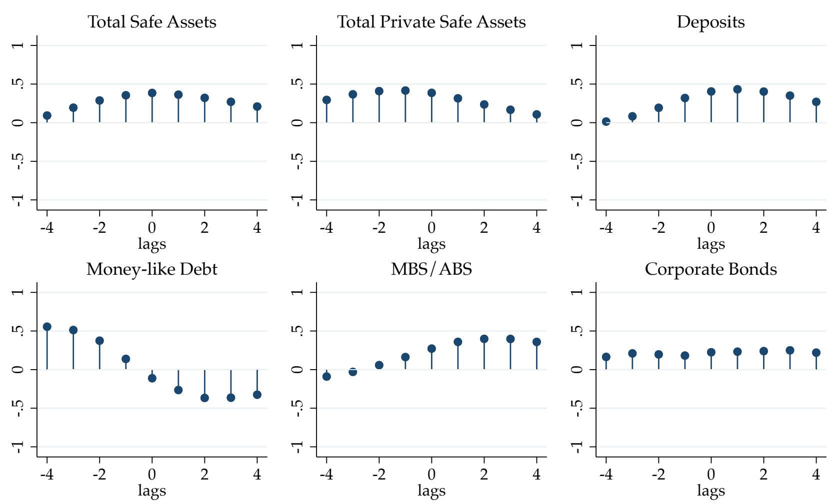
Source: Our calculations using US Financial Accounts data retrieved from FRED, St. Louis Fed. The definitions follow Gorton et al. (2012). Real GDP and all of the safe asset component series are linearly detrended. See text.

Figure 15: Correlations of x_{t+h} (Hamilton filtered) with time-t CFNAI



Source: Our calculations using US Financial Accounts data retrieved from FRED, St. Louis Fed. The definitions follow Gorton et al. (2012). All safe asset component series are detrended with Hamilton filter. See text.

Figure 16: Correlations of x_{t+h} (y-o-y growth) with time-t CFNAI



Source: Our calculations using US Financial Accounts data retrieved from FRED, St. Louis Fed. The definitions follow Gorton et al. (2012). All safe asset component series are plotted in year on year growth rates. See text.

E Robustness for empirical results using instrumental variable techniques

E.1 Countercyclical bond-premia: with instrumental variables

Here, we show the robustness of our results with two different instruments for the cycle: Baumeister and Hamilton (2019)'s structural oil supply shocks, and Fernald (2014)'s utilization-adjusted TFP series. Across, various specifications we find a robust set of results: bond-premia exhibit countercyclicality ($\beta_y < 0$; or $\eta_y - \psi_y > 0$). The estimates obtained in the baseline OLS strategy reported in the paper lie within the confidence band of the estimates obtained from the instrumental variables strategy.

For the instrumental variables approach, we estimate impulse response functions (IRFs) of the financial spreads and log (GDP). We estimate the dynamic IRFs instead of only looking at the contemporaneous effects because the available instruments affect the cycle with some lag. Since this exercise is different from the one reported in Table 1, first we present the IRFs estimated with OLS described below. Note that Fernald (2014)'s utilization-adjusted TFP series is available only at quarterly frequency. Consequently, to keep the exercise comparable across specifications, we present results from IRF estimation on quarterly data.

We directly estimate the cumulative impulse response function using Jordà (2005) local projections method depicted in the following estimating equation. The coefficients α_h denote the cumulative impulse response function, at horizons $h \geq 0$, for a dependent variable b in response to an impulse in variable v at time t:

$$b_{t+h} - b_{t-1} = \alpha_0 + \alpha_h v_t + \Lambda \mathbb{Z}_t + \epsilon_t$$

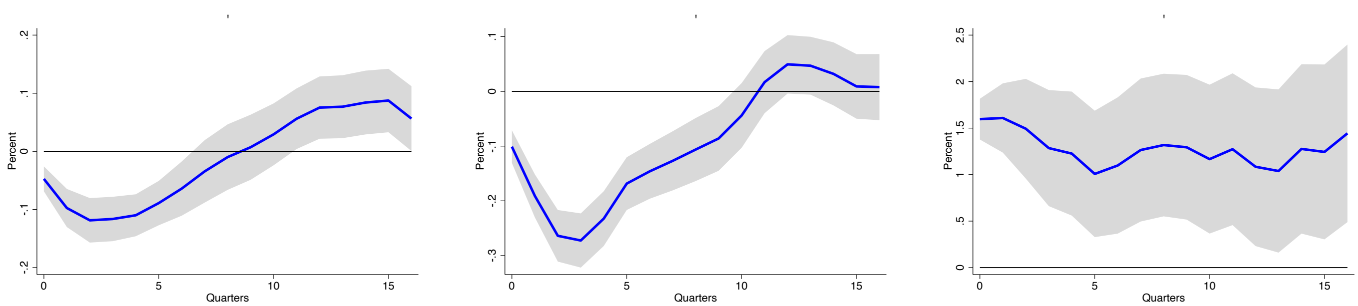
The vector $\mathbb{Z}_t$ includes controls, as in the baseline regression in Table I (VIX, federal funds rate, slope, and T-bill/GDP), as well as two lag terms of the dependent variable. Estimation sample used spans 1952Q1 – 2019Q2.

Figure 17 presents the IRFs to a one standard deviation increase in year on year GDP growth. Shaded areas report one standard error confidence bands. Consistent with the estimates in Table 1, financial spreads (BAA - AAA, AAA - Treasury) fall as GDP increases.

We now consider an instrumental variable strategy. In our preferred specification, we use oil supply shocks from Baumeister and Hamilton (2019). They identify structural oil supply shocks using a state-of-art Bayesian structural vector autoregression framework. We consider their oil supply shocks as an instrument for the business cycle since Baumeister and Hamilton (2019) argue that their identified oil supply shocks are exogenous in the short run. We also believe that the exclusion restriction on oil shocks is valid – that is, oil supply shocks are correlated with the cycle and affect other variables such as the federal funds rate and the supply of public assets only through their effect on the cycle. Nevertheless, in order to ensure that the instruments are

Figure 17: Impulse Response Function to a one standard deviation increase in year-on-year GDP growth: selection-on-observables (or OLS) approach

(a) AAA-Treasury **(b)** BAA-AAA **(c)** log (GDP)



Notes: Shaded areas denote one standard error confidence bands constructed using robust standard errors. IRFs are estimated with Ordinary Least Squares (OLS) or the *selection-on-observables* strategy. The impulse is a one standard deviation increase in year-on-year GDP growth rate. Estimation includes two quarterly lags of the dependent variable, and current controls for VIX, slope, Tbill/GDP ratio, and federal funds rate. Sample (Quarterly): 1952Q1 – 2019Q2

"orthogonal" to contemporary changes in potential confounders, we include a control vector $\mathbb{Z}_t$ in the instrumental variable specification presented below.[52] The vector $\mathbb{Z}_t$ includes controls, as in the baseline regression in Table I (VIX, federal funds rate, slope, and T-bill/GDP), as well as two lag terms of the dependent variable:

$$b_{t+h} - b_{t-1} = \alpha_0 + \alpha_h OilSupplyShock_t + \Lambda \mathbb{Z}_t + \epsilon_t$$

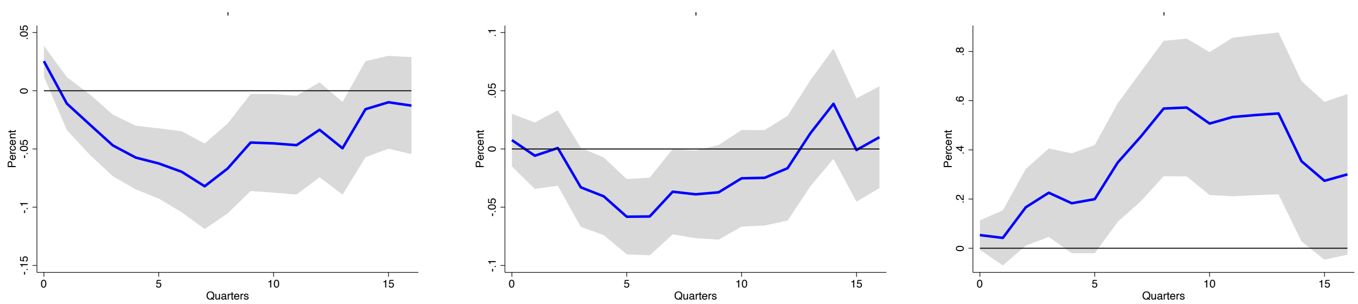
Figure 18 reports the estimated IRFs (α_h) to one standard deviation oil supply shocks. The estimated impulse responses from this Local Projections-Instrumental Variables (LPIV) strategy are consistent with the OLS estimates. One standard deviation shock to oil supply is followed by a fall in GDP and an increase in financial spreads (AAA-Treasury, Baa-Aaa). Since oil supply shocks are instruments for the cycle, we infer from the estimated IRFs that $\beta_y < 0$. That is, the IRFs identify that the bond-premia is counter-cyclical. The peak response of GDP and BAA-AAA spread suggest that a one percentage point increase in cyclical GDP is associated with a 10 basis points reduction in BAA-AAA spread. In other words, $\beta_y = -0.10$. Moreover, the estimates reported in Table 1 lie within a two-standard deviation confidence band of the estimates obtained from using the instrumental variables method.

The data for these oil shocks only starts in 1975, and because of this, the estimates are found to have large confidence bands. For this reason, we also consider an alternative instrument with wider data availability: Fernald (2014)'s utilization adjusted TFP growth rate series. Assuming that the changes in utilization-adjusted TFP are exogenous in the short-run and that the exclusion restriction holds, conditional on a vector of controls, we can consider the TFP growth rate series as an instrument for the cycle. Estimation sample used spans 1952Q1 – 2019Q2. Figure 19 reports

[52]We follow Jordà, Schularick and Taylor (2020*b*) in our implementation of local projections instrumental variable strategy. Controlling for vector $\mathbb{Z}_t$ amounts to embedding a selection-on-observables identification assumption, as formalized by Angrist and Kuersteiner (2011) and utilized by Jordà et al. (2020*b*).

Figure 18: Impulse Response Function to one standard deviation oil supply shock: LPIV

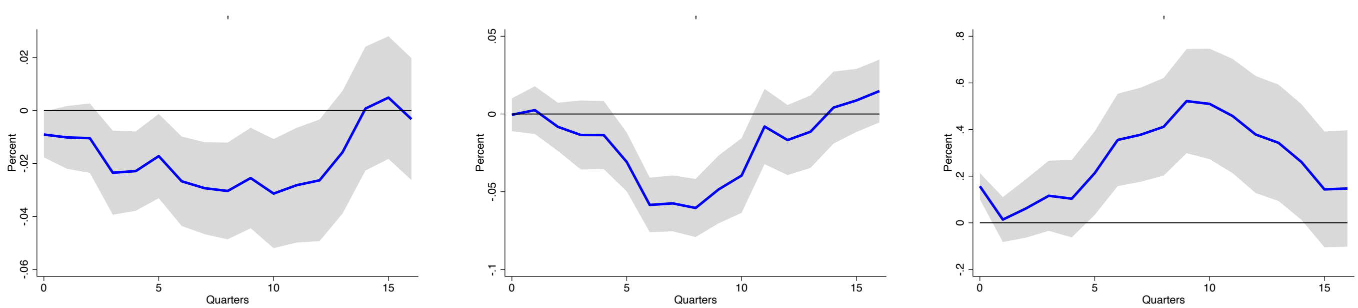
(a) AAA-Treasury **(b)** BAA-AAA **(c)** log (GDP)



Notes: Shaded areas denote one standard error confidence bands constructed using robust standard errors. IRFs are estimated with Baumeister and Hamilton (2019)'s structural oil supply shocks series as instruments for the cycle. The impulse is a one standard deviation oil supply shock. This estimation includes as controls: current VIX, slope, Tbill/GDP ratio, and federal funds rate; and two quarterly lags of the dependent variable. Sample (Quarterly): 1975Q1 – 2019Q2

Figure 19: Impulse Response Function to a one standard deviation increase in utilization-adjusted TFP: LP-IV

(a) AAA-Treasury **(b)** BAA-AAA **(c)** log (GDP)



Notes: Shaded areas denote one standard error confidence bands constructed using robust standard errors. IRFs are estimated with Fernald (2014)'s utilization-adjusted TFP growth rate series as instruments for the cycle. The impulse is a one standard deviation increase in utilization-adjusted TFP growth. This estimation includes as controls: current VIX, slope, Tbill/GDP ratio, and federal funds rate; and two quarterly lags of the dependent variable. Sample (Quarterly): 1952Q1 – 2019Q2

the estimated IRFs (α_h) from the following equation:

$$b_{t+h} - b_{t-1} = \alpha_0 + \alpha_h \Delta TFP_t + \Lambda \mathbb{Z}_t + \epsilon_t$$

A one standard deviation increase in utilization-adjusted TFP growth rate is followed by an increase in GDP and a reduction in financial spreads (AAA-Treasury, Baa-Aaa). Since TFP shocks are considered as instruments for the cycle, we also infer from the estimated IRFs that $\beta_y < 0$. That is, the IRFs identify that the bond-premia is counter-cyclical. Similar to the estimates obtained using oil supply shocks, the peak response of GDP and BAA-AAA spread suggests that a one percentage point increase in cyclical GDP is associated with 10 basis points reduction in BAA-AAA spread.

In sum, our empirical exercises using various empirical strategies confirm that the bond-premia is counter-cyclical.

E.2 Procyclical supply of private safe assets: instrumental variables

We now identify the sign of η_y using an instrumental variable technique. We find robust evidence that $\eta_y > 0$, consistent with the proposed mechanism in the paper.

In order to identify the sign of η_y, we require shifters for safe-asset demand that trace out the safe asset supply curve. One source of these shifters comes from exogenous changes in λ_t, which can be proxied by changes in uncertainty or risk-aversion that are exogenous to the business cycle, the supply of public safe assets, and the federal funds rate.[53] The safe assets data is only available at the quarterly frequency, so we aggregate the constructed instruments to quarterly frequency for use in our estimation. We use the following three instruments, listed in order of highest to lowest data availability:

1. VIX is often considered an indicator of risk-aversion in the financial markets (Bekaert, Hoerova and Lo Duca, 2013). We consider changes in VIX that are not predicted by contemporaneous industrial production, as well as six monthly lags of VIX and industrial production. These VIX residuals are aggregated to quarterly frequency and considered as exogenous safe-asset demand shifters λ_t. Sample: 1952Q1–2019Q2.

2. Ludvigson, Ma and Ng (2021) construct a financial uncertainty index using *shock-based* restrictions in structural vector autoregressions (SVARs) with financial and macroeconomic data. The financial uncertainty index measures a common component in "the time-varying volatilities of h-step ahead forecast errors across a large number of " financial series. This uncertainty index, they argue, is exogenous to economic and policy-related fluctuations, as well as orthogonal to *macro* uncertainty. We consider changes in their financial uncertainty index that are not predictable by six monthly lags of the index. These financial uncertainty residuals are aggregated to quarterly frequency and considered as exogenous safe-asset demand shifters λ_t.[54] Sample: 1961Q1–2019Q2.

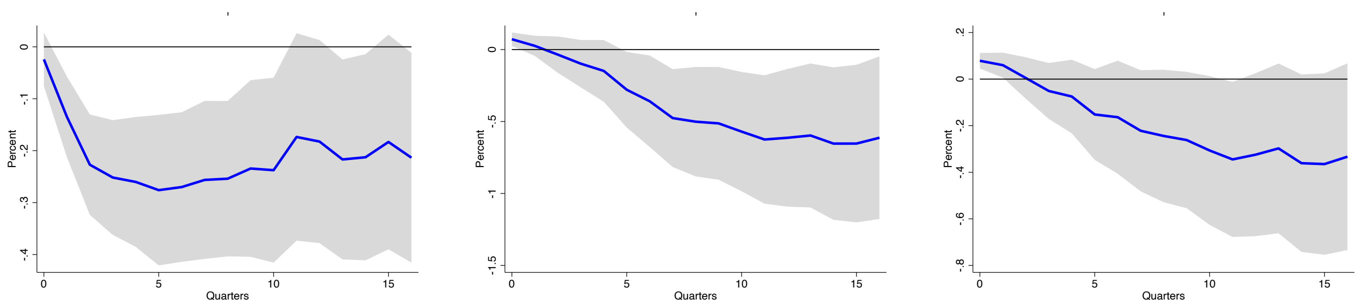
3. Time-variation in risk aversion of agents creates demand for insurance. Bekaert, Engstrom and Xu (forthcoming) construct a time-varying risk aversion index using a dynamic asset pricing model. This measure, they argue, is distinct from time-varying uncertainty in fundamentals. We consider changes in their risk-aversion index that are not predicted by six

[53] Additionally, if we were to take our model seriously, then exogenous changes in federal funds rate affect only the safe asset demand. We also pursued exogenous changes in federal funds rate proxied by Romer & Romer shocks to instrument for shifters in safe-asset demand and found similar results. However, we believe that federal funds rate shocks could also affect safe asset supply through discounting of future firm profits or dividends in more general models. Consequently, our preferred approach is to focus on shifters λ_t.

[54] Another recent work by Pflueger, Siriwardane and Sunderam (2020) also constructs a measure of financial market risk perceptions. We find similar results from using their index: private safe asset supply is procyclical.

Figure 20: Impulse Response Function to a one standard deviation change in VIX residuals

(a) log (GDP) **(b)** log (Private Safe Assets) **(c)** log (Total Safe Assets)



Notes: Shaded areas denote one standard error confidence bands constructed using robust standard errors. IRFs are estimated with VIX residuals series as instruments for the safe asset demand shifter λ_t. VIX residuals are obtained from a regression of VIX on current and six monthly lags of industrial production, and six monthly lags of the VIX. The monthly residuals are aggregated to quarterly frequency. The impulse is a one standard deviation increase in VIX residual. This estimation includes as controls: current Tbill/GDP ratio, and federal funds rate; and two quarterly lags of the dependent variable. Sample (Quarterly): 1952Q1 – 2019Q2.

monthly lags of the index. The resulting risk aversion shocks are exogenous demand shifters λ_t.[55] Sample: 1990Q1–2019Q2.

As before, we estimate the impulse response functions using a local projections instrumental variable strategy:

$$b_{t+h} - b_{t-1} = \alpha_0 + \alpha_h \lambda_t + \Lambda \tilde{\mathbb{C}}_t + \epsilon_t$$

For a dependent variable b, the coefficients α_h directly estimate the cumulative IRF at horizon $h \geq 0$ to a one standard deviation change in demand shifter λ_t. We include a vector of controls, $\tilde{\mathbb{C}}_t$, comprising of the current federal funds rate and T-bill/GDP, as well as two lag terms of the dependent variable.

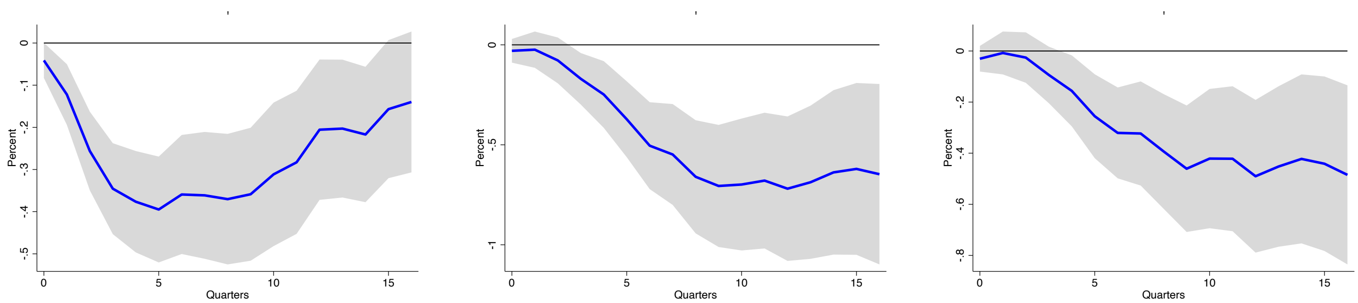
Figure 20 reports the estimated IRFs to one standard deviation change in VIX residuals. Shaded areas denote one standard error confidence bands constructed using robust standard errors.

Economic activity falls, and there is a reduction in private safe assets. We find similar results using residuals to Ludvigson et al. (2021)'s financial uncertainty index (Figure 21), as well as residuals to Bekaert et al. (forthcoming)'s risk aversion index (Figure 22). Since the sample only starts in 1990 for Bekaert et al. (forthcoming) risk aversion index, we estimate large confidence bands but obtain qualitatively similar results. Estimated peak responses of GDP and Private Safe Assets suggest values of η_y greater than one.

[55] We also conducted exercises with another measure of risk aversion that was constructed by Bekaert et al. (2013) from VIX. They decomposed VIX into uncertainty measure and a variance risk-premium, which they interpret as reflecting factors such as risk aversion. However, that measure is found endogenous to the policy rate, and consequently, we prefer the Bekaert et al. (forthcoming) measure. Results are robust to using residuals obtained from a predictive regression on the 2013 index.

Figure 21: Impulse Response Functions to a one standard deviation change in financial uncertainty index residuals

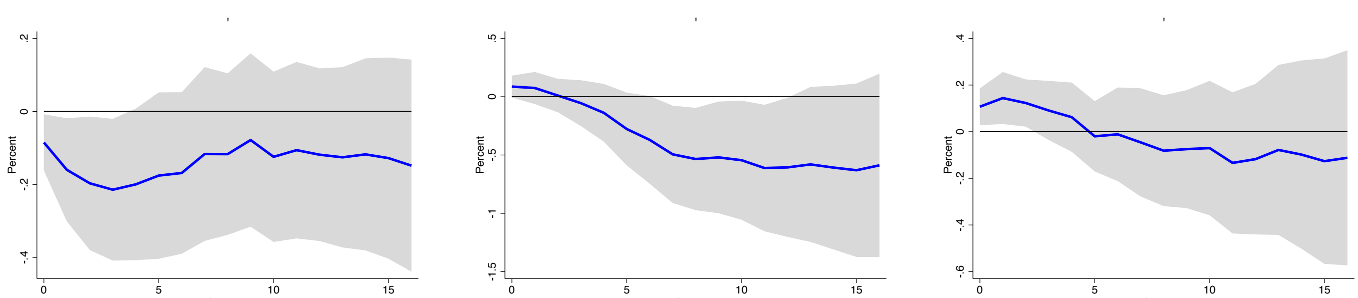
(a) log (GDP) **(b)** log (Private Safe Assets) **(c)** log (Total Safe Assets)



Notes: Shaded areas denote one standard error confidence bands constructed using robust standard errors. IRFs are estimated with Financial Uncertainty Index residuals series as instruments for the safe asset demand shifter λ_t. Financial Uncertainty Index residuals are obtained from a regression of Financial Uncertainty Index on six monthly lags of the Financial Uncertainty Index. Financial Uncertainty Index is obtained from Ludvigson et al. (2021). The monthly residuals are aggregated to quarterly frequency. The impulse is a one standard deviation increase in the Financial Uncertainty Index residual. This estimation includes as controls: current Tbill/GDP ratio, and federal funds rate; and two quarterly lags of the dependent variable. Sample (Quarterly): 1961Q1 – 2019Q2.

Figure 22: Impulse Response Function to a one standard deviation change in risk aversion index residuals

(a) log (GDP) **(b)** log (Private Safe Assets) **(c)** log (Total Safe Assets)



Notes: Shaded areas denote one standard error confidence bands constructed using robust standard errors. IRFs are estimated with time-varying risk-aversion residuals series as instruments for the safe asset demand shifter λ_t. Risk Aversion Index residuals are obtained from a regression of Risk Aversion Index on six monthly lags of the Risk Aversion Index. Risk Aversion Index is obtained from Bekaert et al. (forthcoming) . The monthly residuals are aggregated to quarterly frequency. The impulse is a one standard deviation increase in Risk Aversion Index residual. This estimation includes as controls: current Tbill/GDP ratio, and federal funds rate; and two quarterly lags of the dependent variable. Sample (Quarterly): 1990Q1 – 2019Q2.